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# Bulletin of the American Association of Jesuit Scientists

EASTERN STATES DIVISION

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## NOTICE TO AUTHORS

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## Obituary

REV. GEORGE A. O'DONNELL, S.J.

(Nov. 22, 1899—Jan. 1, 1952)

Father O'Donnell S.J. died on the morning of January 1, 1952 in the fifty-third year of his life. While he was making his annual retreat at the Boston College summer house in Cohasset, Mass., he caught cold and came down with pneumonia. This was early in September 1951. He was taken to St. Elizabeth's Hospital in Brighton where he remained until the time of his death. May his soul rest in eternal peace.

Of late years Father O'Donnell did not enjoy good health. He had a heart condition which forced him to relinquish gradually his teaching schedule, but he attended to the duties of his office as Dean of the Boston College Graduate School until the time he left for his last annual retreat. During the flu epidemic in 1918, he was afflicted with the dreadful illness and never seemed to regain his strength sufficiently. He had pneumonia several times, and these periods of illness taxed his heart to its capacity. During recent years he had to be careful not to exert himself because of the poor condition of his heart.

He was born in South Boston on November 22, 1899. He attended the local elementary school and after his graduation from Boston Latin High School he attended Boston College for one year. At the end of his freshman year, he answered the whisperings of grace in his soul and entered the Jesuit novitiate at St. Andrew-on-the-Hudson in the summer of 1917. Here he made his classical and philosophical studies. During this period he showed outstanding ability in Mathematics, so for his regency he was assigned to teach Freshman and Sophomore Mathematics at Georgetown University in Washington, D. C. In later years, he referred to those years at Georgetown as the happiest two years of his life. Shortly before his death he requested Superiors to allow him to leave the hospital in order to recuperate at his beloved Georgetown. After his regency he went to Weston College to make his theological studies. He was in one of the first classes to be ordained at Weston. The date of his ordination to the Sacred Priesthood was June 22, 1929. From 1931-35 he did doctoral studies at St. Louis University which conferred upon him the Ph.D. degree in Seismology. In his doctoral dissertation he presented a new mathematical treatment of seismic waves, entitled "A New Vector Theory of Wave Motion."

Instead of reporting to the Seismological Observatory at Weston

over which he was nominal head for several years, he was asked by Superiors to head the Graduate School of Boston College. At that time the school was revising its curriculum. He threw all his energies into this important task. The present-day high standards of the school are a credit to his executive ability. Besides his office as Dean, he taught Mathematics in the Graduate School and also was chairman of that department. From 1935 to 1952 he built up a mathematical library that has nearly 5000 volumes which include all the important treatises on every branch of Mathematics, several serial publications, and the complete sets of twenty-four journals, American, English and foreign. The library is a memorial to his vast knowledge of the literature as well as to his deep and intense interest in his fascinating field of Mathematics.

Because of so many duties other than those of teaching, he never had the time for publication. He directed many graduate students in the writing of their M.A. thesis. He held membership in the American Mathematical Society, the London Mathematical Society, the American Association of Jesuit Scientists, the Phi Rho Alpha Physics Research Academy, the Institute of Mathematics Teachers of New England, and the American Association for the Advancement of Science. He was a former president of the American Association of Jesuit Scientists, and was a member of the Entertainment Committee of the International Congress of Mathematicians which was held at Harvard University August 30-September 6, 1950.

As Dean of the Graduate School he belonged to several conferences and associations pertaining to that office. He was a prominent member of The New England Conference on Graduate Education, an association of Deans, and of which he was president for the academic year 1950-51. He faithfully attended the meetings of the Deans of Graduate Schools representing Catholic Colleges and Universities throughout the United States, and of the National Catholic Educational Association, College and University Division.

Father O'Donnell, S.J., never spared himself. He threw himself into his work. He expended himself for others. The large numbers who attended his wake and funeral Mass attest to the good he did for others. He never was too busy to be of assistance to others whether it be a problem in Mathematics, the establishment of a college, or the securing of a position. Nothing was too big or too small for his efforts if it was for a good cause. In the early part of August each year he acted as host to the Knights of Columbus who annually conduct a three-day convention of boy scouts on the college campus. He was easy to approach. He always had a bit of pleasantry to mix with the serious.

During World War II Boston College had a large number of her students in the Naval Reserve as well as in the other branches of the service. It was decided to offer the students courses that would prepare them for their military service. Father O'Donnell, S.J., im-



mediately started a course in Navigation and from 1942-1944 he prepared hundreds of students so well that the grade they received in Navigation at the Midshipmen's school was sufficiently high to excuse them from the final examination. He built up a library in Navigation that would be a credit to any college. This was characteristic of the thorough manner in which he did what he set out to do.

This extra duty exhausted him. He never seemed to quite recover from it. His health continued on the decline until he realized that teaching was too much of a strain upon his heart. He gave up teaching entirely a year ago.

Father O'Donnell, S.J., was a grand priest, a zealous educator, and a person with charity to all. May he rest in peace.

*Anthony J. Eiardi, S.J.*

## Biology

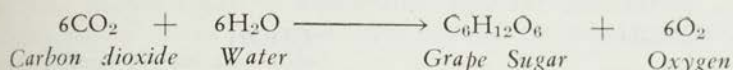
### CARBOHYDRATES AND LIPIDS

WILLIAM D. SULLIVAN, S.J.

Carbohydrates (so called because they are hydrates of carbon) make up 50 to 60% of the American diet. All living substances are known to contain carbohydrates in greater or lesser degrees. Because of their structural complexity, they are difficult to define and so the following definition is not as accurate as it might be. Carbohydrates are the hydrates of carbon in which hydrogen and oxygen are in proportion of 2:1. This is not entirely true, since there are many chemicals, which, on the basis of this definition and their own chemical formulas, should come under the classification of carbohydrates, such as acetic acid ( $\text{CH}_3\text{—COOH}$ ), lactic acid ( $\text{CH}_3\text{—CHOH—CHOH}$ ), and formaldehyde ( $\text{CH}_2\text{O}$ ). In these substances the proportion of hydrogen to oxygen is the same as that of the carbohydrates, yet they are not carbohydrates. There are still other substances which are classified as carbohydrates and which do not have this proportion of 2:1, such as rhamnos ( $\text{C}_6\text{H}_{12}\text{O}_5$ ). This much, at least, is fundamental in the structure of carbohydrates, they are composed of carbon, hydrogen and oxygen and are the very important sources of energy in all animal and vegetable cells. They are the all important constituents of the cell walls in plants, being the supporting skeletons of these walls. It has been shown by some investigators that 70% of the vegetable is carbohydrate.

Ultimately, the energy of all living organisms comes from the

sun. In the presence of sunlight, the chloroplasts combine carbon dioxide and water in such a way that a carbohydrate is manufactured and free oxygen is liberated. The following equation represents the process of *photosynthesis*:



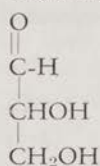
This means that from the combination of 6 molecules of carbon dioxide and 6 molecules of water there result grape sugar and oxygen. Due to the fact that the intermediary reaction is not thoroughly understood, the above equation is not as accurate as it may seem. The end products, however, are correctly represented. There is one theory of the intermediate reaction which says that the carbon dioxide and water are first decomposed by the energy absorbed from the sun in the chloroplasts. The atoms constituting the carbon dioxide and water are then recombined to form formaldehyde. Six molecules of formaldehyde immediately combine to form one molecule of grape sugar. Since formaldehyde is poisonous to the protoplasm of the plant, the combination of six molecules of this substance must be immediate and rapid. The energy, which has been absorbed by the chloroplasts and used to combine carbon dioxide and water so as to form the carbohydrate, is preserved in the carbohydrate as potential energy. This storing of potential energy is, perhaps, the most important phase of photosynthesis, since it is this energy which sustains life in the plant and other living organisms.

The carbohydrates are polyhydroxy aldehydes and polyhydroxy ketones, or chemicals, which, on hydrolysis, will yield either aldehydes or ketones or both. Some of the carbohydrate in animal tissue and plant tissue is tied into the protein molecule since nucleic acid and the nucleoproteins contain a carbohydrate group as the prosthetic group; some more of it is free and not bound in with the protein molecule. Carbohydrates have been found mainly in the plant kingdom, though many of them have been found in the animal kingdom as well.

The general division of carbohydrates classes them as sweet and crystalline compounds called sugars, and as tasteless and noncrystalline compounds called starches, cellulose and products allied to these compounds. They have been classified into aldehyde and ketones, the former being known as aldoses and the latter as ketoses. Finally they have been classified into the monosaccharides, disaccharides, trisaccharides and polysaccharides according to their behavior with hydrolytic agents. A monosaccharide of three carbon atoms, according to this terminology, receives the name aldotriose if the carbohydrate is an aldehyde of three carbon atoms; or it may be a ketotriose if the carbohydrate is a keto alcohol of three carbon atoms such as:

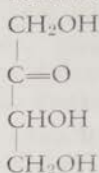
*Aldehyde*

*Aldotriose*



*Ketone*

*Ketotriose*

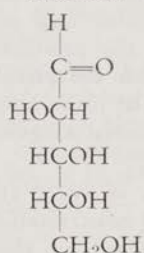


The *monosaccharides* are the simplest of sugars which cannot be hydrolyzed further. Due to the fact that glycoaldehyde ( $\text{CH}_2\text{OH}-\text{CHO}$ ) contains no asymmetric carbon atoms and therefore is not optically active, glyceraldehyde ( $\text{CH}_2\text{OH}-\text{CHOH}-\text{CHO}$ ) is recognized as the simplest of the carbohydrates, or as some of the authors call it, the 'mother substance'.

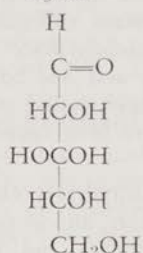
These monosaccharides, therefore, include the *trioses*,  $\text{C}_3\text{H}_6\text{O}_3$ , such as glyceraldehyde, *tetroses*,  $\text{C}_4\text{H}_8\text{O}_4$ , such as erythrose and threose, *pentoses*,  $\text{C}_5\text{H}_{10}\text{O}_5$ , such as ribose, arabinose, xylose and lyxose, *hexoses*,  $\text{C}_6\text{H}_{12}\text{O}_6$ , and other sugars up to the *decoses*. Actually the decoses have been synthesized in the laboratory, but such compounds have not as yet been found in nature. The more common among the monosaccharides are the pentoses and the hexoses, containing 5 and 6 carbon atoms respectively.

Among the more common pentoses are arabinose, the result of hydrolyzing gum arabic, xylose, the result of hydrolyzing straw, and ribose, a constituent of nucleic acid found in yeast. All of these carbohydrates are aldoses and may be represented in the following manner.

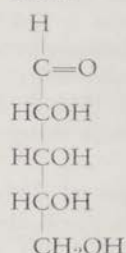
*Arabinose*



*Xylose*



*Ribose*



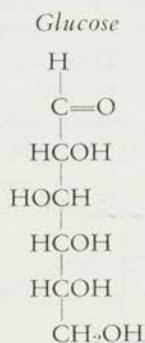
The pentoses and the hexoses differ in that yeast causes the fermentation of the former. They usually appear in their more complex forms called pentosans, which are the anhydrides of the pentose molecules which have lost water molecules. In the same way hexose molecules appear as hexosans. To obtain a pentose from its corresponding pentosan, hydrolysis with acids is used. To estimate the presence of pentose in any tissue, the tissue may be treated with concentrated hydrochloric acid yielding a substance



called furfural. The presence of furfural may then be detected by holding a piece of paper moistened with aniline acetate in the escaping vapor. A red color is produced on the paper.

The more common hexoses are *glucose* and *fructose*. Actually these two hexoses are the only ones which occur free in nature. All the other hexoses appear in combination with the disaccharides and polysaccharides. Some of these other hexoses are talose, galactose, idose, gulose, altrose and allose.

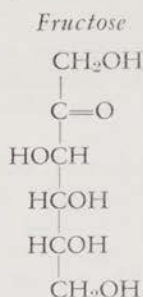
*Glucose* is the most important and most widely used hexose. It is also called dextrose, because it is dextrorotatory. It is found in grapes and hence it is also called grape sugar. It is in the juices of many fruits, in honey and it is in the blood of humans to about 0.1%. In diabetic cases the percentage of glucose in the blood is much higher, and can be easily detected in the urine. Commercially, glucose is prepared from the hydrolysis of starch in the presence of dilute acids and is used as a sweetening agent in syrup, chewing gum and many other food products, such as candy and jellies. Its formula may be written as follows:



There is an aldehyde group at the top of the chain and there then follow four asymmetrical carbon atoms. According to the Le Bel-van't theory there should be sixteen possible stereoisomers of glucose because of these asymmetrical carbon atoms. Though all sixteen are known, only four have been found to exist in nature. Since glucose is an aldehyde, one of the carbon atoms of the compound is combined with hydrogen and oxygen in the aldehyde linkage. For an excellent treatise on the steps which led to the adoption of this formula confer Wertheim (1947).

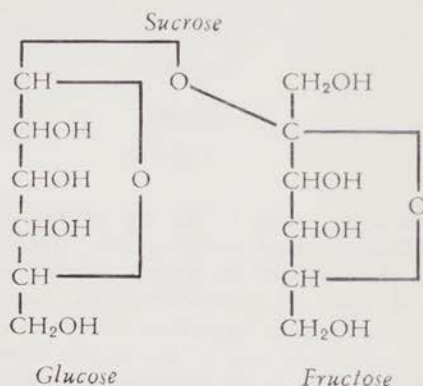
*Fructose*, which is also called levulose or fruit sugar, is a ketohexose. Like glucose it is readily oxidized. The oxidation of fructose and glucose is taken advantage of in the most used tests for sugars, the Fehling's test and the Benedict-Fehling test, wherein cupric hydroxide is reduced to cuprous hydroxide or cuprous oxide. These same tests are used for the quantitative determinations of sugar

content in tissues. It may be obtained by hydrolysis of cane sugar or sucrose and has the following formula:



The *disaccharides* ( $\text{C}_{12}\text{H}_{22}\text{O}_{12}$ ) hydrolyze to yield two molecules of monosaccharides, or they may be said to be a combination of two molecules of monosaccharides with the loss of one molecule of water. These two monosaccharides may be the same or different. Examples of disaccharides are sucrose, maltose and lactose, the former two being present in the vegetable kingdom and the latter in the animal kingdom. They are crystalline solids which react like the simple sugars in solvents.

*Sucrose*, on hydrolysis, yields glucose and fructose, and is represented by the following formula:



Sucrose, along with maltose and lactose, is of tremendous importance physiologically. It is the well-known cane sugar used on the table, and is a constituent of the sap of many trees, especially of the sugar maple. It is also called saccharose and sometimes beet sugar. The structure of sucrose shows it to be composed of two monosaccharides joined at the number one and two carbons. It is the sweetest of all sugars and does not reduce Fehling's solution. Historically, sucrose is known from the earliest times. In 1746 it

was called sugar by Maggrof in Berlin. It has become the basis of the present day sugar industry. The most outstanding property of sucrose is its capacity to caramelize.

When treated with enzymes or with dilute  $H_2SO_4$  and then heated, there is produced molecular units of glucose and fructose of equal quantities. The direction of optical activity is reversed in the chemical reactions of sucrose. If hydrolyzed under polarized light invert products are produced. This means that while sucrose is dextrorotatory, in hydrolytic reactions of sucrose the equal amounts of glucose and fructose produce a sum of negative rotations. For this reason the sugar itself is said to be an invert sugar and its process of hydrolysis is said to be inversion. Hydrolysis, or inversion, may be brought about by invertase, an enzyme, or by acids and alkalies. Another peculiarity of sucrose is that in itself it is not fermentable, but its end products on hydrolysis are fermentable and so one may have fermentation following hydrolysis. Besides invertase, which is found in some yeasts, zymase and sucrose, the latter being found in the small intestine, may also hydrolyze sucrose.

*Maltose*, on hydrolysis, yields two molecules of glucose. It is sometimes called malt sugar, because it is found in malt, a sprouted grain of barley. It is a hydrolytic product of starch produced by an enzyme found in barley called diastase. It may also be hydrolyzed by acid and alkalies or by another enzyme, maltase, which is found in yeasts. Its importance industrially comes from the fact that yeasts so readily attack it.

There is also good evidence for the formula of milk sugar or *lactose*. By juggling the formula a little, the properties of this sugar can be varied. Lactose is not known to come from the vegetable kingdom. In mammalian milk there is six to seven percent lactose, while in cow's milk there is about four percent. It can be obtained by allowing milk to evaporate and then separating the fat from casein which is precipitated by means of rennet. On hydrolysis glucose and galactose are produced. Its commercial source is whey, and can be purified by repeated crystallizations. It can easily be synthesized, having been first synthesized by Hudson in 1942. Its chemical reactions resemble those of the hexoses. It acts like a reducing sugar and is very slowly attacked by yeast.

The *trisaccharides* ( $C_{18}H_{32}O_{16}$ ) yield, on hydrolysis, three molecules of monosaccharides, or they may be defined as the result of a union of three molecules of monosaccharides with the loss of two molecules of water. An example of this class of carbohydrates is *raffinose*, which is found in the cotton seed.

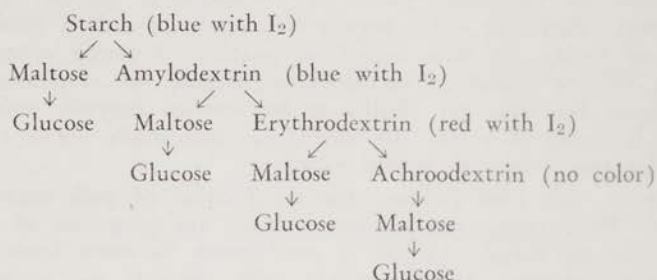
The monosaccharides, the disaccharides and the trisaccharides belong to the class of sugars because of their sweet tastes. They are

soluble in water and alcohol; they crystallize easily and pass through dialyzing membranes.

The *polysaccharides*  $((C_6H_{10}O_5)_x)$  yield on hydrolysis more than three molecules of monosaccharides, or they are the result of a condensation of many molecules of monosaccharides with a corresponding loss of water molecules. Included in this group are the gums, dextrans, pectins, starch, glycogen and cellulose.

The polyoses form colloidal solutions with water; they do not crystallize; and they do not pass through dialyzing membranes. They do not reduce Fehling's solution, though they may be hydrolyzed into the simple sugars which do reduce Fehling's solution.

The best known polysaccharide is *starch*. It is found mainly in fruits, in grains and in tubers where it forms a reserve food supply. Though insoluble in water, if boiled the cell wall bursts and a portion of starch is then dissolved. If the starch is boiled long enough, it is broken down into substances which are simpler in structure and called *dextrins*. If hydrolyzed with diastase, the result is maltose. Some authors say that a progressive hydrolysis of starch produces maltose at each step of its breakdown. Wertheim pictures the progressive steps as follows:



Starch is a product of photosynthesis which appears as two substances. They may or may not be two distinct chemical substances. The cortical area of the starch grain is called *amylopectin*, or alpha amylose, and it has a branched polymer for its molecular structure. The central area is called *amylose*, or beta amylose, which is a linear polymer. Amylopectin is insoluble in water. In hot water it swells and shows no coloration with iodine. Amylose is soluble in water and shows a blue coloration with iodine. The blue color is not a chemical reaction but rather a physical phenomenon of adsorption. By continued heating of the amylose this blue color will disappear. In this linear polymer there are about 200 to 500 glucopyranose forming the chain.

The quantity of starch in a given tissue may be determined by the colorimetric method or it may be hydrolyzed completely with  $H_2SO_4$  and then the amount of glucose and fructose which is produced may be estimated.



*Glycogen*, or as it is sometimes called, animal starch, is a polymer composed of many rings of glucose and is the important source of energy in the animal body. It is found chiefly in the liver (where it is formed and contributes 3% of its total weight) and in the muscles where it is reserved as food supply. In an emergency it can quickly be reverted to glucose. On hydrolysis of glycogen, glucose is produced. It does not reduce Fehling's solution. It can and has been synthesized from proteins and the amino acids.

Another polysaccharide is *inulin* which can be hydrolyzed to fructose. It is found in many plants. Its structure is similar to starch in as much as it is composed of a linear polymer. It does not reduce Fehling's solution, and on reacting with iodine a yellow color is produced. It is said to be found in the Jerusalem artichoke, dahlia, etc. Though it is similar to starch in structure, it differs in that it does not become soluble in hot water.

*Cellulose* is the chief constituent of the walls of cells in plants. Cotton fiber is said to be almost pure cellulose. On hydrolysis cellulose produces glucose. It is said to be composed of cellobiose ( $C_{12}H_{22}O_{11}$ ), a disaccharide isomer of maltose. Another substance, lignocellulose, in all probability, is a combination of cellulose with resins and gums. Lignins, whose chemical structure is not known, cements the fibers of cellulose together forming wood, the supporting skeleton of plants. *Pectin*, in combination with cellulose, forms pectocellulose, and it is that substance responsible for the formation of jellies from fruits.

The industrial uses of cellulose are almost innumerable today. Some of the more common uses are the following. The cellulose found in cotton forms esters and gels when they are treated with nitrate and from this is made artificial silk, films, lacquers, plastics and explosives; when treated with acetate, artificial silk, films and lacquers are formed; when treated with viscose, artificial silk and films are formed. The cellulose found in wood, as decomposition products when distilled, produce methanol, formaldehyde, acetic acid, acetone, acetone oils, charcoal, tar, pitch and creosote; when hydrolyzed, alcohol, cattle food, acetic acid and furfural are formed; when treated chemically, sulfite pulps are produced from which writing paper, tissue paper and wrapping paper are produced.

The *gums*, *pectins*, and *mucilages* are also polysaccharides which yield pentose and hexose sugars when hydrolyzed. From the gums are produced xylose, arabinose, and galactose; from the mucilages there are produced molecules of galactose. Gum arabic is used in medicinal and pharmaceutical work for the making of tablets and pills. They are probably carbohydrates combined with acids, and are used as adhesives as well as to suspend insoluble substances in liquids. The mucilages are used for constipation treatments because they are insoluble, tasteless, non-digestible, odorless and swell when placed in water. Fruits, deficient in pectin, do not have the power to form jellies. Commercial pectin has been added to such

fruits and jellies have resulted. Commercially, pectin is derived almost exclusively from apples and lemons.

Intracellular carbohydrate inclusions may be seen in almost every cell in many different forms and shapes. Sometimes it can be seen as fine granules, sometimes as irregular flakes, sometimes it is localized in certain specific parts of the cytoplasm, while at other times it may be seen to be diffused throughout the cytoplasm.

The role glucose plays in the formation of carbohydrates is a very important one. Almost all the carbohydrates, higher than the monosaccharides, yield glucose on hydrolysis. Sucrose yields glucose and fructose. Lactose yields glucose and galactose. Starch yields dextrans, which, in turn, yield maltose, and finally hydrolyze to glucose. Cellulose yields cellobiose and finally glucose.

### LIPIDS

The lipids are esters of fatty acids plus a trihydric alcohol, glycerol. They are a general non-polar, hydrophobic group of substances found in the living system, relatively insoluble in water and soluble in organic solvents, such as chloroform, benzene and petroleum due to the long aliphatic hydrocarbon chains or benzene rings. The aliphatic chains in some of the lipids may be attached to a polar group at one end and hence may become hydrophilic.

According to Bleuret (1943) they are a group of naturally occurring substances consisting of fatty acids (true fats), substances chemically related to the true fats and substances related to the true fats by reason of their common solubilities.

The lipids are classified into the simple, the compound, the steroids, carotenoids (according to some of the authors) and allied lipoidal substances. The *simple* lipids are esters of fatty acids plus alcohol. Among these are the triglycerides which are the triesters of fatty acids and glycerol. The triglycerides have been divided into the fats and oils, although there is no essential difference between these two. The fats are solid at ordinary temperatures, i.e. 20° C, while the oils are liquid at these temperatures. Some examples of these two are human fat and lard, and olive oil, castor oil and the fish oils. The waxes are also simple lipids, and are esters of fatty acid plus alcohol other than glycerol. Beeswax is an example of this type of lipid.

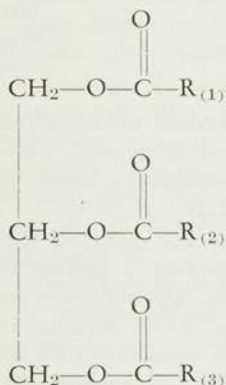
The *compound* lipids are those which, on hydrolysis, yield not only the acids and alcohol but other substances as well. Among these are the phospholipids, containing phosphoric acid and nitrogen (lecithin, cephalin and spingomyelin), the glycolipids, containing glycogen, and the sulfolipids, containing sulfuric acid, and the cerebrosides, containing nitrogenous carbohydrates, such as phenosin, kerosin and nervone. These latter are the principle constituent of myelin in the nerve fiber.

The *steroids* are a complex aliphatic ring system of molecules. Sex

and adrenal glands (the adrenal cortical hormones), bile acids and vitamin D belong to this class. They are hydrogenated phenanthrene derivatives of which cholesterol and ergosterol are the most common. Cholesterol is the fundamental constituent of wool fat or lanolin and is often found linked to one of the fatty acids in an ester linkage.

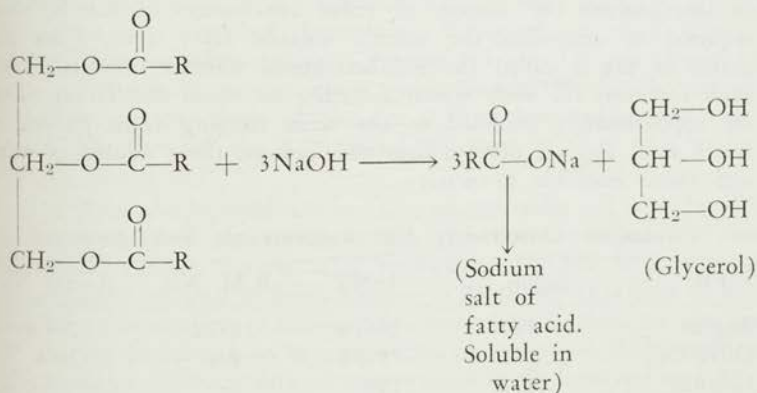
The *carotenoids* are the colored (red or orange) pigments found in the cell. They are insoluble in water and soluble in organic solvents. Examples of these lipids are the carotenes which are found in carrots and grass, vitamin A and xanthophyll found in leaves.

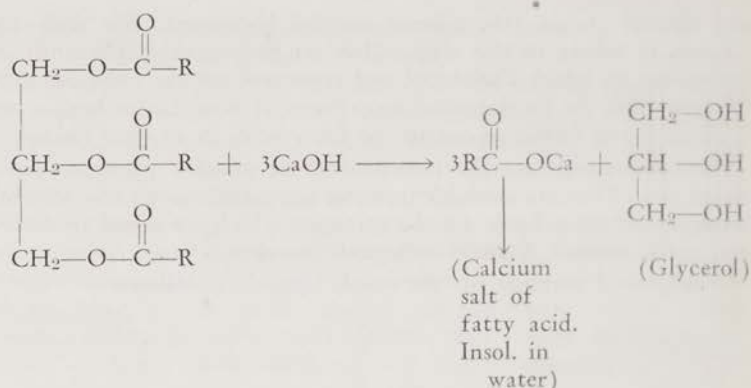
The general formula for the simple lipids is as follows:



In the formula above R represents the fatty acid. When R is the same throughout, the lipid is a simple glyceride; if R is different, it is a mixed glyceride.

The hydrolysis of a simple lipid may be represented by the following equation:





As a result of the breakdown of these fatty acids, glycerol may be seen to be a trihydroxyl compound. This compound joined to three fatty acid molecules produces a simple lipid or a triester, sometimes known as a triglyceride.

Aside from the many physical methods, the usual analysis of a fat depends on the determination of a certain chemical constant. The first analysis depends on the *saponification number*. This represents the number of milligrams of KOH required to saponify one gram of fat. Roughly speaking, this number varies inversely with the molecular weight of the fat. In this analysis the fats and oils are hydrolyzed with the alkali; the fatty acids separate from the glycerol and form a metallic salt. The process is called saponification and the resulting product is a soap. Saponification takes place in the small intestine due to the presence of an enzyme, lipase, which is manufactured in the pancreas.

The second fat analysis depends on the *Reichert-Meissl Number*. In this analysis the number of cubic centimeters of 0.1 N alkali required to neutralize the soluble volatile fatty acids from five grams of fat is called the Reichert-Meissl number. Volatile fatty acids represent the acids which volatilize on steam distillation. They are approximately confined to the series ranging from butyric to lauric acid, and are divided into two groups, those soluble in water and those insoluble in water.

CHEMICAL CONSTANTS DETERMINED BY FAT ANALYSIS

Fat	Sapon. No.	I No.	R.M. No.	Acetyl No.
Butter	220	25-50	26-32	2-8.6
Olive Oil	185-196	79-88	0.6	10.6
Calanus	132	185	—	—
Blue Whale	190	135	—	—



Another method of fat analysis is called the *acetyl number*. It consists in the number of milligrams of KOH required to neutralize the acetic acid resulting from the hydrolysis of one gram of the acetylated fat. A fatty acid containing an hydroxyl (and this reaction requires the presence of hydroxyl groups) will react with an acetic anhydride to form the acetylated compound.

The *iodine number* is a fourth way to determine fat analysis and it consists in the number of grams of iodine absorbed by 100 grams of fat. This number depends on the extent of unsaturation in the molecule of fat. To accelerate the absorption process, halogenating agents, such as iodine monobromide or iodine monochloride, are added. These agents produce diiodo compounds which allow the process to take place more rapidly.

Unsaturated fatty acids can be saturated by the addition of hydrogen and this process is called hydrogenation. This process can be accelerated by using a catalyst such as nickel. By this means, not only unsaturated fatty acids, but also unsaturated fats can be made to add hydrogen. By heating oil to a very high temperature in the presence of nickel, it is possible to reduce and hydrogenate the double bond of the acid and so, in cooling, the oil becomes a solid. If hydrogenation is allowed to go to completion a solid and brittle product would result.

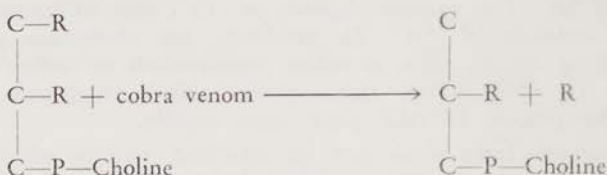
*Rancidity* is a property peculiar to fats and it is easily detected by the unpleasant odor and taste of fat which has been allowed to stand. It is due primarily to chemical changes of the unsaturated fatty acids present. The chemical change involves the oxygen of the air, which attacks the double bond, and the fat is then catalyzed by moisture, heat and light. Peroxides are probably first formed and these are then broken down to aldehydes. Ketones are also formed from the action of microorganisms.

Fats in the human body are hydrolyzed into fatty acids and glycerol. These are then absorbed through the lining of the small intestine. After absorption they are again synthesized into fat and slowly pass into the lymphatic system, from which system they finally pass into the blood stream. Some of this fat is then deposited in the adipose tissue and wherever else it is needed and some more of it is oxidized to  $\text{CO}_2$  in the cells.

The three *phospholipids* are lecithin, spingomyelin and cephalin. The former two have choline as a base and the latter has cholamine as a base. Choline has the same strength as NaOH. It requires the same amount of acid to bring choline to neutrality as it does to bring NaOH to neutrality. Choline has a stimulating effect on the fats. If it is not present in an organism then the organism finds itself unable to make use of the fats. It accumulates in the liver and its activity is still a mystery and can be known only through its effect on the diet of the individual cell. Choline is derived from acetylcholine, from the

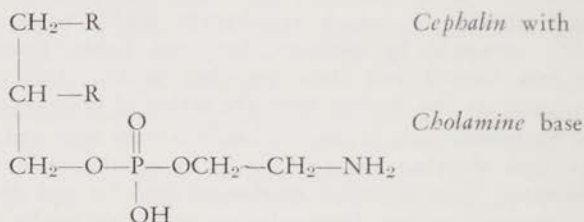
parasynthetic nerve endings. Acetylcholine is the causative factor in nerve transitions.

*Lecithin* is a combination of fatty acid, glycerol, phosphoric acid and choline and plays a very important role in fat metabolism. An increase in fat metabolism indicates an increase in phospholipid content. Lecithin is derived from eggs and has been found in the brain tissue as well. All the phospholipids are insoluble in acetone. Cobra venom will attack lecithin in the following manner:



Cobra venom, on attacking lecithin, knocks one of the fatty acids (R) from the chain and the products which result are lyslecithin and fatty acid. Lyslecithin attacks the red blood cells causing haemolysis. In other words, the individual overcome by cobra venom suffocates to death, because the red blood cells will not pick up oxygen, no matter how much oxygen may be present.

*Cephalin*, similar to lecithin in that it contains the fatty acids, glycerol and phosphoric acid, differs from lecithin in as much as it contains as a base cholamine, or aminoethyl alcohol, whose formula is as follows:



R represents the fatty acids, either saturated or unsaturated. The N compound is the difference between cephalin and lecithin. Cephalin is connected with blood clotting, and unlike lecithin it is insoluble in alcohol.

*Sphingomyelin* is a complex compound, composed of phosphoric acid, choline and sphingosine, another nitrogenous compound, and lignoceric acid. It differs from both lecithin and cephalin in that it is insoluble in ether. It is also insoluble in acetone and cold alcohol.

The *sterols* are monohydroxy alcohols found in all living cells and may occur in two forms, either as free sterols or as combinations of higher fatty acids, such as cholesterol esters. Chemically the sterols are hydrogenated phenanthrene derivatives such as are found among

the bile acids and sex hormones. Vitamin D is related to the sterols through ergosterol,  $C_{27}H_{41}OH$ . Cholesterol is by far the best known sterol.

*Cholesterol* is found in all animal and plant cells except in the one cell animal *Colpidium campylum* (Sullivan, 1948). It is especially present in the nervous tissue, and it is from the nervous tissue and animal brain tissue that it is prepared. It does not saponify. There are two tests which are widely used for the determination of the presence of cholesterol. One of these is the Leibermann-Burshard test; acetic anhydride with concentrated sulfuric acid yields a green colored solution. In the Salkowski test, chloroform with concentrated sulfuric acid yields a bluish-red to purple colored solution. The intensity of the colors will vary with the amount of cholesterol present. Both tests, therefore, are quantitative as well as qualitative. In reacting with digitonum, cholesterol will precipitate yielding cholesterol digitonin. By weighing out the cholesterol digitonin, a direct quantitative test can be made. Cholesterol is synthesized by most animals (Wilber and Seaman, 1948). In plants the sterols are called phytosterols, and such sterols are not used by the animal body. The precise function of the sterols in the vegetable kingdom is unknown, though it is probably a water binding agent.

In general the function of the lipid depends greatly on its location and its disposition in the organism. For example the simple lipids and the glycerides serve as stores of energy or they may serve as protection against cold or other harmful agents as is certainly the case with regard to the lipids and fats in the whale. The bile acids are protein denaturants and serve as aids to digestion. In particular, cholesterol, because of its close association with vitamin D, is a stimulating agent for epidermis and helps in the growth of hair. The sex hormones regulate the metabolic processes as well as the reproductive processes. Myelin of the nervous tissue and brain tissue has as a chief constituent the phospholipids, and lecithin functions in the methylation process going on within the liver.

In recent years protein, carbohydrate and fat inclusions have been found in various types of cells. Adipose tissue has been the specific object of many investigations since this tissue is the store house of reserved fat for many animals and in this tissue a tremendous amount of fat can be accumulated. In the adrenal cortex cells various mixtures of sterols, phospholipids and fatty acids have been discovered.

In the adsorption of fats by the individual cells, the fatty acid and glycerol, which must be absorbed separately through the cell membrane, undergo saponification previous to absorption. Lipase, an enzyme within the cell, reunites the fatty acid and glycerol after absorption.

The carotenoids are a pigmentary inclusion of some cells. These pigmentary inclusions may be of two types, endogenous pigments (products of cellular metabolism) and exogenous pigments (ac-



accumulated by the cells from the outside). The carotenoids are exogenous.

*Carotene* is a carotenoid found in the hepatic cells and is eventually transformed into vitamin A. *Lutein* is the pigment giving color to the yolk of eggs. *Xanthophyll* is the pigment found in leaves and giving to the leaves their peculiar pigmentation. Because these pigments readily go into solution with fats, they spread their color throughout the fatty substance, giving the entire fatty compound a colored effect.

The *lipochromes* have been called by some authors carotenoids. However other authors more accurately classify them as endogenous pigmentary inclusions, distinguishing them from the exogenous pigments of which the carotenoids are the most important. About all that is known of the lipochromes is that they are lipid derivatives. They manifest lipid characteristics and can be seen as yellow or light brown granules in the cell.

*Lipophanerosis*, or as it is sometimes called 'fatty degeneration', is the upsetting of the equilibrium between fats and proteins by a particular toxic condition in the cell. Within the cell, fat is discovered to be in the form of an emulsion with protein. In some cases the fats are the dispersed phase and the proteins are the dispersion phase. In other instances the proteins are the dispersed phase and fats are the dispersion phase. Whenever this apparent colloidal condition of fats and proteins within the cell is altered so as to destroy the emulsoid condition, fatty degeneration or lipophanerosis takes place.

#### BIBLIOGRAPHY

- Lowy, A., Harrow, B., Apfelbaum, P. *An Introduction to Organic Chemistry*. John Wiley & Sons, Inc. New York (1947).
- Sullivan, W. The Effect of Cholesterol On Growth In *Colpidium campylum*. *Trans. of the Amer. Micros. Society* **67**, 262 (1948).
- Wertheim, E. *Textbook of Organic Chemistry*. The Blakiston Company. Philadelphia and London (1947).
- Wilber, C., and Seaman, G. The Lipids in *Colpidium campylum*. *Biological Bulletin*. **94**, 29 (1948).

## EVALUATION OF EVIDENCE FOR SEXUALITY IN BACTERIA

ALWYN HARRY, S.J.

The occurrence of fusion and sexuality in bacteria has been claimed often, and as often disputed. It is a much debated question, and the conception that bacteria have no sexual mode of reproduction is widely entertained. Morphological evidence of the existence of sexes as inferred from the apparent fusion of cells, the formation of protusions and of bodies resembling zygospores, is



very unreliable because of various optical limitations of the microscope, even the electron microscope. However, there are numerous claims that bacteria reproduce sexually like higher organisms. According to Dienes (1946), the question of whether bacteria multiply exclusively by binary fission, or whether, in addition, they have more complex reproductive processes, has more than academic interest. Binary fission makes possible the segregation of hereditary combinations and rearrangements of them. It is difficult, Dienes concludes, to believe that in bacteria no mechanism exists by which new combinations of hereditary characteristics may be produced which are advantageous in a given situation.

The study of inheritance in bacteria was for the most part confined to the investigation of mutational changes in the course of asexual reproduction. With the exception of experiments on pneumococcus type transformations there have been few studies on the direct hereditary interaction of one bacterial type with another. Griffiths (1928), and Avery, McLeod and McCarthy (1944) worked on the transformation of pneumococcal types. Their studies revealed that, under special experimental conditions, a product isolated from a serologically specific, smooth, pneumococcus culture will convert cells of a non-specific rough culture to the smooth type characteristic of the source of the transforming principle.

Genetical studies have produced some evidence for a sexual process in bacteria. Early workers in this field were Lohnis (1921), Almquist (1924), and Mellon (1925), who studied the phenomenon of paragglutination in the colontyphoid dysentery group. (Paragglutination refers to the development of new types which react with antisera for each of two distinct strains when these are grown together in a mixed culture.) Luria (1947), however, maintains that the material presented by these investigators in support of the hypothesis of sexuality in bacteria cannot be used as genetic evidence because of the lack of information on the exchange of recombination of discrete hereditary characters in the course of the alleged sexual fusion.

Sherman and Wing (1937) described experiments designed to detect recombinations of fermentative characters in mixed cultures of various *Escherichia coli* and *Aerobacter aerogenes* strains. Although various combinations of biochemical characters were found, similar types were found in an equal extent in pure cultures so that these authors could not infer the occurrence of sexual fusion.

Gowen and Lincoln (1942) performed similar experiments with strains of *Phytomonas stewartii*, using cultures differing in morphological and pigment characteristics. As in Sherman and Wing's studies, these authors were unable to differentiate the new types they found in their mixed cultures from types which arose spontaneously in single cultures.

Klienberger-Nobel (1946) described automictic, sexual processes in bacteria preceding the formation of the endospore. The micro-

organisms *Clostridium welchii* and *C. septicum* were used in these investigations. Changes in nuclear structure in these bacteria were studied by means of the hydrochloric acid-Giemsa method which produces brilliantly stained specimens. The investigators observed and took photomicrographs of the nuclear changes in the organisms. She concluded that the two main features in spore formation of bacteria appear to be (1) a fusion of the dumbbell bodies into an axial chromatin rod (autogamy), and (2) a reduction partition which is reminiscent of, though not corresponding to, the more complicated phenomenon of meiosis in the higher organisms. Knaysi (1950) does not consider Kleinberger-Nobel's work to be conclusive.

The findings of Lederberg and Tatum (1947) present the strongest supporting evidence for the occurrence in a bacterium of a process of gene recombination, from which the existence of a sexual stage may be inferred. According to Luria (1947), these studies, though still in the preliminary stage, appear to be among the most fundamental advances in the whole history of bacteriological science.

Lederberg and Tatum used, for their experiments, mutants of a single strain of *Escherichia coli*. These strains, characterized by specific growth factor requirements, were obtained after treatment with x-rays, ultraviolet rays and mustard gas. By successive treatments, multiple mutant strains with several genetically and biochemically independent nutritional requirements were produced. When samples of mutant strains were plated into minimal medium with samples of other mutant strains, colonies developed, which possessed the characteristics of the original *E. coli* strain from which all the mutants had been derived. These results strongly suggest that there had occurred fusion followed by exchange of genetic determinants (genes), similar to crossing-over, followed by separation of the fused cells. These all represent a true form of sexuality in a very simple bacterium. It must be emphasized, however, that no other bacterial strain has yet been found to show behavior similar to the *E. coli* strain used by Lederberg and Tatum.

These findings of Lederberg and Tatum have been confirmed by more recent investigators including Haas, Wyss and Stone (1948), and Cavalli and Heslot (1949), using the K12 strain of *E. coli* also.

The work of Dienes (1947) also offers evidence in favor of sexuality in bacteria. This investigator observed the development of viable large bodies in the zone of contact of 2 strains of *Proteus*. The reproduction of these large bodies at the contact of the 2 strains presents at least a superficial similarity to the conjugation and production of sexual spores in the zone of contact of fungus cultures.

#### CONCLUSIONS

1. The occurrence of fusion and sexuality in bacteria is a disputed one.

2. Morphological evidence for the existence of sexuality in bacteria is often unreliable.
3. The most powerful evidence of a sexual process in bacteria has been obtained from genetical studies.
4. The essence of sexuality is the exchange of hereditary properties between different strains—there are observations which indicate that such a process occurs in bacteria by fusion of cells.

#### BIBLIOGRAPHY

- Almquist, E. *J. Infectious Diseases*, **35**, 341 (1924).  
 Avery, O. T., McLeod, C. M., and McCarthy, M. *J. Exptl. Med.* **79**, 137 (1944).  
 Cavalli & Heslott. *Nature*, **164**, 1057 (1949).  
 Dienes, L. *Cold Spring Harbor Symp. on Quant. Biol.* **21**, 1 (1946).  
 Gowen, J. W., and Lincoln, R. E. *J. Bact.* **44**, 551 (1942).  
 Griffiths, F. *J. Hyg.* **27**, 113 (1928).  
 Haas, Wyss, & Stone. *Nat. Acad. Science Proceedings*, **34**, 229 (1948).  
 Klienberger-Nobel. *J. Hyg.* **44**, 99 (1946).  
 Knaysi, Georges, 1951. *Elements of Bacterial Cytology*. W. B. Saunders Co., Phila.  
 Lohnis, F. *Memoirs Natl. Acad. Sc., U. S.* **16**, 1 (1921).  
 Luria, S. E. *Bact. Rev.* **11**, 1 (1947).  
 Mellon, R. R. *J. Bact.* **10**, 481 (1925).  
 Sherman, J. M., and Wing, H. U. *J. Bact.* **33** (3), 315 (1937).

## Mathematics

### NECESSARY AND SUFFICIENT CONDITIONS

JOHN GREEN, S.J.

In theorems for the convergence of series there occur three groups of conditions for convergence. The first group of theorems contain Necessary and Sufficient Conditions; the second group contain Necessary Conditions and the third group contain Sufficient Conditions for the convergence of series.

The statement that a theorem contains a Necessary and Sufficient Condition means that if the conclusion of the theorem exists then the condition is satisfied and conversely if the condition is satisfied then the conclusion exists. A Necessary and Sufficient Condition for a theorem means that the theorem is true and also the converse of the theorem is true.

A Necessary and Sufficient Condition for a limit would be:

- 1) If the limit exists, then the condition is satisfied.



2) If the condition is satisfied, then the limit exists.

Examples of Theorems with Necessary and Sufficient Conditions are:

- 1) The Necessary and Sufficient Condition for the existence of a limiting value of a sequence of numbers  $x_1, x_2, x_3, \dots$  is that corresponding to any given positive number  $\epsilon$ , however small, it shall be possible to find a number  $n$  such that  $|x_{n+p} - x_n| < \epsilon$  for all positive integral values of  $p$ .
- 2) In order that the infinite product  $(1 + a_1)(1 + a_2)\dots$  may be absolutely convergent, it is necessary and sufficient that the series  $a_1 + a_2 + a_3 + \dots$  should be absolutely convergent.
- 3) A Necessary and Sufficient Condition for the convergence of a bounded sequence is that the upper limit  $L$  be equal to the lower limit  $l$ .

Proof: If  $L = l = A$  then for any  $\epsilon > 0$  there is at most a finite number of terms of  $\{x_n\}$  such that  $x_n > L + \epsilon \equiv A + \epsilon$  and also at most a finite number of  $x_n$  for which  $x_n < l - \epsilon \equiv A - \epsilon$ .

Therefore  $A - \epsilon < x_n < A + \epsilon$  if  $n \geq p$  or  $|x_n - A| < \epsilon$  if  $n \geq p$  which is the statement that  $\lim_{n \rightarrow \infty} x_n = A$ .

Conversely, if  $\lim_{n \rightarrow \infty} x_n = A$  then for any  $\epsilon > 0$  and for  $n \geq P$ ,  $|x_n - A| < \epsilon$  or  $A - \epsilon < x_n < A + \epsilon$ . Therefore  $x_n < A + \epsilon$  for an infinite number of  $x_n$  and the inequality  $x_n < A - \epsilon$  is true for at most a finite number of the  $x_n$ . Therefore  $l = A$ .

But  $|x_n - A| < \epsilon$  also states that  $x_n > A - \epsilon$  is true for an infinite number of  $x_n$  and that there are at most a finite number of  $x_n$  for which  $x_n > A + \epsilon$ . Consequently  $L = A$ . Since  $l = A$  and  $L = A$  then  $l = L$ .

Theorems which contain Necessary and Sufficient Conditions for convergence are of so general a nature that in particular series the convergence can only rarely be determined by their application. Therefore more practical tests for convergence are found in the theorems containing either a Necessary or a Sufficient Condition for convergence.

A Necessary Condition for convergence means that if this series is convergent then the condition is true. For example, if the condition  $na_n \rightarrow 0$  is given for the convergence of a series, it is only a Necessary, not a Sufficient Condition for convergence. For if  $na_n$  does not tend to zero, then the series in question is certainly divergent, while  $na_n \rightarrow 0$  does not necessarily imply anything as to the possible convergence of the series.



Examples of Theorems with a Necessary Condition:

- 1) A Necessary Condition for the convergence of a series  $\sum_{n=1}^{\infty} u_n$  is  $\lim_{n \rightarrow \infty} u_n = 0$ . This condition is not sufficient for convergence, as is shown by the fact that in the divergent harmonic series

$$1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \quad \lim_{n \rightarrow \infty} \frac{1}{n} = 0.$$

However the hypergeometric series  $1 + \frac{1}{2^p} + \frac{1}{3^p} + \dots + \frac{1}{n^p}$  converges for all values of  $p > 1$  and diverges for all values of  $p \leq 1$ .

- 2) Raabe's convergence test fails if the numbers  $a_n$  in the expression  $\frac{a_{n+1}}{a_n} = 1 - \frac{a_n}{n}$  though constantly  $> 1$  have the value 1 for a lower limit. In that case, writing  $a_n = 1 + \beta_n$  the condition  $\lim_{n \rightarrow \infty} n\beta_n = \infty$  is a necessary condition for the convergence of  $\sum_{n=1}^{\infty} a_n$ . In fact if  $n\beta_n$  were bounded, then

$$\frac{a_{n+1}}{a_n} \text{ would equal } 1 - \frac{1}{n} - \frac{\theta_n}{n^2} \text{ and } \sum_{n=1}^{\infty} a_n \text{ would be}$$

divergent by Gauss's test.

A Sufficient Condition for convergence means that if this condition is satisfied then the series is convergent. For example, for a given condition a series has absolute convergence but this condition is not necessary for absolute convergence since absolutely convergent series can be found which do not satisfy this condition.

Examples of Theorems with a Sufficient Condition:

- 1) For a series  $\sum_{n=1}^{\infty} \mu_n$  to converge, it is sufficient that the series of moduli  $\sum_{n=1}^{\infty} |\mu_n|$  should converge. This condition

is not necessary for writing  $f_n = \frac{1}{n}$  in  $f_1 - f_2 + f_3 - f_4 + \dots$  a convergent series, it is seen that  $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$  converges, although the series of moduli  $1 + \frac{1}{2}$

$+\frac{1}{3} + \frac{1}{4} + \dots$  is known to be a divergent series.

- 2) The series  $\sum_{n=1}^{\infty} a_n$  is absolutely convergent if from a certain term onward a relation of the form  $|a_n| < c q^n$  (1) holds where  $c$  is a positive number less than 1. This test is usually expressed in one of the following forms: the series  $\sum_{n=1}^{\infty} a_n$  converges absolutely if from a certain term on-

ward, a relation of the form  $|\frac{a_{n+1}}{a_n}| < q$  (2a) holds,

where  $q$  is again a positive number less than 1 and independent of  $n$ ; or if from a certain term onward a relation of the

form  $\sqrt[n]{|a_n|} < q$  (2b) holds, where  $q$  is a positive number less than 1. In particular the conditions of these

tests are satisfied if a relation of the form  $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}$

$= k < 1$  (3a) or  $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = k < 1$  (3b) is true.

The four tests derived from the original criterion  $|a_n| < c q^n$  are not equivalent to one another or to the original, that is, they cannot be derived from one another in both directions. More exactly if (3a) is fulfilled, then (2a) is fulfilled; if (3b) then (2b); if (3a) then (3b); if (2a) then (2b); and if any of the four is satisfied, then so is (1). None of these statements can be reversed. These tests furnish sufficient conditions for the absolute convergence of a series; that is, when they are satisfied then the series converges absolutely. They are definitely not necessary conditions however because absolutely convergent series can be found which do not satisfy these conditions. In the previous example for the Necessary and Sufficient Conditions for a limit, the Necessary Condition would be: if the limit exists then the condition is satisfied; the Sufficient Condition would be: if the condition is satisfied then the limit exists.

In theorems containing either a Necessary or a Sufficient Condition the converse of the theorem is not always true, as it is in the case of theorems containing Necessary and Sufficient Conditions.

## RELATIONS BETWEEN TRIGONOMETRIC AND EXPONENTIAL FUNCTIONS

ROBERT O. BRENNAN, S.J.

The complex exponential function is more convenient than the trigonometric functions as a means of representing harmonic motion and wave phenomena and is much more common in the literature. The physics teacher would like to make his students familiar with the properties of  $e^{ix}$  as soon as possible. The principal task is to prove the relation

$$e^{ix} = \cos x + i \sin x. \quad (1)$$

For this proof one needs, as a minimum, a knowledge of differentiation (or at least some knowledge of certain limits) and in addition one other bit of knowledge or skill. In terms of three different starting points, we shall present here three approaches, recommending the third but offering the other two as possible exercises or auxiliary discussions.

I. The extra bit of knowledge required for the first approach concerns the existence and convergence of power series. Usually the student will not meet power series until he is finishing the integral calculus. Hence sometimes it might be necessary for him to make an act of faith *pro tem*. Once he has met McLaurin's series it is only necessary to compare the series developments of  $e^{ix}$ ,  $\sin x$  and  $\cos x$  to verify Eq. 1.

II. The second approach requires a little more maturity and might be useful as a way of helping the development of maturity. It has the advantage that it can be motivated in terms of de Moivre's theorem. The additional knowledge required here is that it is possible to replace one definition of a mathematical function by another definition in terms of a suitable set of properties. To illustrate this notion, we consider first the trigonometric functions.

The student usually thinks of the sine and cosine as certain geometric relations. Actually, however, there are only a few theorems concerning angles (not triangles) in trigonometry which actually require geometric proofs. If we start with these theorems and, instead of proving them, make them serve as definitions, all the remaining properties of angles follow exactly as they did before. These theorems which are necessary and sufficient propositions to characterize the sine and cosine are, first, the identity

$$\cos^2 x + \sin^2 x = 1; \quad (2)$$

secondly, the addition formulas

$$\cos(x + y) = \cos x \cos y - \sin x \sin y \quad (3)$$

$$\sin(x + y) = \cos x \sin y + \sin x \cos y; \quad (4)$$

and, thirdly, in order to discuss the derivative, either the limit

$$\lim_{x \rightarrow 0} \sin x / x = 1$$

or, more conveniently, the derivative itself

$$d/dx(\cos x) = -\sin x. \quad (5)$$

Once these properties (Eq. 2-5) are known, all the other properties of  $\sin x$  and  $\cos x$  may be derived from them. Hence they may be taken as a definition of the sine and cosine.

The reader may find it interesting to show from Eq. 2-5 that the functions

$$\frac{e^{ix} + e^{-ix}}{2} \quad \frac{e^{ix} - e^{-ix}}{2i} \quad (6)$$

are the  $\cos x$  and  $\sin x$  respectively. If he substitutes an arbitrary base  $a$  for  $e$ , he will see the importance of the derivative in characterizing the sine and cosine.

Once the expressions (6) are identified as the cosine and sine, Eq. 1 follows easily by addition. We wish to apply the same method directly to the exponential function since it has a certain heuristic value in view of de Moivre's theorem.

In considering the trigonometric functions, we took the geometric properties as more familiar to the student and indicated that Eq. 2-5 could be derived from the geometry. The exponential function is not met first in terms of geometry, but as the inverse of the logarithm. However, when we remember that the student is introduced to the logarithm as a way of manipulating exponents, we realize that the exponential is in fact a particular base raised to a power. It has then two properties. The first is the multiplication law which characterizes it as a power:

$$e^x e^y = e^{(x+y)} \quad (7)$$

or, symbolically, so as to emphasize the property in the abstract:

$$f(x)f(y) = f(x+y). \quad (8)$$

We might as well notice here the extension of this property:

$$f(x)^m = f(mx) \quad (9)$$

which follows from Eq. 8 by setting  $y = x$  and using mathematical induction.

It will help to clarify what is to follow if we remark here also that Eq. 7 or 8 is not sufficient to determine uniquely the argument of a function which is known to be an exponential. For if Eq. 7 is true for a given  $x$  and  $y$ , it is true also for  $px$ ,  $py$  and  $p(x+y)$ .

The second property of the exponential not only serves to specify its base but also removes the ambiguity of the argument just mentioned. The choice of  $e$  as a base reduces the general formula for powers



$$\begin{aligned} d/dx(a^u) &= a^u \ln a \, du/dx \text{ to the very simple form} \\ d/dx(e^u) &= e^u \, du/dx \text{ or} \end{aligned} \quad (10)$$

again to isolate the property:

$$df(u)/dx = f(u) \, du/dx. \quad (11)$$

$du/dx$  in Eq. 11 will determine  $u$ , i.e., will distinguish  $e^{ux}$  from  $e^x$  to remove the ambiguity of the argument.

The properties indicated in Eq. 8 and 11 are sufficient to identify an unknown function as an exponential. We may use them to prove Eq. 1.

The student may have met de Moivre's theorem in algebra where it is often proposed as an exercise in mathematical induction. The theorem reads:

$$(\cos x + i \sin x)^m = \cos mx + i \sin mx \quad (12)$$

$$\text{If we define } g(x) = \cos x + i \sin x \quad (13)$$

Then Eq. 12 states that

$$g(x)^m = g(mx).$$

This is the special case of Eq. 8 noted in Eq. 9 and suggests that we try to verify that  $g(x)$  has the property of Eq. 8. This is very simple. We need only show that

$$(\cos x + i \sin x)(\cos y + i \sin y) = \cos(x+y) + i \sin(x+y)$$

which follows very easily by expanding the left side and making use of Eq. 3.

We are now very close to identifying  $g(x)$ . If we can show that it has the property of Eq. 11, the job is done. This is also easy for

$$d/dx g(x) = -\sin x + i \cos x = i[\cos x + i \sin x] = i g(x).$$

The factor  $i$  here shows us that  $f(u)$  of Eq. 11 is  $f(ix)$  or  $g(x) = e^{ix}$ .

III. The third approach is the shortest and best, and has been saved until now as a reward for the patient reader. I am indebted to Fr. Sohon for the method which he mentioned once in conversation.

We define

$$y = \cos x + i \sin x, \quad (14)$$

and calculate

$$dy/dx = i(\cos x + i \sin x) = iy.$$

Now, in addition to differentiation, we need only a little skill, enough to solve this differential equation. There is no trouble in acquiring it just for this occasion. We write

$$dy/y = i dx$$

and by integration have

$$\ln y = ix + C.$$

We now have the relation  $y = Ae^{ix}$  and need only determine  $A$ .

This again is easy since from the definition of  $y$  (Eq. 14), we have  $y = 1$  for  $x = 0$ . Hence  $A$  must be 1.

If one has reached the point in differential equations where he needs this relation and it is unknown to the students, this third method has the advantage of brevity and homogeneity with the rest of the course.

## THE SQUARING OF NUMBERS MENTALLY

JOSEPH A. PERSICH, S.J.

### 1. SQUARING A NUMBER ENDING IN "5"

N.B. The last two numbers of the square are always "25".

To find the number before the "25", multiply the number before the "5" by one more than itself. Place the "25" after the result of this multiplication.

e.g.  $35^2 = 1225$       since  $3(4) = 12$

e.g.  $75^2 = 5625$       since  $7(8) = 56$

### 2. SQUARING NUMBERS BETWEEN 25 AND 75

N.B. You must know by memory the squares of all the numbers between 1 and 25.

METHOD: (Difference between the number and 25) add two zeros mentally

+ (Difference between the number and 50).

e.g.  $47^2 = (47 - 25) 100 + (50 - 47)^2 = 2200 + 9 = 2209$

e.g.  $62^2 = (62 - 25) 100 + (62 - 50)^2 = 3700 + 144 = 3844$

### 3. SQUARING A NUMBER BETWEEN 75 AND 125

a. Between 76 and 99:

[Number - (Difference between the number and 100)] add two zeros mentally

+ (Difference between the number and 100)<sup>2</sup>

e.g.  $96^2 = (96 - 4) 100 + 4^2 = 9216$

e.g.  $88^2 = (88 - 12) 100 + 12^2 = 7744$

b. Between 101 and 125:

[Number + (Difference between the number and 100)] add two zeros mentally

+ (Difference between the number and 100)<sup>2</sup>

e.g.  $108^2 = (108 + 8) 100 + 8^2 = 11664$

e.g.  $122^2 = (122 + 22) 100 + 22^2 = 14884$

### 4. SQUARING A NUMBER BETWEEN 126 AND 1000

1. Express the number in terms of the nearest multiple of 50.

e.g.  $141^2 = (150 - 9)^2$

e.g.  $267^2 = (250 + 17)^2$

2. Represent the difference between the number and the nearest multiple of 50 by "x".

3. Notice these results:

$$\begin{aligned}(50 + x)^2 &= (5^2 + x) 100 + x^2 \\(100 + x)^2 &= (10^2 + 2x) 100 + x^2 \\(150 + x)^2 &= (15^2 + 3x) 100 + x^2 \\(350 - x)^2 &= (35^2 - 7x) 100 + x^2 \\(600 - x)^2 &= (60^2 - 12x) 100 + x^2 \\(850 - x)^2 &= (85^2 - 17x) 100 + x^2\end{aligned}$$

4. Notice in all these cases that our square can be represented by these 2 formulas:

$$\begin{aligned}(\text{Nearest multiple of } 50 + x)^2 &= (a^2 + bx) 100 + x^2 \\(\text{Nearest multiple of } 50 - x)^2 &= (a^2 - bx) 100 + x^2\end{aligned}$$

$$\text{where } a = \frac{\text{nearest multiple of } 50}{10} \quad \text{and } b = \frac{a}{5}$$

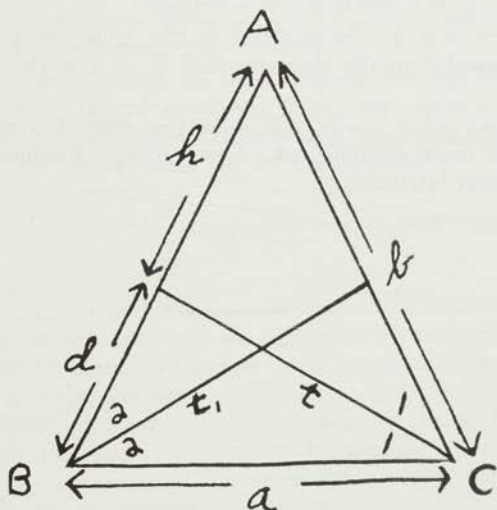
$$\text{e.g. } 213^2 = (200 + 13)^2 = [20^2 + 4(13)] 100 + 13^2 = 45200 + 169 = 45369$$

$$\text{e.g. } (441)^2 = (450 - 9)^2 = [45^2 - 9(9)] 100 + 9^2 = 194400 + 81 = 194481$$

## A THEOREM IN GEOMETRY

JOSEPH P. MERRICK, S.J.

As proofs of this theorem are rarely seen, the following may be of interest.



*Theorem:* If the bisectors of two angles of a triangle are equal, then the triangle is isosceles.

*Given:* In the triangle ABC angle 1 = angle 1, angle 2 = angle 2,  
 $t = t_1$

*To Prove:*  $b = c$  where  $c = h + d$

*Proof:* Since  $t$  is the bisector of angle C then  $b : a = h : d$

Hence  $(b + a) : a = (h + d) : d = c : d$

So  $(b + a) : c = a : d = b : h$

Thus  $d = \frac{ac}{b+a}$  ;  $h = \frac{bc}{b+a}$

But  $t^2 = ab - hd$  (Wentworth, p. 179)

$$\text{Hence } t^2 = ab - \frac{abc^2}{(a+b)^2} = \frac{ab(a+b)^2 - abc^2}{(a+b)^2} = \frac{ab(a+b+c)(a+b-c)}{(a+b)^2} = \frac{4abs(s-c)}{(a+b)^2}$$

where  $s$  is the semiperimeter.

$$\text{Likewise } t_1^2 = \frac{4acs(s-b)}{(a+c)^2}$$

$$\text{Since } t = t_1 \text{ by hypothesis, } \frac{b(s-c)}{(a+b)^2} = \frac{c(s-b)}{(a+c)^2}$$

$$\text{Hence } a^2s(b-c) - sbc(b-c) + 2abc(b-c) + bc(b+c)(b-c) = 0$$

Thus  $b - c = 0$  and  $b = c$  is a solution.

Since  $a^3 + a^2b + a^2c + 3abc + b^2c + bc^2 = 0$  has no solutions applicable to the theorem, then  $b = c$  is the only possible solution.

N.B. Two other proofs, one geometric the other trigonometric, are a bit more complicated. Even college Freshmen will find this theorem intriguing.



# Physics

## SOME APPLICATIONS OF THE FERMI-DIRAC STATISTICS TO ATOMIC AND MOLECULAR PROBLEMS<sup>1</sup>

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L. H. Thomas<sup>2</sup> and, independently, E. Fermi<sup>3</sup> developed a semi-classical method of calculating approximately the charge distribution of isolated atoms; it is known as the Fermi-Thomas method and has been extended to diatomic molecules by F. Hund<sup>4</sup> and to the atoms in a metal by J. C. Slater.<sup>5</sup>

The fundamental assumption of this method is to consider the cloud of electrons surrounding the nucleus or nuclei, as a gas, or ensemble of particles which obey Fermi-Dirac statistics. These electrons are regarded as being non-relativistic particles. Exchange effects are ignored. It is further assumed that the absolute temperature of the ensemble equals zero; this, according to Fermi-Dirac statistics, means that the electron gas occupies the region of phase space of lowest possible energy, and at this region is saturated with two electrons in each cell of phase space of volume  $h^3$  ( $h$  being Planck's constant) but the outside is empty. In other words, the number of electrons per unit volume in phase space is equal to  $2/h^3$  at all points corresponding to an energy less than a certain maximum  $E_0$  (which depends on how many electrons there are in the atom) but zero at points corresponding to an energy greater than  $E_0$ .

Three main steps, based on the foregoing assumptions, lead to the so called Fermi-Thomas equation. First one obtains a relationship between the electron density in ordinary space and the electrostatic potential at any point within the atomic or molecular field,

$$\rho_e = \frac{8\pi}{3h^3} [2m(E_0 + eV)]^{3/2} \quad (1)$$

$V$  takes into account only the potential due to the nucleus alone and the Coulomb interaction between the electrons themselves. ( $E_0 + eV$ ) is the maximum kinetic energy of any one electron at any point in the atomic or molecular field. The electron density is seen to be considered as effectively continuous throughout the field. It is important to notice that the field is assumed to be sensibly the same at a point whether empty or occupied by an electron; such an assumption is plausible only in the interior of a heavy atom.

In the second step Poisson's equation is used in order to obtain a differential equation in  $V$ ,

$$\nabla^2 V = 4\pi e \rho_e \quad (2)$$

whose solution, in the case of an isolated atom, is subject to the boundary condition  $\lim_{r \rightarrow 0} rV = Ze$

Finally, by introducing a convenient transformation of variables, and making use of the fact that the atomic field can be taken to be spherically symmetrical, equation (2) can be put in the form known as the Fermi-Thomas equation

$$\frac{d^2 \phi}{dx^2} = \phi^{3/2} / x^{1/2} \quad (3)$$

For an isolated atom, boundary condition  $\phi(x) = 1$  should be imposed on its solutions.

Equation (3) has been solved numerically by several authors.<sup>5</sup> Many different solutions are found but  $\phi$  approached the  $x$ -axis asymptotically for only one of them; this is the solution for the neutral, isolated atom. When the appropriate solutions of (3) are substituted in expression (1), one obtains the charge distribution of the atom.

The Fermi-Thomas method has been used with success to calculate the number of electrons in the completed atomic shells,<sup>7</sup> the Rydberg correction of the  $s$ -terms<sup>8</sup> and other spectroscopic quantities.

Hund<sup>4</sup> adapted the Fermi-Thomas method to isolated diatomic, homonuclear molecules. This type of molecule can be pictured as a cloud of electrons in the field of two identical nuclei which are supposed to be motionless; the charge in each nucleus equals  $eZ$ ; the distance between them can be put, for convenience, equal to  $2a$ . The significant difference between this type of molecules and the isolated atom is that the field of the former is not spherically symmetrical throughout, but it is assumed to be so in the near vicinity of each nucleus (where it differs slightly from that of each single atom) and also at large distances from both nuclei (where it differs slightly from that of an atom with twice the nuclear charge of one of the component atoms of the molecule).

One can start with the same electron density expression as in the atomic case [equation (1)] and then introduce a new dependent variable  $u = a/eZV$ . Next, using Poisson's equation, the following differential equation in  $u$  is obtained

$$\nabla^2 u = \gamma u^{3/2} \quad (\gamma \text{ is a constant factor}) \quad (4)$$

upon whose solutions the following boundary conditions should be imposed:  $ur_1 \rightarrow 1$  as  $r_1 \rightarrow 0$  and  $ur_2 \rightarrow 1$  as  $r_2 \rightarrow 0$ , where  $r_1$ ,  $r_2$  are distances from nucleus 1 and 2 respectively.

Hund<sup>4</sup> found that, to a first approximation a satisfactory solution for the whole molecular field can be written in terms of the solutions for the individual component atoms, as follows

$$u = \frac{1}{r_1} \phi [\gamma^{2/3} r_1 f(r_1)] + \frac{1}{r_2} \phi [\gamma^{2/3} r_2 f(r_2)] \quad (5)$$

where  $f$  is a suitable interpolation function.

Once the charge density of a molecule is found, one can use it to study molecular properties and calculate molecular constants. For instance, the author attempted, with fair success, to calculate the diamagnetic term in the susceptibility of  $N_2$  using a combination of the Fermi-Thomas method as adapted to molecules by Hund and the Slater atomic orbital method.

By merely imposing a new boundary condition on the solutions of equation (3) (besides the one already mentioned in connection with the isolated atom), the Fermi-Thomas model can be used to investigate the behavior of atoms in a metal. Each nucleus in the metal is assumed to be enclosed in a spherical cell which contains just enough electrons to neutralize the nuclear charge. The potential inside cell arises from the charges within it only, and it is assumed to be spherically symmetrical. At the boundary of the cell the electric field must vanish, since the net charge contained within must be zero; this is the new boundary condition and can be formulated as follows  $\frac{d\phi}{dx} = \frac{\phi}{x}$ , at the surface of the atom. According to this model, one is allowed to speak of atomic radius and volume.

Of the many possible solutions of equation (3), those which satisfy boundary conditions  $\phi(x) = 1$  and  $\frac{d\phi}{dx} = \frac{\phi}{x}$  as  $x \rightarrow 0$  are the solutions for the atoms in a metal. With the help of these solutions one can arrive at interesting results. For instance, once the potential and charge density are known one can readily obtain the potential, kinetic and total energy of each individual atom. Then it is found that the virial theorem holds for individual atoms, as it should if our fundamental assumption of regarding the atom as a statistical ensemble is correct. Finally, from the virial theorem one can pass on to establish an equation of state for individual atoms. These ideas can be found clearly explained in an article by R. P. Feynman, N. Metropolis and E. Teller.<sup>9</sup>

The procedure followed by E. Fermi and L. H. Thomas to establish their solutions of the many-electron atom has been appropriately called semiclassical. A more rigorous approach, that is, one based explicitly on the fundamental postulates of quantum mechanics is desirable; this approach was adopted by P. A. M. Dirac.<sup>10</sup>



From the antisymmetric linear combination of products of one-electron eigenfunctions which, according to quantum mechanics, describes the whole many-electron system, Dirac derives, by purely quantum mechanical methods, an expression for the total electron density of the system,

$$\rho_e = \frac{8\pi}{3h^3} \left\{ \frac{2e^2m}{h} + \left[ \frac{4e^4m^2}{h^2} + 2m(E_0 + eV_0) \right]^{1/2} \right\}^3 \quad (6)$$

This expression takes into account the exchange effects which were ignored in the simple Fermi-Thomas theory. Then by using again Poisson's equation, one obtains a differential equation which can be simplified to a form which bears a remarkable resemblance to the Fermi-Thomas equation,

$$\frac{d^2\psi}{dx^2} = x(\epsilon + \psi^{1/2}/x^{1/2})^3 \quad (7)$$

The term  $\epsilon = (3/32\pi^2)^{1/3} Z^{-2/3}$  represents the exchange phenomena; it is clear that for  $\epsilon = 0$  Dirac's equation reduces exactly to the Fermi-Thomas equation. Thus Dirac provides a theoretical justification to the Fermi-Thomas method besides generalizing it so as to include exchange effects. An unpleasant feature of the Fermi-Thomas-Dirac equation should be pointed out: because of the correction term  $\epsilon$ , the equation requires a different set of solutions for each value of  $Z$ , while the Fermi-Thomas equation requires only one set of solutions for all values of  $Z$ .

The Fermi-Thomas-Dirac equation has not been adapted to diatomic molecules as yet. An attempt to do that is in progress in St. Louis University.

Both the Fermi-Thomas and the Fermi-Thomas-Dirac methods have been employed to study crystals, metals at low and high temperatures, matter under very high pressures as found in the stars, and other topics of interest. However, the numerical results thus obtained are, in most cases, valuable only as first approximations.

#### REFERENCES

1. For a complete discussion of the topics discussed in this paper, the reader is referred to P. Gombás, *Die Statistische Theorie des Atoms und ihre Anwendungen*, Springer, Wien, 1949.
2. L. H. Thomas, "The Calculation of Atomic Fields", *Proc. Camb. Phil. Soc.*, **23**, 542 (1927).
3. E. Fermi, "Un metodo statistico per la determinazione di alcune proprietà dell'atomo", *Atti R. Accad. Naz. Lincei (R. C. Cl. Sci. Fis. Mat. Nat.)*, **7**, 66, (1928).
4. F. Hund, "Berechnung der Elektronenverteilung in einer zweiatomigen Molekel nach der Methode von Thomas und Fermi", *Zeits. f. Phys.*, **77**, 12 (1932).



5. J. C. Slater and H. Krutter, "The Fermi-Thomas method for Metals", *Phys. Rev.*, **47**, 559 (1935).
6. V. Bush and S. H. Caldwell, "Fermi-Thomas Equation Solution by the Differential Analyzer", *Phys. Rev.*, **38**, 1898 (1931) and others.
7. E. Fermi, "Sulla deduzione statistica di alcuna proprietà dell'atomo.— Applicazione alla teoria del sistema periodico degli elementi", *Atti R. Accad. Naz. Lincei* (R. C. Cl. Sci. Fis. Mat. Nat.) **7**, 342 (1928).
8. E. Fermi, "Statistische Berechnung der Rydbergkorrectionem der s-Terme", *Zeits. f. Phys.* **49**, 550 (1928).
9. R. P. Feynman, N. Metropolis and E. Teller, "Equations of State of Elements Based on the Generalized Fermi-Thomas Theory", *Phys. Rev.* **75**, 1561 (1949).
10. P. A. M. Dirac, "Note on Exchange Phenomena in the Thomas Atom", *Proc. Camb. Phil. Soc.*, **26**, 376 (1930).

## THE THEORY OF FINITE STRAIN OF AN ELASTIC BODY

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The subject of Finite Elastic Strain is one in which there is a growing interest today. The fall of 1951 marks, as it were, the coming of age of this offspring of Applied Mathematics, its debut as an independent and legitimate science. The material hitherto dispersed through the journals and discussed only in esoteric conclaves has been collected by, perhaps the most outstanding investigator in this field, Professor Francis Murnaghan, and has been published, with the results of his own research in a book entitled "Finite Deformations of an Elastic Solid", by John Wiley and Sons, Inc. It is therefore appropriate at this time, to review the problem and the literature dealing with it and to try to form some idea of the length, breadth, height and implications of such a problem.

First of all, I want to say a few words on the meaning of strain and stress, which are technical terms in this science of Elasticity. Strain is the measure of the deformation of a body, i.e. the measure of its change of shape or size when subjected to certain forces. Strain is always associated in an elastic body with an internal system of forces, called stresses which are induced in the body by the deformation. They are a sort of sympathetic response of the whole body to the strain imposed upon it. In general, they grow or fade as the strain grows or fades and their nature provides physical criteria for distinguishing a solid from a liquid and a liquid from a gas.

In a perfectly elastic body, the stress components are algebraic functions of the strain components. I say, in a perfectly elastic body, because this simple relationship is only true when we agree to neglect such effects as elastic hysteresis, elastic afterworking, internal friction, and thermoelastic effects.

Now, as it happens, neither the components of strain nor, in

general, the components of stress are linear in the derivatives of the coordinates. Consequently, the differential equations of elastic motion are non-linear. Here the crux of the problem lies. For up to recent years, little was known about the properties and solutions of non-linear differential equations. Some of the fruits of modern research on relatively simple non-linear differential equations are sufficiently surprising and unexpected to make us pause before the problem provided by the complete partial differential equations of elasticity. Here indeed, is a major obstacle.

However, when the exact solution was unattainable, approximate methods were devised. The approximation used consisted in omitting all the non-linear terms in the differential equations and retaining only those linear in the derivatives of the coordinates. The set of equations so obtained are the basis of the Classical or Infinitesimal Theory of Elasticity. In vector notation, they are,

$$\rho \partial^2 \mathbf{u} / \partial t^2 = \nabla \cdot \Phi \quad (\text{after Sohon})$$

where  $\Phi$  is the stress tensor, which assuming a linear relationship between stress and strain (Hooke's Law) and that the medium is isotropic, becomes,

$$\lambda \nabla \cdot \mathbf{u} \mathbf{I} + \mu (\nabla \mathbf{u} + \nabla \mathbf{u}_e) \quad (\text{after Sohon})$$

The approximation corresponds to the case of infinitesimal deformations and the whole traditional theory of elasticity embodies this assumption. Love's classic work on elasticity, and all subsequent treatises embody this assumption.

However, in actual problems where the strain is by no means infinitesimal, as for example, the elastic state of the earth's core or mantle, or in problems involving immense mechanical stresses such as are produced in some modern manufacturing processes and in armaments, the simple linear equations no longer hold. In these cases then, it is necessary to face the problem of finite strain squarely.

I do not intend here, to outline the mathematical theory of finite strain. I do however, want to point out some new ideas and new perspectives that the theory has opened up.

More than anything else, the theory of finite strain has been responsible for a great clarification in our ideas of the nature of stress and strain and elastic energy.

A second point of interest arises in the case of an isotropic medium. A rigorous analysis of the stress-strain relations of a perfectly elastic medium shows that a medium can be isotropic with respect to additional deformations from a given state, only when the initial state of the body is one of hydrostatic stress. Thus no body subject to a shearing force, is "a priori" isotropic with respect to further deformations from this state. This conclusion has immediate reference to the problem of the propagation of elastic waves near the epi-

center of an earthquake, or through a sheared zone of the earth's crust. An apparent variation of velocity with orientation, a phenomenon which has been observed in Southern California and on the Greenland icecap, is usually explained by an appeal to the crystalline properties of the medium. We know now, that the initial state of stress could have a like effect on the medium. We can coin a word and say that the initial state of stress may render a medium "pseudo-crystalline".

Another important result of research on finite strain is to clarify the meaning and number of the so-called "elastic constants". An "elastic constant" is, by definition, any parameter in the stress-strain relation that is independent of the initial strain. It is not, however, as the name would imply, a constant, but is a function strongly dependent upon the initial state of stress. In the theory of finite strain the initial stress plays an important part in the stress-strain relation. For example, in the case of a body subject to an initial uniform hydrostatic pressure  $P_0$ , the following formulae hold for departures from this state:

1. if the hydrostatic pressure is increased, the PV (pressure vs. volume) relation is,

$$P = (V_0/V)^{1/3} \left\{ P_0 + \beta/2(1 - V/V_0)^{3/4} + \gamma/4(1 - V/V_0)^{4/3} + \dots \right\} \quad (A)$$

where  $V$  = volume of the deformed body when subjected to a pressure  $P$ , the subscript zero refers to the initial state and  $\beta$  and  $\gamma$  are "elastic constants".

Stopping the series where we do, and omitting terms of higher order in  $(1 - V/V_0)$ , we see that three "elastic constants" are required, viz.  $P_0$  = initial hydrostatic pressure, and  $\beta$  and  $\gamma$  which are, in general, functions of  $P_0$ . In the corresponding infinitesimal problem, besides the initial pressure, there is only one "elastic constant", the bulk modulus, and the corresponding PV relation (in the infinitesimal theory) is

$$P = P_0 - k(1 - V/V_0)$$

Murnaghan found that the formula (A), cut off at the third term, was able to express completely, within the margin of experimental error, Bridgeman's results on the compressibility of sodium, which were obtained in a series of experiments performed at Harvard using pressures up to  $10^5$  atmospheres.

2. when applied to the case of small periodic elastic vibrations about this state, it is found that the velocity of propagation of longitudinal type waves is

$$v = \sqrt{\frac{\lambda + 2\mu - P_0}{\rho_0}}$$



while the velocity of propagation of transverse type waves is

$$\sqrt{\frac{\mu - P_0}{\rho_0}}$$

expressions which differ from the classical formulae by the appearance of the quantity  $P_0$  which is the initial hydrostatic stress. When  $P_0$  is zero, these expressions become equivalent to the classical expressions and  $\lambda$  and  $\mu$  become the well known Lamè constants.

I should like to close this paper with a few words on the importance of the theory of finite strain in seismology. It is apparent from what I have said, that the approach to the problem of elastic deformations of the earth's crust, or in the body of the earth should be by means of the theory of finite strain. The initial state of stress of the earth's mantle and core is far from infinitesimal. However, transitory elastic disturbances, like seismic waves propagated outward from the focus of an earthquake, are, outside the immediate area of focus, virtually infinitesimal in magnitude compared with the state of initial strain. Thus it would seem that the problem of seismic wave propagation can be treated by a combination of the old and new theories.

I want to make brief reference to the work done in applying the theory of finite strain to seismology. Francis Birch of Harvard has published the results of an interesting investigation on the effects of pressure on the elastic parameters of a solid, approaching the question from the point of view of finite strain. He has applied these to the calculation of the variation in seismic velocities within a simplified earth model. Bullen has utilized these investigations of Birch in his revision of the calculated density distribution within the earth. Apart from these interesting endeavors, little has been done to bring the theory of finite strain to bear on seismological problems. It is an application that may in future years yield many interesting results.

Much has yet to be done in the field of finite strain theory. Relatively few problems have been solved. Even the apparently simple problems, like that of the uniform extension of a rod or wire, bristle with unsuspected difficulties when the new approach is made. Furthermore much work has yet to be done on non-linear partial differential equations of the type that occur in this theory.

The theory of finite strain is relatively new territory and relatively undeveloped. At present, little more than the trailways have been blazed. It is up to the coming generation of mathematicians to inhabit the land.

#### BIBLIOGRAPHY

- Murnaghan, F. D., "Finite Deformations of an Elastic Solid", *Am. Jour. Math.*, vol. 59, (1937), 235-260.
- . Von Karman Anniversary Volume, Pasadena, Calif. (1941), 121-136.



- . "Foundations of the Theory of Elasticity", *Proc. Symposia Appl. Math.* I, Amer. Math. Soc., N. Y., 158-174.
- . *Anais Acad. Brazil. Ci.*, vol. 21, (1949), 329-336.
- . "Lectures on Elasticity", Dublin Instit. for Advanced Studies, mimeo. notes, 1948.
- Richter, Hans, "The Isotropic Elasticity Law", *Z. Angew. Math. Mech.*, vol. 28, (1948), 205-209.
- . "Distortion Tensor, Distortion Deviator, and Stress Tensor for Finite Deformations", *Z. Angew. Math. Mech.*, vol. 29, (1949), 65-75.
- Signorini, A., "On Finite Deformations of an Elastic Solid", *Proc. Seventh Inter. Congr. Appl. Mech.*, vol. 4, (1948), 237-247.
- Swainger, K. H., "Severe Deformations", *Proc. Seventh Inter. Congr. Appl. Mech.*, vol. 1, (1948), 49-60.
- Green and Shield, "Finite Elastic Deformations of Incompressible Isotropic Bodies", *Proc. Roy. Soc. London, A*, vol. 202, (1950), 1070, pp. 407-419.
- Green and Zerna, "Theory of Elasticity in General Coordinates", *Phil. Mag.*, vol. 41, (1950), 313-337.
- Rivlin, R. S., "Large Elastic Deformations of Isotropic Materials", *Phil. Trans. Roy. Soc. London, Ser. A*, 241, (1948), 379-397.
- . "Large Elastic Deformations of Isotropic Materials VI, Further Results in the Theory of Tortion, Shear and Flexure", *Phil. Trans. Roy. Soc., London, Ser. A*, 845, (1949), 173-195.
- Deuker, E. A., *Deutsche Math.*, vol. 5, (1941), 546.
- Neuber, H., *Z. Angew. Math. Mech.*, vol. 23, (1943), 321.
- Brillouin, L., *Les Tenseurs en Mecanique et en Elasticite*, (Masson, Paris, 1938; also Dover, N. Y., 1946).
- Seth, R. B., "Finite Strain in Elastic Problems", *Phil. Trans. Roy. Soc., Ser. A*, 234, (1935), 231.
- Oldroyd, J. G., "Finite Strains in an Anisotropic Elastic Continuum", *Proc. Roy. Soc., Ser. A, Math. Phys. Ser.* 202, 1070, (1950), 345-358.
- Signorini, A., "Finite Thermoelastic Transformations 11.", *Ann. Mat. Pura Appl.* (4), Vol. 30, (1949), 1-72; *Ann. Mat. Pura Appl.* (4), Vol. 22, (1943), 33-143.
- Birch, F., *J. Appl. Math.*, vol. 8, (1937), 129.
- . "Effect of Pressure upon the Elastic Parameters of Isotropic Solids According to Murnaghan's Theory of Finite Strain", *J. Appl. Math.*, vol. 9, (1938), 279-288.
- . "The Variation of Seismic Velocities Within a Simplified Earth Model, In Accordance with the Theory of Finite Strain", *Bull. Seis. Soc. Am.*, vol. 29, (1939), 463-479.
- Birch, F. and Bancroft, *J. Geol.*, vol. 46, (1938), 39.
- Bullen, K. E., *Bull. Seis. Soc. Am.*, vol. 30, (1940), 235.

## AN AETHERIAL PROPHECY

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On January 15, 1944 Karl K. Darrow delivered the third annual Richtmyer Memorial Lecture of the American Association of Physics Teachers before a joint meeting of this Association and the American Physical Society at Columbia University in New York. The subject of his talk was "The Future of Physics, Past and Present," and after reviewing the rise and decline of the aether theory, Darrow said:

"Will the aether remain a ghost? It is foolhardy to be a prophet, but I suppose that by the title of this speech I have committed myself to making at least one prophecy. Therefore, I foretell that the aether will come back. This old and faithful comrade of the human mind in its imaginative flights cannot be dead forever. It arose to satisfy a deep-implanted appetite of the intellect; the appetite seems dormant now, but it will reawaken demanding to be fed. I do not predict the return of the aether for tomorrow, nor perhaps for my time, but I think that some of you will live to welcome it back. The place of its return is equally beyond my foresight. In the eighteenth century the aether resided on the continent of Europe, and was spurned in England; in the early nineteenth century it was banished from the continent and found a happy home in the British Isles. Perhaps it will come back in 1980 in Australia." [*American Journal of Physics*, 12, 60 (1944)].

It now appears that Darrow was a better prophet than he realized, for the aether has come back once again—and this not in 1980 in Australia, but in 1951 in Great Britain. The November 24, 1951 issue of *Nature* contains a letter from Professor P. A. M. Dirac, of Cambridge University, in which Dirac, world-famous for his work in the field of relativistic quantum mechanics, indicates that there now exist both negative and positive arguments for the existence of an aether.

Einstein's discovery of the theory of relativity in 1905 led to the abandonment of the aether by physicists because it could not be fitted in with relativity. Dirac claims that the advent of quantum mechanics has again changed this situation, and that now the aether is no longer ruled out by relativity. The existence of an aether, moving with a definite velocity even in a perfect vacuum, appeared to be inconsistent with the requirements of relativity that there be no preferred direction in such a region of space-time, but that all directions must be equivalent to one another. Now, however, the aether must be treated from the viewpoint of quantum mechanics. When this is done the velocity of the aether at a certain point of space-time will not usually be a well-defined quantity, but will be distributed over various probable values which are obtained by taking the square of the modulus of the wave function representing the aether. Dirac believes that it is possible to find such a wave function which will preserve the proper-

ties of the aether, and still represent the perfect vacuum state in accordance with the principle of relativity. Such a wave function, however, cannot be normalized, i.e., the integral of the square of its modulus over all space-time is not finite. Since states corresponding to wave functions that can be normalized are the only states that can be attained in practice, Dirac reaches the interesting conclusion that the perfect vacuum is an idealized state, not attainable in practice. As he says: "We must make some profound alterations in our theoretical ideas of the vacuum. It is no longer a trivial state, but needs elaborate mathematics for its description."

Dirac's positive argument for the existence of an aether is based on his new theory of electrodynamics. This theory predicts that in any region of space in which there is any electric charge, this charge must flow with a definite velocity  $v$ ; and in regions where there is no charge,  $v$  is the velocity with which a small charge would have to flow if it were introduced. As Dirac says: "It is natural to regard it as the velocity of some real physical thing. Thus with the new theory of electrodynamics we are rather forced to have an aether."

Dirac's defense of the aether has received support from a number of British physicists and mathematicians, including Sir Edmund Whittaker. The Jan. 26, 1952 issue of *Nature* contains a number of letters supporting Dirac's position, including one from Bondi and Gold, in which they say that Dirac's conclusions fit in nicely with their own theory of continuous creation. In the same issue Dirac discusses the relationship between these two theories. He says: "Where matter exists, both theories require it to have the aether velocity. Where there is no matter, Bondi and Gold interpret the aether velocity as the velocity of the matter which gets created by their process of continual creation. I interpret it as the velocity which a small electric charge would have if it were introduced."

It is unlikely that Dirac's conclusions will be immediately accepted by all physicists, especially in the United States where the aether concept has been anathema for many years now. Dirac's theory will have to stand the test of future theoretical developments and experimental verifications. But the outstanding position Dirac holds in the world of theoretical physics would seem to indicate that there is a good chance that the aether concept will eventually return to popularity, as prophesized by Darrow.

DYSON, G. MALCOLM, *Short Guide to the Chemical Literature*, Longmans, Green and Co., London, N. Y., Toronto 1951. 8°, 144 pp., \$1.75. The outstanding virtue of this book is its conciseness. It bids fair to introduce the student to the maze of chemical literature in an effective way.

The first five chapters are bibliographic in nature, consisting of enumeration, along with historical notes and discussion, of dictionaries,



journals, abstracts and other reference works according to their type, the countries in which they are published and the various fields of interest. The heart of the work is to be found in the sixth and final chapter. It discusses actual literature searches. In addition, there are three appendices: 1. a handy reference list of items in the older (nineteenth century) literature, 2. a sample of literature searching in the field of organic chemistry, and 3. a table which coordinates years and volume numbers for the common journals. Author and subject indexes are also provided.

Like most works of this sort, the book is at its best in discussing the literature of organic chemistry. Other branches are also treated. A fresh and interesting approach to the literature of inorganic chemistry is presented. A frank discussion of the limitations of literature searches in the other fields is included.

In style, the book is addressed directly to the student; not indirectly through the teacher, who is apt to determine sales and royalties. It is didactic in the best sense of the word and breathes with every phrase the overtones of actual experience. It seems to have been written by a practising chemist, rather than by a professional bibliographer.

This citation of actual examples of literature searches is satisfying. The direct approach through handbooks like Gmelin and Beilstein into key collections like *Chemical*, and *British Chemical Abstracts*, seems eminently practicable. Pitfalls of literature search are confided with the student and impressed on him through actual examples. Thus the necessity for checking down two lines of sources, American and British for example, is urged. Undoubtedly the author has acquired his information the "hard way" and is happy to pass the findings on.

With all of these virtues, the shortcomings can be overlooked or remedied in teaching practice. It seems to the reviewer that the author might have toned down the reliability of *Chemical Abstracts* for the run 1907-1919. Lange's *Handbook of Chemistry* (7th ed., Handbook Publishers Inc., Sandusky, Ohio, 1949) is not mentioned in this work. Yet it is the fastest key for getting at quick, "spot" information in Beilstein. Certain items of bibliographic information are not up to date: for example, there are now four volumes of the *Memorial Lectures of the Chemical Society*, not two (p. 39). The *Journal of the American Chemical Society* commenced publication in 1879, not 1897 (p. 7). It is to be regretted that the names of the publishers are generally omitted from the bibliographic data of texts and monographs.

In the last analysis, a student only learns literature search by searching the literature. But this book will save him many hours in his introduction to it. *bafSJ*