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THE EDITOR'S PAGE

With the publication of this Number, the BULLETIN enters its 23rd year, and it is hoped, its last year under war time conditions. Also, with the publication of this Issue science, with the rest of the world, enters a new era, the era of atomic energy. The atomic bomb is a reality, having left the realm of theory at the fated cities of Japan, and the control of nuclear fission for peace time use is only a matter of time, long or short. It is all part of the same era we have entered. Words like atom, electron, proton, neutron, nucleus and nuclear disintegration, which not long ago were part of the technical language of the scientist are now part of the everyday language of the people. And sudden application has succeeded in making these words understood where the teacher of scientific novices has had only partial success. Anyone may find descriptions of the subject cut to his fancy and ability to understand if only he will take the patience to search. The scientist will find satisfaction in the book "Atomic Energy for Military Purposes" by Smyth. Those only slightly less technically minded will enjoy the explanations given in recent numbers of CHEMISTRY. (Vol. 19, 1, 2.) The ordinary citizen desiring merely to keep up with modern developments of common interest will enjoy and be satisfied with such excellent explanations and diagrams as are found in the New York Times Magazine section for August 12th 1945 and in Life and Time for the same Issues. Editorials and special articles in all newspapers will keep him posted of the bomb and its countless implications in modern life.

Yes, the atomic bomb and atomic energy have ceased to be the exclusive property of the scientist hidden in his laboratory. But in a more serious sense the atomic energy has passed from the hands of the scientist into those of the men who direct the destinies of nations. The scientist has revealed one more of the marvelous secrets hidden by the Creator in nature for men to find. Someone has cynically remarked that the atomic bomb is the end of the quest for a substitute for God. Others with a different point of view have rejoiced that we are brought one step nearer to understanding the meaning of Infinite Power. Through all their discussions from various angles, with selfish and humanitarian considerations, leaders must ultimately choose

one of these two viewpoints, and this choice alone is of supreme importance. Before long, at longest, the atomic bomb will be the secret and property of no one nation or group of nations. When it belongs to the world, will men consider it the development which has made God and his law unnecessary, or will it bring back to them an understanding and respect and awe, in the true meaning of the word, of the power which governs this universe. These are the two arms of the balance; that choice is the knife edge on which future peace rests. Many scientists regret the discovery of so potent a weapon because they see in it only the destructive force of the world. They do not know the principles which can lead men to put this force to peaceful humane uses. We, as Catholic scientists, know that only the spiritual direction of Christian philosophy can control the atom and preserve man himself from destruction. We do not fear the bomb. We do fear the controlling principles. It is ours to give to our profession and to our students the principles which can give us life, safety and peace.

Commenting on the opposition to Nazi anti-religious and anti-intellectual policies, the Editor of the "Month" cites the following from Dr. Albert Einstein: (c.f. "Month," Jan.-Feb., 1945, p. 53.)

"When the Revolution came in Germany, I looked to the Universities to defend it, knowing that they always had boasted their devotion to the cause of truth. But no, the Universities immediately were silenced. Then I looked to the great editors of the newspapers whose flaring editorials in days gone by had proclaimed their love of freedom, and they, like the Universities were silenced in a few short weeks. Then I looked to the individual writers who as literary guides of Germany had written much and often concerning the praise of freedom in modern life; they too were mute. Only the Church stood squarely in the path of Hitler's campaign for suppressing truth. I never had any special interest in the Church before now, but I feel a great affection and admiration, because the Church alone had the courage and persistence to stand for intellectual truth and moral freedom. I am forced, then, to confess that what I once despised I now praise undeservedly."

ASTRONOMY

OCCULTATIONS OF STARS BY THE MOON

REV. JAMES K. CONNOLLY, S.J.

(Continued from May-June issue)

The preceding section of this article (Vol. xxii, No. 4, p. 104) has described the process of predicting and observing an occultation. The purpose of the observation is to determine the difference between the actual and predicted positions of the moon at the instant of occultation. The determination of this difference is called the reduction of the occultation. The process starts with an observed time, an identified star and a known position of the observer. What to do with these is determined by the source from which the occultation was predicted. The prediction may have come from

- 1) NA for stars brighter than 7.5 Magnitude
- 2) one's own predictions for stars fainter than 7.5 Magnitude
- 3) or a few cases may arise in which a star is observed which is contained in the AE and not in the NA; e.g., a sixth magnitude star twenty four hours before full moon or a good observation of a fifth magnitude star at a bright limb immersion. There will be three methods of reduction corresponding to these three sources of prediction.

CLASS 1. STARS PREDICTED BY NA.

To reduce to standard form the results of the observation of such a star requires only ten or fifteen minutes per star. There is, however, a certain amount of preliminary work to be done for each place of observation. Once this work is done for any given station, it is applicable to all stars observed at that station.

NOTE. Henceforth letter symbols commonly given in Greek letters, such as alpha for right ascension, will be shown as the corresponding English letter (capital or small) but underlined. Thus \underline{E} or \underline{e} stands for eta capital and small; $\underline{r} \sin \underline{f}'$ may be read, rho sin phi prime. \underline{SADa} originally read: Capital Sigma-A-Capital Delta-small alpha.)

Let \underline{r} be the observer's distance from the center of the earth and \underline{f}' , his geocentric latitude. One must know $\underline{r} \sin \underline{f}'$ and $\underline{r} \cos \underline{f}'$. These quantities can be determined in a few minutes with the help of a table given annually by AE in the Eclipse section of "Use of Tables" near page 600. The use of this table requires that the geographical latitude (\underline{f}) of the observer be known. This is the latitude shown on a map and may, to a sufficient accuracy, be picked off a Coast and Geodetic Survey sheet for the observer's locality. It is desirable to know this geographical latitude to a second of arc or better.

Next, one should have a copy of "Prediction and Reduction of Occultations" which is a supplement to the NA for 1938. Henceforth it will be referred to as NASupp. One can secure it from H. M. Stationery Office, Adastral House, Kingsway, London, W. C. 2. Its publication price was 2s. 0d.

The NASupp. contains instructions for the prediction and reduction of NA predicted occultations plus four tables used in the reductions. Instructions are included for the computing of a fifth table of two quantities: \underline{x} and Q .

$$\underline{x} = (\underline{r} \cos \underline{f}' \sin h) / k$$

$Q = (\underline{r} \cos \underline{f}' \cos h) / k$. $\underline{r} \cos \underline{f}'$ has, presumably, been previously found. $k = 0.2724953$ (NASupp.) Values of \underline{x} should be computed and tabulated for different values of "h" from 0^h to 6^h at intervals of 1^m (i.e., 15 minutes of arc.) Values of Q have, of course, been found at the same time as the values of \underline{x} , since \underline{x} for any angle equals Q for the complementary angle. Thus \underline{x} for $4^h 5^m$ is the same as Q for $1^h 55^m$. This will be referred to as Table V.

A change in Table I, (NASupp. p. 42) will facilitate computation. In place of the second column, headed "h m" type a substitute and paste it in, over the given column. The value given on each line of the substitute column consists of the value on the corresponding line of the given column less the longitude of the observer in hours minutes and thousandths of minutes. An example of this change is found in NASupp. p. 40, where the column is adapted to an observer at Strasbourg in longitude $-0^h 31.^m 071$. (Minus longitude is longitude east.)

With this preliminary work done, once for all, the reduction of individual stars may be begun. A NA for the year of occultation is required. To find the data necessary for reduction, proceed in this

way. One has already found the occultation predicted in the NA for some date and hour. This prediction gives the name of the star. Turn to the section of the NA, "Elements of Occultations" and look up this star name for the date of occultation. When found, the first column of the table will be seen to give a serial number. Look up this serial number in the "Occultation Reduction Elements" section in the NA. Under the serial number will be found a set of unlabelled numbers arranged in a regular pattern. The different numbers of the set may be identified by their places in the pattern, according to the key given in NASupp. p. 33. The key is reproduced here with the previously noted substitution of English for Greek characters.

Serial No.	\underline{m}_0		r	s	v	\underline{p}_y
T_1	x_1	$\frac{D_1''}{D_1''} + \frac{D_2''}{D_2''}$	y_1	$\frac{D_1''}{D_1''} + \frac{D_2''}{D_2''}$	F_1	G_1
T_2	x_2	$\frac{D_2''}{D_2''} + \frac{D_3''}{D_3''}$	y_2	$\frac{D_2''}{D_2''} + \frac{D_3''}{D_3''}$	F_2	G_2
T_3	x_3		y_3		F_3	G_3

The reduction can best be computed on a special form which is reproduced as Fig. 1. These forms have been available from the Astronomical Department of Yale University.

One can immediately fill in the places marked 1 in Fig. 1. Serial number is given at top left of set of numbers just found; place of observation; date of observation; star observed (whose name has already been noted in the prediction); observer's name; phase; limb of moon at which occultation took place, i.e., bright or dark; Universal time at which occultation was observed; $r \sin f'$ as previously computed; r, s, \underline{m}_0 , \underline{p}_y from top line of the set of numbers; magnitude of star, from data of prediction; name of computer; remarks concerning computation, e.g., that they were done in duplicate.

Next fill in sections marked 2 to 5 in Fig. 1, reducing UT of occultation to hours, minutes and three decimals, then to hours and five decimals. The next term down (4) is a correction term which is changed from time to time. New values have been published in Popular Astronomy, The Astronomical Journal and elsewhere. Current value can be found from the Astronomy Department of Yale University. Sum of 3 and 4 gives 5, the corrected time with which to enter the set of numbers to interpolate the values of x and of y for spaces 6.

The formula for finding x is

$$x = (1-n)x_0 + nx_1 + B''(D''_0 + D''_1)$$

If time, (space δ) falls between two tabulated times, say T_0 and T_1 , n equals the decimal part of an hour from T_0 to the time of occultation. x_0 is x value corresponding to T_0 . x_1 corresponds to T_1 . The third term is found, in every case, without interpolation, from Table II, (NASupp. p. 43) for proper value of n and of $D''_0 + D''_1$ given in the set numbers between the two values of x which are used.

REDUCTION OF OCCULTATION. (A. J. 1062; OCCULTATION SUPPLEMENT OF N.A.)	SERIAL NO. <u>1</u>		PLACE <u>1</u>		DATE <u>1</u>				
	STAR <u>1</u>		OBSERVER <u>1</u>		PHASE <u>1</u> LIMB <u>1</u>				
	MAGN <u>1</u>		COMPUTER <u>1</u>		REMARKS				
	U.T. (h,m,s)	<u>1</u> h <u> </u> m <u> </u> s	$\rho \sin \phi'$	<u>1</u>	r	<u>1</u>	η <u>15</u>		
	U.T. (h,m)	<u>2</u> h <u> </u> m	s	<u>1</u>	q	<u>12</u>	η <u>16</u>		
	U.T. (h)	<u>3</u> h <u> </u> <u> </u>							
	CORR ^o .	<u>4</u>	x	<u>6</u>			SUM = η <u>17</u>		
	SUM	<u>5</u>	ξ	<u>11</u>			y <u>6</u>		
	μ_0	<u>1</u> h <u> </u> m	$f = x - \xi$	<u>18</u>			$g = y - \eta$ <u>19</u>		
	RED ^o . FOR HRS	<u>7</u>	F	<u>13</u>			G <u>14</u>		
MINUTES	<u>8</u>	$f^2 + g^2$	<u>20</u>			$FF + Gg$ <u>21</u>			
RED ^o . FOR MIN.	<u>9</u>	πy	<u>1</u>			v <u>1</u>			
SUM = h	<u>10</u>	COEFF ^o . OF $\Delta\alpha$	<u>23</u>			COEFF ^o . OF $\Delta\delta$ <u>22</u>			
REMARKS _____									
REFERENCE NO.	p	q	p^2	pq + -	q^2	$\Delta\sigma$ + -	$p\Delta\sigma$ + -	$q\Delta\sigma$ + -	LUNATION

Fig. 1. Form for reduction of NA stars.

If time falls before the first tabulated time, T_0 , n is the hour and fraction from second tabulated value, T_1 to time of occultation. x_0 is second listed value, x_1 is first listed value.

If time falls after the last tabulated time, say T_3 , n is hour and fraction from second last tabulated value, T_2 , x_0 corresponds to T_2 , x_1 to T_3 .

The same method applies to the interpolation of y .

In the first column of Table I (NASupp. p. 42) find number equal to number of integral hours in space 2, Fig. 1. (All spaces referred to henceforth are those of Fig. 1.) Enter corresponding value from column two in space 7. It is supposed that column two has been altered as previously suggested. Next copy minutes and thousandths from space 2 to space 8. Enter correction for these minutes from second part of Table I on space 9. Sum of m_0 , 7, 8 and 9 equals hour angle, h , in space 10. This should not much exceed six hours since it is the angle, in hours, from the meridian to the moon at the instant of occultation.

With this value of h find corresponding values of x and Q by simple interpolation from Table V. Enter values in spaces 11 and 12.

By interpolation at sight, find for the time of occultation (space 5) the values of F and G from the last two columns of the set of numbers. Enter on card in spaces 13 and 14.

Enter on card e equal to $e_1 + e_2$ in space 17.

Enter on card f equal to $x - \underline{x}$ in space 18.

Enter on card g equal to $y - \underline{e}$ in space 19.

Form $f^2 + g^2$ to fourth decimal place and enter in space 20. f and g are themselves in units of the fourth decimal place. A table of squares, such as Barlow's, will be useful in forming these squares.

Form $Ff + Gg$ and enter in space 21. This product sum is taken to the nearest integer, considering F and G as whole numbers and g and f as being given in units of the fourth decimal place.

With this value of $Ff + Gg$ take from Table III (NASupp. p. 44) the quantities p , q , p^2 , pq , q^2 and enter them in their proper places on card.

With given values of \underline{p}_y and $f^2 + g^2$ take from table IV (NASupp. p. 46) value of $\underline{D}s$ and enter on card. This quantity, ordinarily read "Delta sigma" is the distance that the moon is from its predicted position, measured along a line from the star to the predicted position of the moon's center.

Form products $p\underline{D}s$ and $q\underline{D}s$. $\underline{D}s$ is taken as a whole number and p and q as in units of the second decimal place.

Coefficient of \underline{Dd} (space 22) equals the first two figures of g . It is to be remembered that g is given in units of the fourth decimal. Accordingly, if g is given as 862, the number really is 0.0862, so to two figures, 0.09. \underline{Dd} would be -0.09 .

Coefficient of \underline{Da} (space 23) equals $-0.1 v f$. For decimal place consider v to be a whole number and f to be in units of the fourth decimal.

This completes the reduction. The reader may be inclined to contrast the number of explanatory pages with the earlier statement that the reduction takes only ten or fifteen minutes. He can find a similar contrast in the time needed to tie a bowline and the space needed to describe the knot accurately without using pictures.

CLASS 2. STARS PREDICTED BY OBSERVER.

Just as the prediction of such occultations entails additional labor, so will their reduction. The stars must be identified and certain information obtained. An observer who has the catalog for his star chart can identify the stars from the catalog. An observer who lacks catalogs must depend upon catalog owners for the identification and information. Proceed thus: Make a tracing of the rectangle of the chart in which the occulted star is found. On the tracing paper, copy in the occulted star and enough of the neighboring stars to enable another person to recognize the configuration of stars on another chart which may have a quite different scale. Indicate the occulted star with an arrow. Mark the corners of the rectangle with correct values taken from your chart. This traced rectangle should fit the chart directly and is not offset to fit precession marks, as was the prediction tracing. Mark the date of the chart. Send the tracing to some observatory willing to cooperate in this work, requesting: Identity of star; position of star (and epoch of position) precession, secular variation, proper motion and magnitude. When the information is received one can begin the

Reduction of stars to apparent place. Star catalogs give the right ascension and declination of stars for a given year, 1875, 1910 or 1950 for example. The reduction to the position used in the computation is made in two sections: a) The star is brought up to what is called the mean place for the beginning of the year and b) it is then brought up to the apparent place at the instant of occultation.

a) *Reduction to mean place at beginning of year.* Because of precession, the right ascension and declination of a star are always changing. Because the change, called precession, itself depends on the right ascension and declination there is also a continual change in precession. The change in precession is usually given for a hundred years and is

called the secular variation, henceforth abbreviated to SV. Therefore, in order to reduce a star from its catalog position to mean place the following data must be known: Date of catalog, called the epoch; the right ascension, \underline{a} , and declination, \underline{d} , of the star at that epoch, the precession and secular variation for the star in right ascension and declination. If $\underline{a^0}$ is right ascension of mean place, \underline{a} the right ascension of the catalog and T the number of years from the epoch of the catalog,

$\underline{a^0} = \underline{a} + T(\text{Precession in right ascension}) + T^2 \text{SV}/200$ The formula for \underline{d} , the declination of mean place, is the same form. If proper motion is given, multiply it by T and add it to the three terms already added.

b) *Reduction to apparent place at time of occultation.* The AE gives a much simplified method of reducing star positions from mean to apparent place. Formulas and tables are given in the AE immediately following the "Additional Stars Occulted by the Moon" section. First reduce $\underline{a^0}$ and $\underline{d^0}$ to hours and hundredths and to degrees and hundredths, respectively. Next from Tabel A, with $\underline{a^0}$ as variable, find $\underline{d''}$ by simple interpolation, neglecting the second differences. Form $\underline{d^0} - \underline{d''}$. Then turn to Table B for the right month. Here the variable is $\underline{a^0}$ tabulated at half hour intervals. The next step is most easily done on a slide rule. Set the index of the rule to the difference between $\underline{a^0}$ and the nearest half hour. This difference is negative if $\underline{a^0}$ is less than the nearest half hour. To find \underline{SAa} (and the same method applies to $\underline{SAa'}$) multiply the slide rule setting by the *difference* listed in the column "Var. per hour" between the two values of $\underline{a^0}$ bracketing your value. Add this product to the variation listed in the same column opposite the nearest list value of $\underline{a^0}$. Multiply this sum by the slide rule setting and add this product to the quantity listed in the Column \underline{SAa} opposite the nearest half hour. One has to be careful of signs. Throughout this process the slide rule setting remains unchanged, index at the difference between $\underline{a^0}$ and the nearest listed half hour. This same setting is used to find \underline{SADa} and $\underline{SCDc'}$ by simple interpolation between the listed values. These last two quantities are multiplied by $\underline{d^0} - \underline{d''}$. The apparent place is then given by

$$\underline{a} = \underline{a^0} + \underline{SAa} + (\underline{d^0} - \underline{d''}) \underline{SADa} + \underline{tm} \quad \text{and} \\ \underline{d} = \underline{d^0} + \underline{SAa'} + (\underline{d^0} - \underline{d''}) \underline{SCDc'} + \underline{tm'}$$

\underline{m} is proper motion in right ascension, if known. $\underline{m'}$ is proper motion in declination. \underline{t} is fraction of year up to date of occultation given in calendar section of AE.

(To be concluded)

CHEMISTRY

THE CHEMISTRY OF SYNTHETIC RESINS AND PLASTICS

LOURDU M. YEDDANAPALLI, S.J.

HISTORICAL

What is a plastic? Bakelite would have been the answer of the man in the street a few years ago, and now lucite, plexiglass and a number of others would be the answer. Cellulose plastic would not probably come to his mind as a plastic. In point of fact, however, the first consciously developed plastic was cellulose plastic, celluloid. In 1868, the cost of billiard balls became exorbitantly high due to the scarcity of world supply of elephant ivory; a prize of \$10,000 was offered to anyone who would bring out a synthetic substitute for ivory. A young printer, John Wesley Hyatt of New England, later of ball-bearing fame, produced the required substitute in the form of celluloid, obtained by nitrating cellulose and compounding the resulting material with camphor (plasticizer) into an easily moldable plastic for billiard balls. It is rather curious to note that neither the latent potentialities nor the principles of celluloid preparation were subsequently exploited to advantage until recent times.

A little later in 1890, Spittler in Germany mixed almost by accident sour cow's milk with formaldehyde and obtained a shiny horn-like material, a casein plastic. In 1909, Dr. Leo Baekeland of Yonkers developed by systematic research the phenol-formaldehyde plastic, generally known as Bakelite. Other investigators in England and in European countries synthesized new plastics and evolved new processes for already known plastics, with the result that the plastics industry experienced a phenomenal growth. Its development especially during World War II has been most marked and startling in the United States.

Plastics industry is not only new and full of immense possibilities of future expansion, but is also of "a basic character well designed to mother innumerable offspring industries", for instance, the chemical industry, the cotton, wood-waste, soybean and oil industries, and the steel industry, as will presently be evident from the nature of the raw materials which enter into the making of synthetic plastics.

DEFINITION

To return to our original question of what is a plastic, it is not easy at the present moment to define it in simple terms. The difficulty is recognized by authors who attempt such a definition in standard treatises or text-books. From a scientific viewpoint, the subject is too young and has not yet been sufficiently investigated to lend itself to

a simple and clear formulation. "A plastic is a complicated material and even technical men in the industry differ as to the limits of the field. On the laboratory side, a highly trained chemist if not familiar with the special chemistry of plastics would have great difficulty today determining from a sample what it is made of."¹

However, a comprehensive description of a plastic is not difficult to offer. A plastic is a synthetic material. Its chief distinguishing component is the binder, namely the resin "which is a complex amorphous organic solid or semi-solid material (usually a mixture) built up by chemical reaction of comparatively simple compounds, approximating the natural resins in luster, fracture, comparative brittleness, insolubility in water, fusibility or plasticity, and some degree of rubber-like extensibility, but commonly deviating from natural resins in chemical composition and behavior with reagents".² At some intermediate stage in its production, it must be plastic, capable of being shaped, or liquid, capable of being cast. At some other subsequent stage it must be brought to a more or less rigid condition (hardens by cooling). A plastic frequently, though not always, achieves its final stage through a rearrangement of molecules by condensation or by polymerization (hardens by heat).

The distinguishing or specific component of a plastic, then, is the resinous binder which holds together all the other components incorporated in the plastic, which components however need not all of them always be present. These are: (a) the *fillers*, like wood flour, mica, metal powder, etc., which not only reduce the cost but impart to the finished product special mechanical properties; (b) the *plasticizers*, which render the brittle dry resins plastic or make them "flow" and therefore moldable; (c) the *hardeners*, which accelerate the final process of setting of the plastic; and (d) *pigments and dyes*, responsible for the beautiful colors and shades of tints which make the plastics so attractive and popular.

RAW MATERIALS

Plastics chemistry is concerned primarily with the preparation of the resins from easily available, cheap and simple organic compounds (organic as opposed to inorganic compounds which do not strictly speaking enter into the composition of modern synthetic plastics, as they do in the manufacture of cements and ceramics), such as phenols, formaldehyde, ureas, glycols, polyhydric alcohols, phthalic anhydride, vinyl chloride, acetate, etc.

Coal-tar is the largest single source of raw materials, from which are obtained phenol, benzene which can be catalytically converted into phenol, cresols and xylenols, naphthalene which can be oxidized to phthalic and maleic anhydrides, styrene, coumarones and indenes. Coal

¹ Simonds: Industrial Plastics, 1944. p. 4

² Plastics Catalog, 1944. p. 909

is another large source for calcium carbide from which can be prepared acetylene, the starting material for vinyl acetate and chloride, and for acetic anhydride. Petroleum is becoming increasingly important as a source of ethylene from cracked gases, used for the preparation of vinyl chloride, acrylic esters and styrene, not to mention butadiene and isobutylene for synthetic rubber. Cellulose from cotton linters and from wood pulp is the raw material for the numerous cellulose plastics.

POLYMERIZATION

The fundamental process by which these simple molecules are built up into very large compounds of molecular weights running up to hundreds of thousands required in order for the material to possess useful mechanical properties, is termed polymerization. Classical organic chemistry attributed this capacity of polymerize (by a process of self addition) only to molecules containing an unsaturated bond like $—C=C—$, $—C=O$, $—C=N$, so that the chemical composition of the resulting polymeric molecule would be the same as that of the starting simple molecule. In fact up to the classical work of W. H. Carothers, such was the generally accepted view of polymerizable substances. In a series of brilliant original researches, Carothers³ put forth a more comprehensive definition of polymerization in view of the inadequacy of the earlier definition when applied to many substances commonly accepted as polymers, both natural such as cellulose, silk fibroin and other natural proteins, and synthetic polymers like polypeptides, poly-oxymethylene, polyethylene glycol, etc. Polymerization was therefore defined as "any chemical combination of a number of similar molecules to form a single [large] molecule". This can be achieved either by simple self-addition, or by a process of condensation involving elimination of some simple molecule like H_2O , NH_3 , etc., so that one could speak of *addition polymerization* and *condensation polymerization*.

ADDITION POLYMERIZATION

Addition polymerization occurs essentially with molecules containing an unsaturated linkage. Thus ethylene can be made to polymerize into the industrially important polyethylene or polythene according to the formula:



Drastic treatment at high temperatures and pressures is needed to bring about this reaction. Substituted ethylenes, however, of the type $CH_2=CHX$ where X is a negative group like phenyl, carboxylic, ester, or a halogen atom, undergo polymerization much more readily.

³ W. H. Carothers: J. Am. Chem. Soc., 51, 2548 (1929)

For other references cf either Che. Reviews, 8, 353-426 (1931) or Collected Papers of W. H. Carothers on Polymerization, edited by H. Mark and G. S. Whitby, Vol. I, High Polymers. Interscience Publishers.

Examples of technically and industrially important polymers are polystyrene from styrene $C_6H_5-CH=CH_2$, polyvinyl chloride from vinyl chloride $CH_2=CHCl$, polyvinylidene chloride from vinylidene chloride $CH_2=Ccl_2$, polyacrylic esters from methacrylate $CH_2=C(CH_3)COOR$, polyvinyl acetate from vinyl acetate $CH_3-COO-CH=CH_2$, etc. This type of polymerization, called also vinyl polymerization because of the compounds generally undergoing it, is in practice carried out with the help of a catalyst, usually an organic compound capable of yielding free-radicals like benzoyl peroxide⁴ etc., though light, electrical discharge could be employed, and appears to be a chain process involving

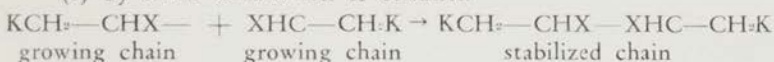
(a) activation of the monomer leading to an activated molecule having the character of a free-radical,



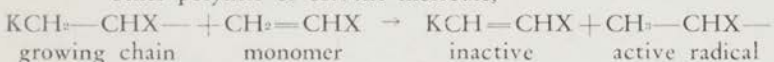
(b) followed by successive additions of monomer molecules,
 $KCH_2-CHX- + CH_2=CHX \rightarrow KCH_2-CHX-CH_2-CHX-$
 active radical monomer growing chain

(c) until the polymer is stabilized either

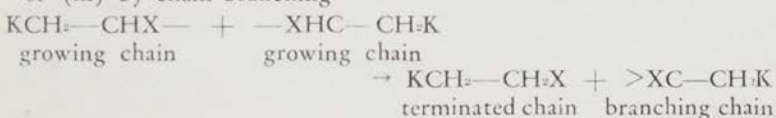
(i) by chain termination or cessation



or (ii) by chain transfer in which the activity is transferred to some other polymer or solvent molecule,



or (iii) by chain branching



It appears that the process (b) has a low activation energy of about 5000 cal per mole and is consequently repeated frequently about 1000 times in one hundredth of a second with the result that the chain grows to a full size, 1000 times the size of the monomer, in a comparatively short time, before the chain is terminated by a cessation step. The consequence of this is that the average chain length or the molecular weight of the polymer will be independent of time or extent of the polymerization, unlike in the other type of polymerization as will presently be seen.

⁴ Cf. H. Mark and R. Raff, *High Polymers* Vol. III, p. 199, ff.
 For mechanism see C. C. Price, *Some Aspects of the Mechanism of Addition Polymerization*, *Ann. N. Y. Acad. Sci.*, Vol. XLIV, Art. 4, p. 351.

POLYMERIZATION IN EMULSION AND SUSPENSION

The various modes of polymerization of vinyl derivatives need at least a passing mention. Polymerization or copolymerization may be carried out in homogeneous media, i.e., in bulk or in solution, as well as in heterogeneous media, i.e., in emulsion and in suspension. These two latter modes have become lately of great technical and practical importance, though their complete theoretical elucidation is not as yet available. Emulsion polymerization⁷ lends itself to easy control of temperature, proceeds at a rapid rate and yields very high molecular weight polymeric material. Its importance may be gauged by the fact that most of the butadiene-styrene copolymer rubber in the United States is being produced by this method. The method is being applied also in the production of plastic materials. Its main disadvantage, however, is that the polymer material is difficultly purified from the dispersing agent which is usually a soap. This particular disadvantage is avoided by polymerization in suspension. By this last method, polymeric material can be prepared in the form of pearls of varying size and degree of transparency, which can be easily purified. In consequence of this peculiar form, the name, pearl polymerization has been given to this method.⁸

CHAIN AND SPACE POLYMERS

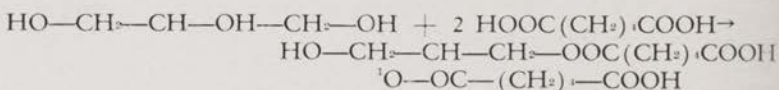
The polymer molecule resulting from either type of polymerization may have one of the two following general structures, namely a straight linear structure, or a highly cross-linked three dimensional structure. It is to be remarked that a linear chain molecule with some degree of branching may be assimilated to the straight linear type, whereas one with a high degree of branching may be regarded as approaching the second three dimensional type. This distinction is of practical importance and serves as a basis of classification of synthetic plastics and at the same time gives an insight into the geometry of the synthetic high polymers.

If the monomer has only two reactive groups, or only a single double bond, the polymer resulting therefrom either through addition or condensation process will essentially be a linear chain molecule, as can be seen from the examples given above. Polyesters from glycols and dicarboxylic acids, polyamides from similar acids and ω , ω' diamines will be linear molecules. Polyethylene, polyisobutylene, polystyrene, polyacrylic esters, polyvinylacetate, polyvinyl chloride, etc., are believed to consist also of linear chains, but with some degree of branching as can be seen from the nature of addition polymerization due to the possibility of branching during the reaction.

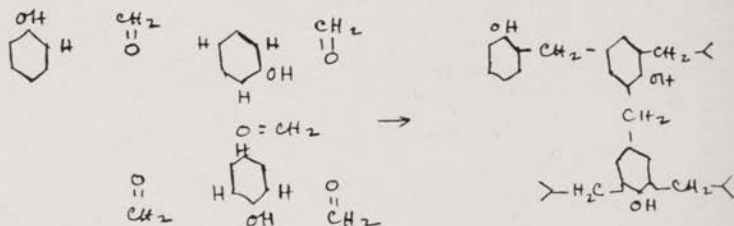
⁷ H. Mark and R. Raff, *op. cit.*, p. 81

⁸ For recent work, cf. C. F. Fryling and E. W. Harrington, *Ind. Eng. Chem.*, 36, 114 (1944). W. P. Hohenstein, F. Vignello and H. Mark, *India Rubber World*, 110, 291 (1944); W. P. Hohenstein, S. Siggia and H. Mark, *ibid.*, 111, 173 (1944); S. Siggia, W. P. Hohenstein and H. Mark, *ibid.*, 111, 436 (1945).

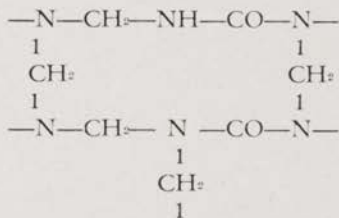
On the other hand, if the monomer or monomers are polyfunctional, namely contain more than two reactive groups, there is definite possibility of growth in the polymeric molecule in more than two directions in space, leading to complex three dimensional structure. Thus if glycerol, instead of glycol, is condensed with adipic acid, the reaction may be expressed by the formula



The secondary hydroxyl group in each glycerol molecule is a possible place to start a branch, and as the reaction proceeds, a very complicated three dimensional network will have been built up by successive stages of condensation. Similarly glycerol will condense with phthalic anhydride to yield valuable glyptal resins which make excellent varnish material because of the highly complex molecular network. Another interesting example is the phenol-formaldehyde resin. Phenol has three reactive H atoms in the benzene nucleus in the ortho- and para- positions to the OH group, and therefore is trifunctional with regard to condensation with formaldehyde. The reaction may be represented by



Urea, possessing four reactive H atoms, can condense with formaldehyde to give, also, complex molecular structures. It is interesting to note



that the soft butadiene, or butadiene-styrene copolymer, on being reacted with sulfur can be converted into stiff and hard rubber, ebonite, due to the many sulfur bridges established between adjacent linear chains at the places of the double bonds in the original soft material.

This division of synthetic high polymers into the linear and the complex cross-linked structures corresponds to that existing in the natural high polymeric materials such as cellulose, rubber, chitin, inulin, etc., which are mostly long chain or thread-like molecules, and keratin, lignin and others which possess rather three dimensional complex structures.

The physical properties of the two types of synthetic polymers are quite distinct and of technical importance. The chain polymers possess an indefinite capacity for swelling in suitable solvents and finally forming homogeneous solutions exhibiting properties of molecularly dispersed solutes. Chain polymers containing reactive side groups along the chain are capable of undergoing permutoid reactions in the swollen state, esterification, chlorination, saponification, etc., without apparently changing their average size or shape. Thus polyvinyl acetate can be hydrolyzed into the corresponding alcohol, which in turn can be condensed with aldehydes to give acetals. For instance, polyvinyl alcohol condenses with butyraldehyde to give the technically important polyvinyl butyral plastic which is used in the making of self-sealing gasoline tanks. Finally chain polymers exhibit a more or less defined softening zone of temperature below which the material possesses all the properties of an amorphous or quasi-crystalline solid, and above which temperature it is a viscous liquid. They are therefore typically *thermoplastic*, softening on heating, and on cooling setting to a solid, the process being indefinitely reproducible.

The net and space polymers show only a limited capacity to swell in a few solvents, but the material does not go into solution even to a slight extent. Phenol-formaldehyde, urea formaldehyde are examples. They do not undergo permutoid reactions, and are therefore chemically inert. They possess no definite softening zone of temperature; and at relatively high temperature, they decompose. Since heating below this decomposition temperature hardens them, they are termed *thermosetting*. Because of their compact and cross-lined structure, they possess a greater resistance to mechanical abrasion and heat than the thermoplastic materials, but are brittle, and consequently need in general an appropriate filling material like wood or some other flour for securing good impact strength and other mechanical properties, as well as a compounding material for molding.

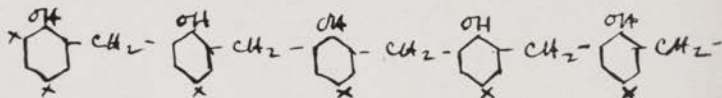
Based on the above considerations, the synthetic plastics can be divided into two main groups, namely thermosetting and thermoplastic substances.

Thermosetting plastics would comprise:

- (1) The phenolic group: phenol- and substituted phenol-formaldehydes, phenol-furfuraldehyde (Bakelite, Durite, etc.)
- (2) The urea group: urea- and thiourea-formaldehyde, melamine-formaldehyde (Bakelite urea, Beetle, Melmac, Catalin Melamine, etc.)

- (3) The alkyd group: glycerol and all types of polybasic acids chief of which is phthalic anhydride (Amberol, Beckacite, Teglac, etc.)

It may be of interest to note in connection with thermosetting resins, that they can be made, and in practice are made, to pass through the thermoplastic stage before they reach the final thermosetting state. Thus on reacting equimolecular amounts of phenol and formaldehyde under proper control of temperature and catalyst, the resin can be made to consist of long linear chain molecules of the type



But if to this more formaldehyde is added, condensation of the other reactive hydrogens marked with an asterisk can be achieved with consequent establishment of cross-links between the chains to give compact complex structures of the type already indicated. In view of this, it is still customary to speak of the A, B, C, stage resins in the case of phenol-formaldehyde. A-stage resin will be the initial condensation product, liquid or solid; B-stage will correspond to the intermediate product with a number of free methylol groups in the benzene rings constituting the chains, still plastic and moldable which under the action of heat and pressure in the mold is converted into the C-stage in which the finished molded article is obtained.

The thermoplastic resins would include:

- (1) The cellulose group: cellulose nitrate (Pyralin, etc.), acetate (Tenite I, etc.), butyrate, acetate-butyrate (Tenite II, etc.), and cellulose ethers (Ethocel, etc.)
- (2) The vinyl group: vinyl resins of all types, acetates (Vinylite), chlorides (Koroseal), alcohols, acetals (Alvar, Butvar, Saflex), methylmethacrylates (Lucite, Plexiglas), styrene, etc., vinylidene chloride (Saran)
- (3) The indene group: indenenes
- (4) The polyamide group: various types of simple polyamides, and mixed polyamides (Nylons)

In general no fillers are used with these resins. To improve their moldability, plasticizers are required. They take on and retain good colors, possess good optical properties and can be made sufficiently elastic and flexible. Also thermoplastic resins are the only ones which can be utilized for extrusion and the very economic and rapid injection molding, though sometimes, a thermosetting resin can be so modified by addition of thermoplastic material as to be used for injection molding, as has been recently reported in the case of phenolic resins.

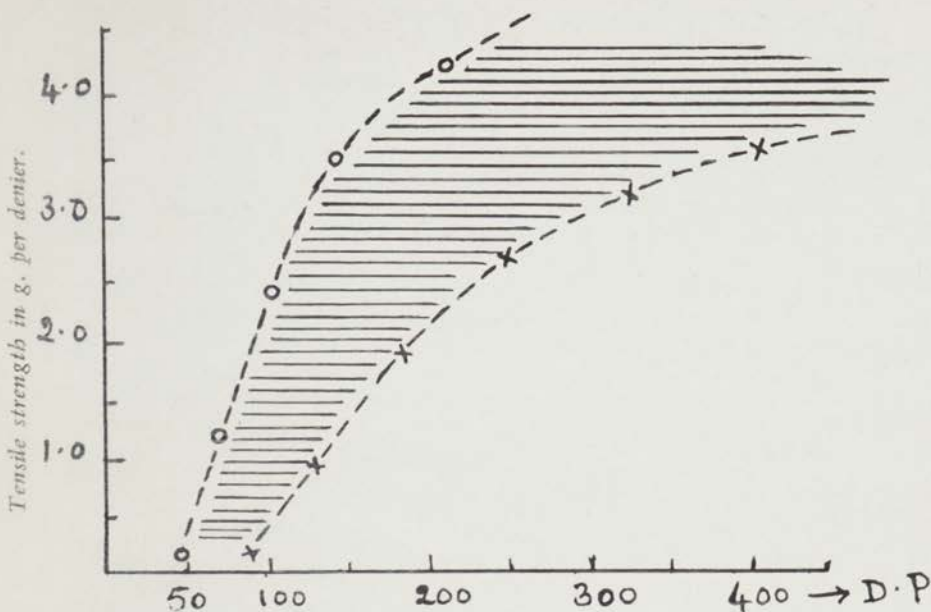


Fig. 1. Diagram connecting schematically tensile strength with average degree of polymerization.

FIBERS, PLASTICS AND RUBBERS

After this summary review of the chemistry of synthetic plastics, it seems advisable and instructive to indicate their relation to other natural and synthetic high polymers such as fibers and rubbers. From the study, during the past 50 years, of the structure of natural products like cellulose, starch, proteins, chitin and rubber, and of synthetic products, during the last ten years, a good deal of information has been secured concerning certain broad general relationships existing between the molecular and intermolecular structure and the mechanical properties of high polymeric substances.⁹

For any high polymeric substance to possess any mechanical strength at all, it must have a certain minimum degree of polymerization, D.P., which denotes the number of monomeric units in a polymer molecule. Above this critical value, the mechanical strength or tenacity of the material increases with increasing D.P. or chain length up to a limiting value of the D.P. beyond which the tenacity improves little with further chain length. These lower and upper limits of D.P. seem to be smaller for polyamides like nylon than for polyhydrocarbons like polythene and polystyrene. Figure 1 shows this relationship diagram-

⁹ Cf. H. Mark: Some Aspects of the Synthetic Rubber Problem, American Scientist, Vol. 31, No. 2, pp 79 ff. 1943.

matically. The curve with circles represents the relation for polyamides and the one with crosses holds for polyhydrocarbons. The shaded area between the two lines indicates the relationship for all other high polymers such as cellulose esters, polyvinyl derivatives.

Besides the average chain length, the uniformity of the chain length in a given high polymer material seems to have some not yet quite clarified influence on the mechanical properties. Any actual synthetic polymer usually consists of molecules of varying chain lengths or molecular weights according to well known distribution curves.¹⁰ The more uniform the chain lengths of the molecules composing a sample, the better the mechanical properties of the material. It appears that a comparatively small amount (between 10 to 15 percent by weight) of constituents having a D.P. below 150 is definitely detrimental to the mechanical properties of the sample, so that removal of such constituents will greatly improve the value of the material.

The effects of a high degree of branching or of cross-linking between the chains of high polymers have already been described. Their effect is a definite stiffening of the material and rendering it thermosetting, whereas their absence leads to a thermoplastic material.

Finally of utmost importance with regard to the mechanical properties of high polymers are the magnitude and the geometrical arrangement of the intermolecular forces between the chains. These intermolecular forces may be due to polar groups in the chains or hydrogen bonding, and the geometrical arrangement may lead to a good fitting of the chains into some sort of regular crystalline pattern, or at least in certain regions along the chains, as is now generally accepted. If *the molecular forces of attraction between chains are small and the chains fit badly into a lattice, the material, at a given temperature and stress, will always have the tendency to return to its original state and therefore will behave as a typical rubber.* Example, butadiene-styrene copolymer. If *the forces are strong, and the regular structure of the chains provides for easy fitting into the lattice, a state of crystallinity is favored and the material will be a typical fiber.* Example, cold-drawn nylon filament, oriented polyvinylidene chloride fiber. Finally, if *the forces are moderate and the geometry of chains is only moderately favorable for crystallization, the material will behave as a typical plastic.* Such are the thermoplastics which have been described.

CONCLUSION

In conclusion it may be stated that as soon as a certain degree of polymerization is reached, usually above 400 or 500, the synthetic material acquires sufficient useful mechanical strength which in special cases might be enhanced by further increase of polymerization in particular cases. The more intimate properties such as softness, flexibility,

¹⁰ Cf. Mark, *ibid.*, pp 105-110

elasticity, etc., depend upon the magnitude and distribution of intermolecular forces between the chains, which are a function of the chemical nature of the constituent monomeric units of the chains. If they are large and regularly distributed along the chains, the material will exhibit a decided tendency to crystallize and therefore be of a fiber type. If the forces are small and (or) irregularly distributed, the material will behave as a typical rubber. Intermediate cases are characteristic of plastic materials. It is this "deeper scientific understanding of how structure determines properties", as Professor H. Mark has remarked, which will enable us "to quit copying natural materials slavishly and without imagination", and to design and create "new types of materials with properties, never before obtained properties, which for certain given purposes excel considerably the qualities of natural materials". Herein lies the secret of the future of synthetic plastics as well as of all synthetic high polymers.

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MATHEMATICS

THE EXPONENTIAL SOLUTION OF THE EQUATIONS OF THE OSCILLATOR

By ROBERT O. BRENNAN, S.J.

In the *Jesuit Science Bulletin* of December 1937 (vol. xv, no. 2) Fathers Love, S.J., and Quigley, S.J., treat of simplifications of trigonometric problems by the use of complex exponentials. We shall apply the same technique to the solution of the differential equations of electric oscillators. This type of solution may be applied with convenience and symmetry to a number of problems, and it is thought that a single treatment of them with a consistent terminology will be useful.

The equations may also, by a change of constants, be applied to mechanical and acoustical oscillators. Mr. Bezuska, S.J., has summed up the relations of the constants in the three oscillators in the December, 1942 *Bulletin* (vol. xx, no. 2).

We shall make use of the following definitions which may be found in the article of Fathers Love and Quigley:

$$\begin{aligned} \sin x &= x - x^3/3! + x^5/5! - x^7/7! + \dots \\ \cos x &= 1 - x^2/2! + x^4/4! - x^6/6! + \dots \\ \exp z &= 1 + z + z^2/2! + z^3/3! + z^4/4! + \dots \end{aligned}$$

From these are derived:

$$\exp(jx) = \cos x + j \sin x \tag{A}$$

$$\exp(-jx) = \cos x - j \sin x \tag{B}$$

and

$$\sin x = \frac{\exp(jx) - \exp(-jx)}{2j}; \quad \cos x = \frac{\exp(jx) + \exp(-jx)}{2}$$

We shall also require definitions of the hyperbolic functions and relations analogous to the above that flow from their definitions.

$$\sinh x = x + x^3/3! + x^5/5! + x^7/7! + \dots$$

$$\cosh x = 1 + x^2/2! + x^4/4! + x^6/6! + \dots$$

$$\exp(x) = \cosh x + \sinh x \tag{C}$$

$$\exp(-x) = \cosh x - \sinh x \tag{D}$$

$$\cosh x = \frac{\exp(x) + \exp(-x)}{2}; \quad \sinh x = \frac{\exp(x) - \exp(-x)}{2} \tag{E}$$

We shall solve first the homogeneous equations of the free oscillator, undamped and damped. Then we shall apply the same method to find a particular integral of the complete equation of the oscillator with an impressed sinusoidal electromotive force. The first two solutions will be treated in the present paper; the third will be the subject of a separate paper.

THE FREE UNDAMPED OSCILLATOR:

The circuit consists of a self-inductance (L henries), a capacitance (C farads), and a key in series. The resistance is negligible. The condenser is given an initial potential (E' volts) and the key is then closed. The voltage drops are then that of the condenser, q/C volts, and that of the inductance, LD^2q volts, where q is the charge, and D^2q is the second time derivative of q . By Kirchhoff's Voltage Law: "If in an electric network a closed path is traversed, the algebraic sum of the voltage drops across the individual elements in the direction of traversal is zero,"

$$LD^2 + q/C = 0 \quad (1)$$

This is a homogeneous linear differential equation of the second order with constant coefficients. Its solution will involve two arbitrary constants. We shall assume a solution of the form

$$q = \exp(mt) \quad (2)$$

where m is a constant whose value will be determined so that the assumed solution, equation (2) satisfies equation (1).

By differentiation of equation (2) we obtain:

$$Dq = m \exp(mt) ; D^2q = m^2 \exp(mt)$$

Substitution of these values for q and D^2q into equation (1) yields:

$$(Lm^2 + 1/C) \exp(mt) = 0. \quad (3)$$

In order that equation (3) be true for all values of t , the coefficient must be 0. Hence we have: $m = \pm \sqrt{-1/LC} = \pm j \sqrt{1/LC}$

Or $m = \pm j\omega$, where $\omega = \sqrt{1/LC}$. Then particular solutions of equation (1) are: $q = \exp(j\omega t)$; $q = \exp(-j\omega t)$. The general solution of equation (1) consists of the sum of these two particular solutions multiplied by arbitrary constants:

$$q = a \exp(j\omega t) + b \exp(-j\omega t) \quad (4)$$

The arbitrary constants a and b may be real or complex. We shall consider them as complex, but since we need only two arbitrary constants, not four as would be contained in the complex numbers: $a = a' + ja''$, $b = b' + jb''$, we let a and b be conjugates, i.e., $a = a' + ja''$, $b = a' - ja''$.

The solution involves imaginary exponentials, which demand further analysis before we can interpret the equation physically. Ap-

plying equations (A) and (B), and collecting terms we have equation (4) in the form: $q = (a + b)\cos \underline{wt} + (a - b)j \sin \underline{wt}$. Since a and b are conjugate complex numbers, $a + b = 2a'$, $(a - b)j = -2a''$. Setting $A = 2a'$, $B = -2a''$, we have our final solution in its first form:

$$q = A \cos \underline{wt} + B \sin \underline{wt} \quad (5)$$

Another form of the solution may be obtained by multiplying and dividing equation (5) by $\sqrt{A^2 + B^2} = C'$. The terms A/C' and B/C' may now be taken as the cosine and sine of some angle, say c' , since the sum of their squares is 1. Making use of the trigonometric identity: $\cos x \cdot \cos y + \sin x \cdot \sin y = \cos(x - y)$, we obtain as the other form of the solution:

$$q = C' \cos(\underline{wt} - c') \quad (6)$$

The relations of the constants in the two forms of the solution are: $C' = \sqrt{A^2 + B^2}$, $c' = \arctan B/A$.

The equation for the current may be obtained by differentiating the solution. We shall choose the form (5). Then

$$Dq = i = -\underline{Aw} \sin \underline{wt} + \underline{Bw} \cos \underline{wt} \quad (7)$$

From equations (5) and (7) we can evaluate the arbitrary constants in terms of the initial conditions (i.e., $t = 0$) of the problem. Let the initial value of the charge be \underline{q} , that of the current be \underline{i} . Then for $t = 0$, in equations (5) and (7):

$$A = \underline{q} ; B = \underline{i}/\underline{w}.$$

The period (T) of the oscillator is the time between successive repetitions of its motion. When, therefore, the argument of the sine and cosine functions of equation (5) changes from \underline{wt} to $\underline{w}(t' + T)$, the motion repeats itself. Since we know that the trigonometric functions repeat their values when their argument changes by 2π radians, then $\underline{w}(t' + T) = \underline{wt} + 2\pi$, and

$$\underline{T} = 2\pi/\underline{w} = 2\pi\sqrt{LC}$$

The frequency, $n = 1/T = \underline{w}/2\pi$.

THE FREE DAMPED OSCILLATOR:

The circuit is the same as in the case of the undamped oscillator, except that the resistance (R ohms) is not negligible. Application of Kirchhoff's law gives as the differential equation of the circuit:

$$LD^2q + RDq + q/C = 0 \quad (8)$$

The solution proceeds as before. We assume the solution, $q = \exp(mt)$, calculate the derivatives and substitute in equation (8). Then

$$(Lm^2 + Rm + 1/C)\exp(mt) = 0. \quad (9)$$

As before, the coefficient must be 0. Placing this condition on m , we obtain

$$m = -R/2L \pm \sqrt{(R/2L)^2 - 1/LC} \quad (10)$$

and the solution of equation (8) is

$$q = a \exp(m't) + b \exp(m''t) \quad (11)$$

where m' is m as defined by equation (10) with the plus sign before the radical; m'' with the minus sign, and a and b are arbitrary constants.

This solution can take three forms, according as m , as defined in equation (10), has complex values, a real repeated value, or is real and double-valued; that is, according as the discriminant $(R/2L)^2 - 1/LC$ is less than, equal to, or greater than 0.

CASE I: $(R/2L)^2 - 1/LC < 0$. (Under-damped). In this case m is complex. We define $w = \sqrt{1/LC - (R/2L)^2}$ and rewrite equation (11):

$$q = a \exp(-Rt/2L + jwt) + b \exp(-Rt/2L - jwt)$$

$$\text{whence } q = \exp(-Rt/2L) [a \exp(jwt) + b \exp(-jwt)] \quad (12)$$

The terms here in the brackets are of the same form as the right hand of equation (4) which we reduced to the forms of equations (5) and (7). Equation (12) may therefore be written:

$$q = \exp(-Rt/2L) (A \cos wt + B \sin wt) \quad (13)$$

$$\text{or } q = C' \exp(-Rt/2L) \cos(wt - c') \quad (14)$$

where the relations $C' = \sqrt{A^2 + B^2}$, and $c' = \arctan B/A$ are still true.

The equation for the current is from equation (13):

$i = Dq = \exp(-Rt/2L) [(Bw - Ra/2L) \cos wt - (Aw + RB/2L) \sin wt]$. The constants A and B in terms of the initial charge and the initial current are: $A = \underline{q}$; $B = (1/w) (\dot{\underline{q}} + Rq/2L)$ (15)

From equations (13) and (14), we see that in this case the discharge is oscillatory with the amplitude dying away as the quantity $\exp(-Rt/2L)$ decreases. The notion of period as the time for repetition of motion is not verified here, as the damped oscillator never repeats its motion. We may define the period as the time interval between successive maxima, provided that this time interval be constant.

The maxima are determined by setting the first time derivative of q equal to 0. Doing this in equation (14), we find that the condition on t for maximum values of q is

$$t = c'/w - (1/w) \arctan R/2Lw \quad (16)$$

Since the arc tangent is multivalued, the time interval between successive maximum values of q will be constant and we have:

$$T = (2\pi)/w = 2\pi/\sqrt{1/LC - (R/2L)^2} \quad (17)$$

CASE II: $(R/2L)^2 - 1/LC = 0$. (Critically damped). In this case we have but one value for m . The solution in equation (11) reduces to $q = A \exp(-Rt/2L)$, where $A = a + b$. This solution contains but a single arbitrary constant while the second order differential equation (8) requires two in its general solution. To obtain another particular solution, then, we multiply the trial solution (2) by t , and proceed as in the previous cases. Then

$$q = t \exp(mt) ; Dq = mt \exp(mt) + \exp(mt)$$

$$D^2q = m^2t \exp(mt) + 2m \exp(mt)$$

Upon substituting into equation (8), we obtain

$$(Lm^2 + Rm + 1/C) \exp(mt) + (2Lm + R) \exp(mt) = 0 \quad (18)$$

The general solution in the second case is therefore:

$$q = A \exp(-Rt/2L) + Bt \exp(-Rt/2L) \quad (19)$$

where A and B are arbitrary constants. The current equation may be obtained by differentiation of equation (19). There is no oscillation but a gradual return to O .

CASE III: $(R/2L)^2 - 1/LC > 0$. (Over-damped). In this case there are two real values for m . Equation (11) becomes

$$q = \exp(-Rt/2L) [a \exp(pt) + b \exp(-pt)]$$

where a and b are arbitrary real constants, and p is defined as $\sqrt{[(R/2L)^2 - 1/LC]}$. Here we transform the equation by use of equations (C) and (D). Then $q = \exp(-Rt/2L) [(a + b) \cosh pt + (a - b) \sinh pt]$. Setting $A = a + b$, $B = a - b$, we have as one form of the final solution in the third case:

$$q = \exp(-Rt/2L) \cdot (A \cosh pt + B \sinh pt) \quad (20)$$

Equation (20) may be obtained in another form by multiplying and dividing by $\sqrt{A^2 - B^2} = C'$. The difference of the squares of the coefficients of $\cosh pt$ and $\sinh pt$ is then equal to one. Application of the formula $\cosh^2 x - \sinh^2 x = 1$, which may be verified by the definitions (E), leads to

$$q = C' \exp(-Rt/2L) \cdot (\cosh pt \cdot \cosh c' + \sinh pt \cdot \sinh c').$$

And again, the relation $\cosh x \cdot \cosh y + \sinh x \cdot \sinh y = \cosh(x + y)$ leads to

$$q = C' \exp(-Rt/2L) \cosh(pt + c') \quad (21)$$

where $C' = \sqrt{A^2 - B^2}$ and $\tanh c' = B/A$.

Again the solution is non-oscillatory, and the charge gradually approaches O . The equation for the current may be found by differentiation of equation (20) or (21), and the constants evaluated in terms of the initial conditions.

ON THE EVALUATION OF $\int_a^b x^{-1} dx$

By T. F. MULCRONE, S.J.

The student of elementary integral calculus is frequently faced with a serious difficulty concerning the formulas

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad (n \neq -1), \quad (1)$$

and
$$\int x^{-1} dx = \log x + c. \quad (2)$$

These closely related formulas are generally presented as independent entities with no other correlation than the statement that (1) is not valid when $\underline{n} = -1$, for in this case the right member involves division by zero. Nor is the intimate connection between (1) and (2) brought out when, regarding integration as the inverse of differentiation, (2) is established by integrating each side of the equation

$$d(\log x + c) = x^{-1} dx,$$

the proof ordinarily given in text-books.

The following simple and direct derivation of (2) from (1) employs the notion of limit and the evaluation of indeterminate forms, with which the student is already familiar. The fact that the definite integral is employed will require that the presentation of this derivation in the classroom be deferred for some time until the definite integral has been mastered. But the instructor will do well to point out at the beginning that (2) is really a limit case of and intimately connected with (1), and that these facts will be demonstrated in class as soon as the constant of integration (C) has been incorporated in the definite integral.

The problem therefore is the derivation of the formula

$$\int_a^b x^{-1} dx = \log x \Big|_a^b, \quad (0 < a < b < \infty),$$

by means of the general formula

$$\int_a^b x^n dx = \left. \frac{x^{n+1}}{n+1} \right|_a^b. \quad (3)$$

Passing to the limit, for $n = -1$, we have

$$\int_a^b x^{-1} dx = \lim_{n \rightarrow -1} \int_a^b x^n dx = \lim_{n \rightarrow -1} \left. \frac{x^{n+1}}{n+1} \right]_a^b \quad (4)$$

$$= \lim_{n \rightarrow -1} \frac{b^{n+1} - a^{n+1}}{n+1}$$

$$= \frac{b^0 - a^0}{0} \quad (5)$$

where the right member of (5) evidently assumes the indeterminate form $0/0$ for all finite non-zero values of a and b .

The limit may be determined by applying L'Hospital's Rule to (4), giving us

$$\int_a^b x^{-1} dx = \lim_{n \rightarrow -1} \left. \frac{D_n(x^{n+1})}{D_n(n+1)} \right]_a^b$$

$$= \lim_{n \rightarrow -1} \left. \frac{x^{n+1} \log x}{1} \right]_a^b$$

$$= \log x \Big|_a^b, \quad (0 < a < b < \infty).$$

Q.E.D.

CHEMISTRY for October, 1945 prints a seven page summary of German Chemical Developments. The article uses such headings as From Water, Air and Coal, Acetylene Utilization, Surface Active Chemicals, Tanning Agents, Polymers, Electrochemicals, Rocket Propulsion Chemicals, Lubricants and Fuel Additives and Substitutes.

The organic chemistry laboratory course at Clark University has a unique roster of organic preparations, some of which are of the continuous type and smack of actual syntheses in industry! Thus agitation and stirring motors become a problem. This has been solved by means of a long shaft with many pulleys on it and driven by a single motor. Students are warned to disengage their apparatus from the driving pulley when quitting laboratory. If a driving cord gets caught in the pulley during some subsequent class in a student's absence, he might as well take up glass-blowing.

OURS IN THE NEWS

A brief obituary notice of Fr. Schmitt, S.J., appears in the September 10th Issue of *Chem. and Eng. News*. The Editor regrets that an appreciation of Fr. Schmitt cannot appear in this Issue of the Bulletin, but it seemed advisable to delay such a notice until a record worthy of Fr. Schmitt's outstanding contributions both to this BULLETIN and to the American Association of Jesuit Scientists could be made. This obituary will appear in the December Issue of this BULLETIN.

The September 14th Issue of *Science* and the September 25th Issue of *Chem. Eng. News* note Fr. Yeddanapalli's return to St. Joseph's College, Trichinopoly, India. Both note Fr. Yeddanapalli's work in High Polymer Research at the Polytechnic Institute of Brooklyn.

The *J. Chem. Ed.* for June 1945 publishes an article by Fr. Fiekers and Mr. Gibson of the Holy Cross Chemistry Department "Illustration of the Gas Laws, A Dynamic Model for Statistical Phenomena." The authors have demonstrated the apparatus and lectured on its use before many school groups, including the New England Association of Chemistry Teachers.

Fr. McGuinn, head of the Chemistry Department of Boston College had published a note on "Collection of Spilled Mercury" in the September 1945 Issue of *J. Chem. Ed.* Fr. McGuinn also gave a paper at the Seventh Summer Conference of the New England Association of Chemistry Teachers held at Massachusetts State College, Amherst, Massachusetts. His subject was "Interpretation of Titration Curves." Fr. Fiekers of Holy Cross, and Fr. Hutchinson of Portland also attended the conference.

A new course entitled "The Principles of Natural Science" got under weigh with the term commencing November 1, 1944 at Holy Cross College, Worcester, Mass. Its purpose is to acquaint students with the method and spirit of modern science and to relate science to religion, philosophy and art. It is an intensive 8 credit course and provides time for laboratory or "workshop." At present its outline is in tentative form. It starts with Genesis, has masterpieces from all the established sciences and closes with a period of "research." In organization it is practically independent of the different science departments. Its development will be interesting to follow.

