CONTENTS

Obituary:
Francis W. Power, S.J. (1893-1944) ......................................................... 68

Chemistry:
The Limits of Error in Volumetric Analysis, with Emphasis on Back Titration.
Rev. Bernard A. Fiekers, S.J.,
Holy Cross College, Worcester, Mass. ...................................................... 71

A Survey of Publication Activity in the Chemistry Department of the College of the Holy Cross.
Rev. Bernard A. Fiekers, S.J.,
Holy Cross College, Worcester, Mass. ...................................................... 77

Mathematics:
The Possible Right Spherical Triangles.
Rev. Walter J. Miller, S.J.,
Woodstock College, Woodstock, Md. ....................................................... 81

Application of the Multinomial Theorem.
C. Frederick Koehler, S.J.,
Weston College, Weston, Mass. .............................................................. 83

Meteorology:
The Pioneer Forecasters of Hurricanes
Rev. Walter M. Drum, S.J. ........................................................................... 87
OBITUARY

FR. FRANCIS W. POWER, S.J.
1893—1944

It is not an ordinary occurrence for the truly professional scientist to divide his energies in heterogeneous fields, and yet, devotees of various branches of the Arts and Sciences met recently on a common ground to mourn the passing of Father Francis W. Power, S.J. Father Power's wide circle of friends and acquaintances testify that he lit tapers at the shrine of many Arts and Sciences. Although his first and most cultivated devotion was to chemistry, nevertheless his interests were as diversified as the contents of a small boy's pocket. He would discuss with equal glibness "H.M.S. Pinafore," the Mean deviation, ballistics, or the habitat of a meadow flower. Inquisitive, alert, observant and blessed with a retentive memory, his excursions into foreign fields broadened his view and deepened his knowledge much beyond the point of superficiality. While his knowledge of some subjects could not be considered profound, nevertheless it was rarely inaccurate. From his high school days, when the cellar of his home was his laboratory, to the last days of his life he was primarily a student and a scholar.

Francis W. Power, S.J., was born June 19, 1893, in Worcester, Mass., where he graduated from the local public high school. He entered Holy Cross college in 1911, and there worked under the late Father George L. Coyle, S.J., until he graduated in 1913. His proficiency and interest in chemistry won for him the respect and encouragement of Father Coyle. This association developed into a friendship which was to influence his whole future in the field of chemistry. With a thorough foundation in chemistry, physics, and mathematics, he entered Clark University in September, 1915. There he came under the influence and guidance of another well known chemist, Dr. Charles A. Kraus. After receiving his Master's degree in 1917 Francis Power entered the industrial field as assistant chief chemist at the DuPont Powder factory, Carney's Point. Here he remained until the end of World War I. In 1919 he entered the Jesuit novitiate, at Yonkers. After his philosophy courses at Woodstock, he was assigned to Fordham University, in 1924, to teach quantitative analysis. He enrolled in the graduate school and carried on advanced studies in chemistry under Dr. Carl P. Sherwin. He completed his doctorate problem on "The quantitative Determination of Phenylacetylglutamine," and was granted the degree of Doctor of Science in 1926; he was ordained a priest at Weston College in 1929.
During theological studies Father Power acted as an instructor of chemistry and in his spare moments continued research in Analytical Physiological Chemistry. In 1930 he returned to Fordham to become acting head of the Chemistry Department. During this year he supervised the remodeling of the Chemistry Building. After a year of tertianship at Poughkeepsie, he returned to Fordham as Professor of Analytical Chemistry and, since 1933, he was Professor of Micro Chemistry. He remained at Fordham as an active member of the faculty until about eighteen months before his death, when poor health forced him to leave the classroom.

Father Power found time during these eleven years to participate in the activities of various chemical societies, as a member, as an officer and as a referee of various publications. He was very active in the first microchemical society formed in this country, and later became the first president of the Metropolitan Microchemical Society. For the last three years of his life he was Secretary-Treasurer for the Division of Analytical and Micro Chemistry of the American Chemical Society.

A host of students will testify that in the classroom and in the quantitative laboratory more than in any other places Father Power made a lasting impression. His offhand manner of delivery, coupled with patience and simplicity, made him an excellent teacher. He often worked side by side with his students in their analyses, and his patience and cooperation engendered a liking for a subject so unattractive to many.

Years of experience and a firsthand knowledge of the imperfections of current texts provoked Father Power to write a new book on quantitative analysis. He actually got as far as the treatise on "Sources of Error," in the third chapter. Dissatisfied with the usual treatment of this topic, he began to study the subject further with the result that he became intensely interested in the whole field of mathematical statistics. For the next four or five years he read and studied extensively. He visited and corresponded with many authorities in the field, and in turn they often discussed with him their papers and contributions before publishing. He spoke frequently before various scientific groups on mathematical statistics, and wrote several papers for the journals on the treatment of chemical data in the light of statistics. He was consulted by industrial firms on problems relating to the interpretation of data obtained from research work. Equipped with the acumen of a philosopher and scientist, Father Power had the ability of discussing abstruse notions of statistics in simple, understandable language. His investigations were so thorough and painstaking in a subject where authorities often disagree, that he passed on to a better life without completing the third chapter of his projected text.
Even the most ardent admirer would not consider Father Power a genius. Few would regard him as brilliant but all who know him will admit that in his death, education is deprived of an excellent teacher, Science an industrious and talented scholar, the Catholic Church a zealous priest and the Society of Jesus a devoted son.

Father Power's publications as listed in Chemical Abstracts Author Index through 1942 are listed as follows:


Father Power contributed to The RETORT, AMERICA and the JESUIT SCIENCE BULLETIN.

A short vita of Father Power is to be found in:

J. B. M.

Correction. Jesuit Science Bulletin. Vol. XXII, Dec. 1944, p. 37. Ninth line from bottom of page: "Motion is a continuous achievement . . . " should read: "Motion is a continuous achievement from an initial term to an end. Successive continuity and duration are the qualities of motion."
Many of the stoichiometrical relationships in volumetric analysis may be expressed as multiplication or division problems:

\[ U = \frac{(x)(y)}{(z)} \]  

Thus they readily lend themselves to logarithmic differentiation:

\[ \frac{dU}{U} = \frac{dx}{x} + \frac{dv}{y} - \frac{dz}{z} \]  

If increments are used instead of differentials, “d” can be replaced by “delta”, which is omitted here for typographical reasons. These increments can be used to express the margin of possible error in measurements. In equation (2), the terms to the right give the relative possible error for direct measure; the term to the left refers to indirect measure. Such deviations are of their nature indifferent to algebraic sign, but if all signs are written alike, an equation for cumulative error is had:

\[ \frac{dU}{U} = \frac{dx}{x} + \frac{dv}{y} + \frac{dz}{z} \]  

Many sources of error may enter such analyses. Here we are exclusively concerned with the error limits in the use of the burette, pipette, measuring flask and chemical balance, not so much for the sake of the calculations that can be made, as for the sake of laying a rational foundation for good volumetric technique. These principles may then be extended to work on other instruments.

**1. COMPARISON OF SOLUTIONS:** In comparing the strength of two solutions (titrant, sub t; back-titrant, sub b) we may not know the absolute concentration of either. Their relation is given by:

\[ v_t = kv_b \]  

where the v terms indicate the volumes used in the determination and k is the ratio of concentration of back-titrant to titrant. The possible error is given by:

\[ \frac{dk}{k} = \frac{dv_t}{v_t} + \frac{dv_b}{v_b} \]
k is a constant of indirect physical measure and thus it is subject to error in its determination. Equation (4) could be written:

\[ v_b = k' v_t \]  

(6)

where

\[ k' = 1/k \]  

(7)

Then likewise

\[ \frac{dk'}{k'} = \frac{dv_t}{v_t} + \frac{dv_b}{v_b} \]  

(8)

and

\[ \frac{dk}{k} = \frac{dk'}{k'} \]  

(9)

but

\[ dk' \neq dk \]  

(10)

unless k has the value of unity or is assumed to be sufficiently close to unity. The relative errors in these measured constants are equal to their reciprocals; the absolute errors are not necessarily equal.

To illustrate this treatment—, in a standard 50 ml. burette, the best approach to true volume reading that we can make is usually assigned the value 0.05 ml. When we compare the strengths of two reacting solutions so that only about 10 ml. of each are used, then:

\[ \frac{dk}{k} = \frac{0.05}{10} + \frac{0.05}{10} = 0.01 \]  

(11)

or the relative possible percent error in measuring k with be 1%. Using larger volumes, however,

\[ \frac{dk}{k} = \frac{0.05}{25} + \frac{0.05}{25} = 0.004 \]  

(12)

or four parts per thousand; where the volumes are 50 ml., the usual two parts per thousand ideal would be assured. Data from the comparison of solutions are used in secondary standardization and in backtitration. It is important then to reduce the margin of error in this determination.

II. STANDARDIZATION OF SOLUTIONS: In standardizing either of the solutions directly we call upon the general relationship:

\[ c_t = \frac{w_s}{V_t} \]  

(13)

where \( w_s \) is the weight of standard; \( V_t \), the volume of titrant required; and \( c_t \) is a term for the concentration of titrant which generally has to be transformed to chemical units by the use of appropriate chemical factors. For example, in the standardization of hydrochloric acid by a well known method:
\[
N_{\text{HCl}} = \frac{(w_{Na \text{ CO}_{3}} \times N_{\text{HCl}})}{(MEW_{Na \text{ CO}_{3}} \times V_{\text{HCl}})}
\]

where, \(N_{\text{HCl}}\) is the normality of hydrochloric acid; \(V_{\text{HCl}}\), the neutralization volume determined by titration; \((w_{Na \text{ CO}_{3}})\) the weight of primary standard, and, \(MEW_{Na \text{ CO}_{3}}\) the milliequivalent weight of anhydrous sodium carbonate, the primary standard. The value of the latter is 0.05300 and this is generally regarded as invariable. The outer limits of error in equations (13) and (14) are then given by:

\[
\frac{dc_t}{c_t} = \frac{dN_t}{N_t} = \frac{dV_t}{V_t} + \frac{dw_s}{w_s}
\]

Sometimes the last term in equation (15) to the right may be neglected. In that case it is tacitly assumed that the weight determination was precise enough to approach constancy and could be neglected. This is neither necessary nor is it always true. Thus this extra term should be kept in mind. Considering, however, that the relative error in the weight determination may be neglected, we may rewrite equation (15) as follows:

\[
\frac{dc_t}{c_t} = \frac{dV_t}{V_t}
\]

This expression has been derived elsewhere \(^1\) for students who are not familiar with the calculus.

**III. CALCULATION OF CONCENTRATION OF BACK-TITRANT:** By the foregoing methods the concentration of back-titrant can now be calculated:

\[
V_t N_t = V_b N_b
\]

wherein the concentration terms \(c_t\) and \(c_b\) cannot be used for normality \(N\) directly, but are converted to \(N\) by dividing in each case by the proper milliequivalent weight value. Whereas:

\[
k = \frac{v}{v_b}
\]

---

\(^1\) THIS BULLETIN, 12, 96 - 97 (1934)
then by equations (4) and (17):
\[ N_b = k N_t \]  
(18)
on differentiation etc., the relative error is given by:
\[ \frac{dN_b}{N_b} = \frac{dk}{k} + \frac{dN_t}{N_t} \]  
(19)
the value of \( N_b \) being less precise than \( N_t \) because the relative error in \( k \) enters into its determination. The determination of \( N_b \) from a primary standard of its own, however, would enhance its precision.

IV. ANALYSIS OF SAMPLE: A stoichiometrical relationship that is often used for determination or analysis of sample is given by:
\[ \%c = \frac{(V_t)(N_t)(\text{MEW}_c)(100)}{(W_s)} \]  
(20)
where \( \%c \) is the percentage value of the constituent sought; \( V_t \), the volume of titrant used in the determination; \( N_t \), the normality of the titrant; \( \text{MEW}_c \) the milliequivalent weight of the constituent sought; and \( W_s \), the weight of the sample. The relative error is given by:
\[ \frac{d\%c}{\%c} = \frac{dV_t}{V_t} + \frac{dN_t}{N_t} + \frac{dW_s}{W_s} \]  
(21)
the other factors in equation (20) being considered constant. The relative error in concentration is already a composite of two terms by equation (15). If partition of the sample with pipette (sub p) and measuring flask (sub m) are included in the procedure of analysis, equation (20) is then adjusted to:
\[ \%c = \frac{(V_t)(N_t)(\text{MEW}_c)(100)(V_m)}{(W_s)} \]  
(22)
and the terms:
\[ + \frac{dV_m}{V_m} + \frac{dV_p}{V_p} \]  
(23)
are added to equation (21).

After inspecting this equation and seeing the accumulation of the possibility of error, one might be sceptical about the accuracy of any analysis. The principle of indifference to algebraic sign, however, must be remembered. Accuracy, viz. the discrepancy between the determined results of an analysis and the true composition of the sample as given in numerical terms, now gives way to precision, viz. the mere probability of having our results express the true composition.
when the procedure is designed as an approach to the true. It is possible that each of the terms in equations (21) and (23) might have algebraic signs that alternate positive and negative. It is improbable that all effects will cancel exactly. But the precision seems in general to be enhanced as the number of terms increases. This is not stated in order to advise an increase in the number of terms in a given experiment by the introduction of partitioning procedures and the like. It is far wiser to increase the number of terms by repeating the experiment, and keeping terms both with regard to their absolute value and in regard to their number as well, in any given experimental procedure, at a minimum.

V. BACK-TITRATION AND INDIRECT ANALYSES: Throughout this discussion little has been said of corrective procedures, such as the use of titrant in the determination of blank, of the back-titrant in the restoration of an end-point that has been passed by, and in indirect analysis such as the commoner Kjeldahl procedure. Stoichiometrical formulations for such procedures are many and varied. All such cases, however, can be grouped together and treated "per modum unius" by merely substituting a correction for the volume terms in equations (14), (20) and (22) and noting the effect on the error equations (15), (21) and (23) respectively. Thus:

\[ V_s = V_t - k V_b \]  

where \( V_s \) is the stoichiometrical volume of titrant required in a determination where back-titratrion is necessary, and is calculated from equation (24), being then substituted for volume terms in equations (14), (20) and (22); \( V_t \) is the actual volume of titrant used; \( V_b \), the actual volume of back-titrant used; and \( k \), the equivalence constant from equation (4).

Inspecting error equations (15), (21) and (23), the \( dV_t \) terms are no longer assigned the value of 0.05 ml., but the outside error limit is now given by differentiating equation (24):

\[ dV_s = dV_t + dV_b k + dk V_b \]  

and the relative error by:

\[ \frac{dV_s}{V_s} = \frac{dV_t}{V_s} + \frac{dV_b}{V_s} k + \frac{dk V_b}{V_s} \]  

In equation (24), the \( k V_b \) term measures the volume of over-titratation (sub ot):

\[ V_b = V_{ot}/k \]  

This allows a very convenient substitution in the last terms to the right of equations (25) and (26). They may be rewritten:

\[ dV_s = dV_t + dV_b k + V_{ot} dk/k \]  

\[ \frac{dV_s}{V_s} = \frac{dV_t}{V_s} + k \frac{dV_b}{V_s} + \frac{dk V_{ot}}{k V_s} \]
The terms to the right of equation (29) show how back-titration and similar procedures add to the possible error in a given determination. A study of the corresponding absolute error equation (28) is simpler to undertake. Equation (28) is linear for any given value of \( k \). In it, \( dV_t \) is the possible absolute error to be encountered in any titration (0.05 ml.); \( dV_b \) is the additional error which is due to back-titration and enters as a "cover charge", so to speak, for even starting back-titration. The sum of these terms for any given value of \( k \) constitutes the intercept on the error axis of a graph for the error plotted against the volume of overtitration. In the third term, the relative error in \( k \), namely \( dk/k \), gives the slope of the line.

A study of the role of \( k \) and its relative error term in these equations is interesting. In the first place, wherever the volumes of titrating solutions are never allowed to fall below 25 ml., the intercept on the error axis cannot fall below \( dV_s/V_s \) value 0.002. At \( k \) value unity, back-titration included, the relative error value is 0.004; at \( k \) value, 2, it is 0.006; at \( k \) value, 5, it is 0.012 and so forth. Thus the error is minimized by avoiding back-titration. If this is not feasible, as dilute a back-titrant as possible should be employed. In this connection, if there is any danger of an exchange of role of titrant and back-titrant from procedure to procedure (E.g. HCl used as a titrant in the standardization procedure, and as a back-titrant in the analysis procedure) it may be more practical to keep the \( k \) value unity. These figures are based on the use of a 50 ml. burette without refill.

Secondly, every milliliter of overtitration tends to increase the error. For every 10 ml. overtitration, the relative error is increased by: 0.0016 at \( dk/k \) value 0.004; by 0.0032 at \( dk/k \) value 0.008 and so forth. It is imperative then, contrary to the common impression, to reduce to a practical minimum whatever overtitrations in ordinary analysis, or whatever calculated excesses in indirect analyses, might be necessary.

Thirdly, the error in the standardization of the back-titrant should likewise be minimized—, for this reduces the relative error in \( k \).

CONCLUSIONS:
1) Burette volumes as large as possible are to be used, provided there is no danger of having to refill the burette. This may be accomplished by selecting proportionately larger weights of sample or by the use of a more dilute titrant of the same relative, possible error.
2) Burette refills are to be avoided. They add a new error term to the procedure, which may be large if small volumes in excess of 50 ml. are used.
3) Overtitration and indirect analyses are to be avoided as far as possible.
4) The volume of overtitration, if such a procedure is necessary, should be reduced to a minimum.
5) Back-titrants that are more concentrated than the titrant should be avoided. Less concentrated back-titrants are to be recommended provided their relative error values are low and the volume required is convenient.

A SURVEY OF PUBLICATION ACTIVITY IN THE CHEMISTRY DEPARTMENT OF THE COLLEGE OF THE HOLY CROSS

REV. BERNARD A. FIEKERS, S.J.

I. Departmental Publications Listed in Chemical Abstracts—Non-Jesuit Authors.


II. Departmental Publications Listed in Chemical Abstracts — Jesuit Authors.


III. Departmental Publications NOT Listed in Chemical Abstracts.


*According to the obituary of Father Coyle, basic, acid and dry analysis went through nearly a dozen editions with a total issue of over 50,000 copies. Nucleus (Northeastern Section of the American Chemical Society), 9 (1), 110-113 (1932). Some of the later editions were published from Georgetown and Fordham Universities.
28) Staff of the Chemistry Department, "Definition of liberal education," COLLEGE NEWSLETTER, (College Department of NCEA), 7 (4), 2 (1944).

IV. Publications that occurred before or during staff membership, but credited to other departments. These items are included in order to render the staff bibliography more complete.


Jesuit members of the staff have written continuously for THIS BULLETIN since its inception in 1922-1923. Since 1927, the HORMONE has been published in this department. At times its articles have been digested and abstracted by the CHEMISTRY (Science) LEAFLET, SCIENCE DIGEST and CHEMICAL ABSTRACTS. A survey of alumni publications through Chemical Abstracts Annual Author Index 1943 indicates approximately one hundred papers and patents or books written by alumni. This list should be substantially complete up to December 1, 1944.

The University of Detroit has a large chart for the Qualitative Analysis of the Metals attached to the laboratory wall for ready reference. The outstanding feature of this chart is the fact that small tubes with liquids and colored precipitates are attached to it at each point where a precipitate appears in the scheme. The student may use this for comparison with his own work, as well as for familiarizing himself with the scheme during review work.

The organic chemistry laboratory at Providence College has neatly disposed of the ring stand question. In one corner of the laboratory there is a rack for storing even the largest ringstands at once compactly and accessibly.

The chemistry lecture hall at Boston College will leave as lasting an impression on the student as on the visitor. Literally one-third of its broad front wall is covered with an atomic chart that is visible in the 242nd seat and beyond.
THE POSSIBLE RIGHT SPHERICAL TRIANGLES

WALTER J. MILLER, S.J.

This summary of the possible right spherical triangles (both general and special) has been found useful in two spherical trigonometry classes. The statements may easily be derived from Napier's Rules of Circular Parts and their Corollaries, and from the solid-geometry proposition stating that if two angles of a spherical triangle are equal, the sides opposite are equal, and conversely. Mutatis mutandis, the letters a and b (as well as A and B) may be interchanged, to cover cases in which a different leg or angle is known than the one specified in the summary.

CASE ONE GIVEN TWO LEGS The triangle is always possible.

\(a = b = \frac{\pi}{2}\) TRIRECTANGULAR, TRIQUADRANTAL triangle.

\(a = b \neq \frac{\pi}{2}\) Triangle is possible

\(a \neq b\) Triangle is possible

but the hypotenuse c:

\begin{align*}
&\text{must be in Quadrant I if } a \text{ and } b \text{ are in the same Q.}, \\
&\text{must be in Quadrant II if } a \text{ and } b \text{ are in different Qs.}
\end{align*}

(In other words, two and only two sides may be in Q. II, while all three sides may be in Quadrant I.)

CASE TWO GIVEN ONE LEG AND ONE ADJACENT ANGLE The triangle is always possible.

\(a = B = \frac{\pi}{2}\) TRIRECTANGULAR, TRIQUADRANTAL triangle.

\(a = B \neq \frac{\pi}{2}\) Triangle is possible.

\(a \neq B\) Triangle is possible.

CASE THREE GIVEN HYPOTENUSE AND ONE LEG

\(c = a = \frac{\pi}{2}\) BIRECTANGULAR, BIQUADRANTAL, ISOSCELES, INDETERMINATE triangle, with \(b = B\) but both unknown.

\(c = a \neq \frac{\pi}{2}\) Triangle is IMPOSSIBLE.

\(c \neq a\) Triangle is possible if \(c\) lies between \(a\) and \((180^\circ - a)\); in other words, \(c\) must lie nearer \(90^\circ\) than \(a\) does.
CASE FOUR  GIVEN HYPOTENUSE AND ONE ADJACENT ANGLE

\[ c = 90^\circ \]
A BIRECTANGULAR, BIQUADRANTAL, ISOSCELES, INDETERMINATE triangle, with \( b = B \) but both unknown.

\[ c = 90^\circ \neq A \]
A BIRECTANGULAR, BIQUADRANTAL, ISOSCELES, INDETERMINATE triangle, with \( a = A \) but both unknown.

\[ c \neq 90^\circ \]
Triangle is possible.

CASE FIVE  GIVEN TWO ANGLES ADJACENT TO THE HYPOTENUSE

\[ A = B = 90^\circ \]
TRIRECTANGULAR, TRIQUADRANTAL triangle.

\[ A = B \neq 90^\circ \]
Triangle is possible, and \( a = b = 90^\circ \) (Isosceles Triangle).

\[ A \neq B \]  
\{ 
A acute: if \( (90^\circ - A) \leq B \leq (90^\circ + A) \)  
A obtuse: if \( (A - 90^\circ) \leq B \leq (270^\circ - A) \)  
the triangles are possible.
\}

CASE SIX  GIVEN ONE LEG AND AN OPPOSITE ANGLE

\[ a = A = 90^\circ \]
BIRECTANGULAR, BIQUADRANTAL, ISOSCELES, INDETERMINATE triangle, with \( b = B \) but both unknown.

\[ a = A \neq 90^\circ \]
\[ b = c = B = C = 90^\circ \]
BIRECT., BIQUAD., ISOS. triangle.

\[ a \neq A \]  
Triangle is possible (ambiguous case, 2 sols.),  
\{ 
1. if \( a \) and \( A \) lie in the same quadrant;  
2. \( A > a \) for \( A \) acute; \( A < a \) for \( A \) obtuse.
\}

John Carroll University possesses an actual scale model of the gram molecular volume and has it on display in the lecture room at all times. The six sides of this cube give information on all kinds of interrelated data.

Canisius College, with an eye to safety in organic laboratory, had a shower arrangement built into the wall that partitions two laboratories, thus making it accessible to the "ardent" student in either laboratory. Frames with copper screening placed over the sinks at the ends of the troughs at Canisius at once catch all solids washed down the trough and provide as well a device to prevent the splashing of water from the faucets above the sink.
APPLICATION OF THE MULTINOMIAL THEOREM

C. Frederick Koehler, S. J.

The Binomial Expansion for positive integral powers can be derived from the Multinomial Theorem using the following method.

Consider: \((A_1 + A_2 + \ldots + A_m)^n\) \((1)\)
a polynomial raised to a positive integral power. Such a polynomial implies the product of \(n\) factors of the type
\((A_1 + A_2 + \ldots + A_m) (A_1 + A_2 + \ldots + A_m) \ldots (A_1 + A_2 + \ldots + A_m)\)
\((2)\)
and the complete expansion will involve the sum of individual terms of the type
\[\frac{a_1 a_2 \ldots a_m}{A_1 A_2 \ldots A_m}\]
\((3)\)
where, for the present, the numerical coefficient is omitted. In every such term, \(a_1 + a_2 + \ldots + a_m = n\) because, as is obvious, the numerical summation of the exponents of each term must equal the exponent to which the whole expansion is to be raised. It remains to establish the particular form of \((3)\) which each individual term will assume for variations of \(a_1, a_2, \ldots, a_m\), and there are as many particular forms of \((3)\) as there are terms in the final expansion of \((1)\). This depends entirely on the numerical values which the exponents \(a_1, a_2, \ldots, a_m\) can have, such as
\[\frac{a_1 a_2}{A_1 A_2}\]
\((4)\)
having always \(A_1^1\) and \(A_2^2\) and so on. With regard to \((4)\), we can understand the partial product as formed by taking \(A_1\) from the first, second, and third parenthesis of \((2)\); \(A_2\) from the fourth and fifth; \(A_3\) from the sixth. Hence, the number of times the partial product of \((3)\) will occur, and the way in which it will occur, depends on the number of permutations which we can make of the \(n\) exponents, e.g. \((4)\).
From the discussion above, the literal coefficients are satisfactorily established. The numerical coefficient appears in the following form.

\[
\frac{n!}{a_1!a_2! \ldots a_m!}
\]

(5)

Therefore, the individual terms of the expansion

\[(A_1 + A_2 + \ldots + A_m)^n\]

with their numerical and literal coefficients are;

\[
\frac{n!}{a_1!a_2! \ldots a_m!} A_1^{a_1} A_2^{a_2} \ldots A_m^{a_m}
\]

(6)

where \(a_1, a_2, \ldots, a_m\) assume all positive integral values consistent with the relation;

\[a_1 + a_2 + \ldots + a_m = n.\]

(7)

Here let us note that \(0! = 1\) by definition. Now, summing all of the individual terms we find the complete expansion to be;

\[(A_1 + A_2 + \ldots + A_m)^n = \sum_{a_1}^{a_2} \ldots \sum_{a_m}^{a_n} \frac{n!}{a_1!a_2! \ldots a_m!} A_1^{a_1} A_2^{a_2} \ldots A_m^{a_m}\]

(8)

\[a_1 + a_2 + \ldots + a_m = n\]

which is the Multinomial Theorem. An example of its utility can be seen by referring to the issue of the BULLETIN for March 1944 where it was employed by the author in developing the \(N^{th}\) derivative of a finite product.

The Binomial Expansion is merely a particular case of (8), having \(m = 2\). Therefore, (8) for the Binominal Expansion takes the form

\[(A_1 + A_2)^n = \sum_{a_1}^{a_2} \frac{n!}{a_1!a_2!} A_1^{a_1} A_2^{a_2}\]

(9)

which is the Binomial Theorem.
Below are a few rules which will facilitate the application of the Multinomial Theorem in the form of (8), and the Binomial Theorem in the form of (9).

Let \( m \) = number of terms to be expanded and hence the number of digits in each permutation, since each utilizes all of the digits.

Let \( n \) = the exponent which each permutation must have as its sum, i.e. \( a_1 + a_2 + \ldots + a_m = n \).

Let \( k \) = number of repeated digits in a combination. e.g. in 300, \( k = 2 \).

In general, to find the number of simple and group permutations the sum of each equaling \( n \),

1. Note the digits from 0, 1, 2, \ldots, \( n \).
2. Note the combinations of the digits whose sums equal \( n \). This must be done experimentally, but the total number of possible combinations can be determined from "Euler's table of partitions".
3. The number of permutations of each combination of \( m \) digits is \( m! \).
   a. If the combinations have repeats, the number of permutations that can be formed from each is \( \frac{m!}{k!} \).

The sum of steps 3 and 3a give the total number of permutations.

As an illustration, we expand \((3x + 2y)^4\).

Step 1. \( n = 4 \); therefore, 0, 1, 2, 3, 4.

2a. \((0 + 4) \quad n = 4\)
2b. \((1 + 3) \quad n = 4\)
2c. \((2 + 2) \quad n = 4,\)

Euler's table showed that when \( n = 4 \), and \( m = 2 \), there were three combinations.

Step 3. The number of permutations of \((0 + 4)\)
   is \( m! = 2 \), i.e. 0, 4; 4, 0.
   The number of permutations of \((1 + 3)\)
   is \( m! = 2 \), i.e. 1, 3; 3, 1.
   The number of permutations of \((2 + 2)\)
   is \( \frac{m!}{k!} = \frac{2!}{2!} = 1 \), i.e. 2, 2.

Therefore, the total of permutations whose individual sums equal 4 is five. Substitute the results of step 3 thus:

1. Euler, Introductio. Lib. 1, chapt. 16.
2. See Chrystals' Algebra, Vol. II for many theorems on Combinations, etc., derived from the Binomial Theorem.
3. This expansion is true for any value of \( n \), even if \( n \) is not an integer. The theorem is used in number theory to show that for every polynomial \( f(x) \), every prime \( P \) and every integer \( m \), we obtain the important theorem

\[
f(x^{fm}) = [f(x)]^{fm} \mod. P.
\]
\[(3x + 2y)^4 = 1. \frac{4!}{4!} (3x)^0 (2y)^4 = 16y^4\]

\[2. \frac{4!}{4!} (3x)^4 (2y)^0 = 81x^4\]

\[3. \frac{4}{1} (3x)^1 (2y)^3 = 96xy^3\]

\[4. \frac{24}{3!} (3x)^3 (2y)^1 = 216x^3y\]

\[5. \frac{12}{2!} (3x)^2 (2y)^2 = 216x^2y^2\]

Summing:
\[(3x + 2y)^4 = 81x^4 + 216x^3y + 216x^2y^2 + 96xy^3 + 16y^4 \quad \text{q.e.i.}\]

The notation of the above steps was employed to facilitate the handling of the Multinomial Theorem. The Binomial Theorem presents no difficulty, for \(m\) is equal to two or can be reduced to two if desirable, and the expansion contains always \((n + 1)\) terms.

NOTICE OF INDEX FOR CHEMICAL AND METALLURGICAL ENGINEERING FOR 1944

Hidden away at the bottom of page 147 of the December issue of CHEM & MET is a notice that indices for 1944 can be had only by writing to the Index Editor, Chem. & Met., 330 West 42nd St. N.Y.C. 18. This announcement is not listed in the table of contents for that issue and is apt to escape the attention of librarians who will be looking for the index when binding schedules commence. There is much talk these days about paper shortage. But the increasing bulk of commercial advertising would give no indication of shortage. Advertising matter in this journal, incidentally is paginated consecutively with the textual content and a great percentage of advertising appears on the same page with the text. It is difficult to reduce the size of the bound annual volume by the elimination of advertising matter at the time of binding. This can be accomplished if a little thought and time is given to the matter before bindery orders are made out.

B.A.F.
THE PIONEER FORECASTERS OF HURRICANES

By the REV. WALTER M. DRUM, S.J.

Shortly after our war with Spain, the United States Weather Bureau established a branch office in Havana. While speaking of the work of this branch office, some one has written the following words:

"At first it was difficult to interest the people in the warning service, since they are by nature very conservative and slow to adopt any change in their accustomed methods and mode of living. The issue of warnings of hurricanes was a most radical change, the inhabitants being accustomed to hear of these phenomena only upon their near approach."

The author of these words meant, of course, only to say a kind word about the Weather Bureau in Havana. His word was neither kind nor true. "The issue of warnings of hurricanes" by the United States Weather Bureau was not a "most radical change." For thirty years such timely warnings of hurricanes have been issued, that the inhabitants of Havana have been "accustomed to hear of these phenomena" long before their near approach. No account of the warning service in Cuba is exact without due credit to Spain's most eminent meteorologists, those pioneer forecasters of hurricanes, the Jesuit Fathers of the Observatory, of Belén.

The Royal College of Belén, in Havana, celebrated its golden jubilee last year and issued a memorial volume entitled "Quincuagésimo Aniversario del Colegio de Belén." The greater part of the volume is made up of a sketch and vindication of the Observatory of Belén; and this part has also been published by itself under the title "Anotes Historicos acerca del Observatorio del Colegio de Belén, Habana, por el P. M. Gutiérrez-Lanza, S. J." We draw our materials from the work of Fr. Gutiérrez-Lanza.

The Observatory of Belén is astronomical, magnetic and meteorological in its purpose. Lack of sufficient personnel necessitates the limitation of routine work to meteorology and whatever subserves thereto. Regular observations are made in the astronomical department to obtain the geographical and astronomical coordinates and the exact time. In the magnetic department, daily observations have been made since July, 1862; and these observations have been of much service to

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1 This paper by Fr. Drum was first published by the Press of Stormont and Jackson, Washington, D. C., for the Observatory of Belén, Havana, Cuba, August 1905. The pamphlet has been kindly loaned for republication by the Holy Cross Library, and is published here because of its extraordinary historical interest, especially to members of the Society of Jesus.
our Coast and Geodetic Survey. The chief department is that of meteorology. It was founded in 1857 by Fr. Antonio Cabre. On March 1, 1858, were started the regular meteorological observations that have not since been interrupted. The science of meteorology was yet in its infancy, when, in December, 1859, the Jesuits of Belén published a report to show the variations of meteorological and climatological elements in Havana from March 1, 1858, till the date of publication of the report. From that time, with few omissions, annual reports have been published by the Observatory. Fr. Gutiérrez-Lanza announces that the omissions will soon be supplied, and the Observatory will have given to the learned world a set of meteorological reports that cover a period which is probably longer than any other observatory may boast to have covered with accuracy and care. By means of these reports, a comprehensive study may now be made of all meteorological phenomena in Havana during well nigh fifty years past. W. F. R. Phillips, of the Weather Bureau of Washington, in the annual report of the Weather Bureau for 1897-1898, draws his information about the climate of Havana almost entirely from the published volumes of Belén. The worth of this collection may be estimated by the awards it has won the Observatory. The meteorological publications of Belén received a diploma of honor at the Philadelphia Exposition of 1876; a diploma and silver medal, at the Paris Exposition of 1878; a diploma and gold medal at the Barcelona Exposition of 1888; a diploma and bronze medal, at the Chicago Exposition of 1893; a gold medal, at the St. Louis Exposition of 1904.

Fr. José Reynal, who was in charge from 1864 till 1867, began a monthly meteorological review, which has since been discontinued.

In the year 1870, the golden age of the Observatory began. Father Benito Vines, a man of rare talent, masterful knowledge of the sciences, and a will-power which pushed through with indomitable energy whatever plan be proposed, became director of the Observatory, and at once undertook to grapple with the problem of the storms of the Antilles. The havoc wrought by cyclones had been terrible. “The inhabitants being accustomed to hear of these phenomena only upon their near approach,” could not make ready for the scourge. Heartrending was the loss of life and property. In the name of science and humanity, Fr. Vines set himself to discover ways and means of issuing timely warnings of the oncoming storm.

For twenty three years the heroic meteorologist took the readings of various instruments and worked at his problem how to organize a defence against hurricanes and to lessen the loss of life therefrom. He had not the corps of able assistants that is seen in a liberally founded observatory, but did the work almost single handed and alone. The daily routine was tremendous. It included observations of atmospheric phenomena every even hour from 4 A. M. till 10 P. M., an extraordinary international observation at 7:30 P. M., and, in times of special need, observations every hour of the day and night. The phenomena noted at each observation were readings of the declinometer.
bifilar, barometer, thermometer (normal, maximum and minimum, wet and dry bulb); tension of water vapor, relative humidity, direction and velocity of the wind, direction of upper and lower and middle strata of clouds, solar or lunar halo, rainfall, and other magnetic, electric and optical elements. The wonderful accuracy and care with which Fr. Vines noted all these phenomena from 1870 until his death shows the conviction he had of the importance of the work. His annual report gives accurate details of all these weather phenomena, the averages of all the observations of each phenomenon during each month, the daily mean of each phenomenon, the monthly mean of each hour, and the averages of both these means. The results are plotted on large monthly diagrams by means of curves which are determined not only by the records of instruments that chart automatically, but also by the ten daily readings of each normal instrument. Among the automatic recorders of Belén, the most remarkable is the Meteorograph of Fr. Secchi. This invention was the gem of the exposition of Paris and won more prizes than did anything else. For thirty years the Belén Meteorograph has been uninterruptedly and most accurately registering the various meteorological elements.

The monthly diagrams of Fr. Vines met with hearty approval from the world of science. *Nature* (vol. 31, p. 361) says: "As each day has one tenth of an inch devoted to it, the two-hourly observations of all the different elements can be readily seen and compared with each other, and this part of the work is done with a scrupulous care and accuracy it would not be easy to surpass." The *Diario de la Marina* (25 June, 1887) says: "The publications are an honor not only to the College of Belén and to their learned author, but to Spanish science that counts among its most illustrious representatives the wise and modest Jesuit, Fr. Vines."

The Meteorological Reporter to the Government of India and Director General of Indian Observatories wrote: "I am extremely anxious to have a complete set of all the Havana Observations, and hence the trouble I am giving you." The chief observatories and scientific societies all the world over have shown like anxiety and appreciation.

These annual reports represent only part of the work of Fr. Vines. He studied the structure of the cyclone, the phenomena that preceded it, and the havoc left in its wake. In fact, he noted with ardor and painstaking care all meteorological signs and data which preceded, accompanied and followed up the storm,—and that, too, without neglecting the ordinary and toilsome routine work of the Observatory. Fortunately the records of the Observatory gave full information about the hurricanes of twelve years past, and Fr. Vines was laboring right in the trajectory of most of the West Indian storms. Nearly every year he had an opportunity of observing a cyclone either whirling around almost in a circle, or losing its force at the well nigh complete calm of the treacherous turning point, or rushing along with ever increasing velocity on the second branch of its parabolic trajectory. The purpose of his study was fourfold:
1st. To find some sign or group of signs that would invariably prove the existence of a cyclone while it was yet at a great distance from the observer;

2nd. To get his bearing in regard to the whirlwind, i.e., to determine what part of the horizon it was coming from;

3d. To locate the trajectory or curve along which the cyclone would move, and to do this on time for the observer to betake himself to a safe distance from that fatal curve;

4th. To determine the distance of the hurricane from the observer, its intensity, area and velocity.

Fr. Vines was the first to work with any success at this fourfold problem; to him belongs the credit of whatsoever has been accomplished by the issue of hurricane warnings.

He first worked especially at barometric variations, and published the Havana readings, regular and irregular, from 1858 till 1871, together with a study of the connection of these variations with storms. He was then attracted by the singular form, structure, stability, and direction of the clouds that he calls *cirro-stratus plumiformes*. The conviction was brought strongly home to him that maybe after all this marked direction had something to do with the position of the cyclone, and he could by means of these clouds find out the bearing of the storm. Others before him had noticed the connection of *cirro-stratus plumiformes* with the hurricane, but one and all had said or implied that from the appearance of these clouds nothing at all could be learned about the nature and bearing of the whirlwind. No one before Fr. Vines even suspected that these clouds were fleet messengers sent high in air and from the very heart of the cyclone to give timely notice of its oncoming violence. He made observations and discovered that his conjectures were right,—the directions of these light upper clouds corresponded almost exactly with the radius of the cyclone; and the vanishing point, at which these directions converged, corresponded with that part of the horizon in which the storm was. The discovery was simple yet wonderful, and spurred on Fr. Vines to further investigation. He observed that the different clouds in different altitudes revealed the directions of the different currents of air, and found in the relative altitudes of these clouds a means of establishing his "law of cyclonic currents at different altitudes,"—a law unknown before it was formulated by Fr. Vines. In the years and centuries to come, it will ever be the glory of this pioneer forecaster of hurricanes that he was the first meteorologist to forecast the cyclone by observation of the upper as well as lower clouds, and the first to explain with scholarly and scientific accuracy the hitherto mysterious gradation of the various currents that go to make up the scourge of the tropics.

Fr. Federico Faura, S. J., predecessor to Fr. Algué in the Observatory of Manila, writes, in a pamphlet entitled *Senales Precursoras del Temporal en el Archipielago Filipino*, that the discovery of the connection of the upper clouds with the hurricane is "one of the greatest triumphs brought about in the past few years."
That others might take advantage of his discovery, Fr. Vines invented two instruments, the Cyclonoscope and the Cyclononephoscope, for observing from afar the existence and bearing of an hurricane. These two instruments were awarded a diploma and a medal at the Chicago Exposition.

The people and government of Cuba were not so "conservative and slow to adopt any change," as the above quoted writer makes them out to be. They took the greatest interest in the work of Fr. Vines. All knew that their interests were wrapped up in his work. After the three cyclones of 1875 and 1876, the Cubans were most anxious to have the great meteorologist study the storm-tracks left in their beautiful island. The Governor-General and subordinate officials, the Royal Academy of Science in Havana and other learned societies all gave him letters of enthusiastic recommendation. Steamship companies offered free transportation. Railroad companies showed such marked courtesy and consideration as several times to put a special express on the road for his convenience and at their own expense. All telegraphic service was given free. Fr. Vines did not find it "difficult to interest the people in the warning service" that he hoped to establish. Great was the joy in Havana, when it was found that these excursions had proven to the eminent scientist the truth of the theories he had formulated in his Observatory.

At last, in 1877, after six years of study, Fr. Vines gave to meteorologists his great work "Apuntes Relativos á los Huracanes de las Antíllas." This remarkable treatise was incorporated into the "Anales de la Academia de Ciencias de la Habana." The Signal Office of Washington published a digest of it as "Practical Hints in Regard to West Indian Hurricanes," whereof three large editions were exhausted in a couple of years. The Pilot Chart of the Hydrographic Office of Washington, during 1888 and 1889, published the laws of Fr. Vines a few at a time, and often reprinted the same laws month after month, so as to have them fixed in the memories of navigators. In its issue for May, 1889, the Pilot Chart says of certain of Fr. Vines' laws: "These important laws, established by the study and long experience of Fr. Vines, should be thoroughly understood by every navigator and utilized by shaping his course so as to avoid a hurricane." In April, 1889, the same publication speaks of "an officer detailed from the Hydrographic Office to visit Havana in order to consult with Fr. Vines, the leading authority on West Indian hurricanes." This officer was Mr. Everett Hayden, Chief of the Division of Meteorology of the Department of the Navy and editor of the Pilot Chart. After his return from the trip on which the Secretary of the Navy had sent him, Mr. Hayden, in a lecture before the Naval Academy, spoke of "Padre Vines, the eminent Havana meteorologist, whose scientific ability, indefatigable energy, and unrivalled opportunities have linked his name so inseparably with all recent advances in our knowledge of these storms. Indeed, the lecturer has himself suggested coined the word Vinesa, to express for our tropical storms what the word typhoon
does for those of China, in honor and recognition of the distinguished services of his esteemed friend, the Director of the Meteorological Observatory of the Colegio de Belén.” Professor Willis L. Moore, Chief of the Weather Bureau of the United States, wrote in Collier’s Weekly, October 6, 1900: “Reverend Father Vines, who probably gave more intelligent study and investigation to tropical storms than any other scientist, says that it must be admitted that tropical cyclones do not form at any point within the tropical zones, but that they single out for their formation and development definite regions in these zones.”

Across the Atlantic there was the same enthusiasm over the great discovery and the epoch-making work of Fr. Vines. One abstract of the work appeared in the Ann. Hydr. Berlin, under the title Neuere Forschungen über Westindische Orkane, another in the Wochenschrift. In the Meteor. Zeitschrift of Berlin, there appeared a lengthy article Die Untersuchungen von Pater Vines über Westindische Orkane. Abbé Moigno made a thorough study of the work of Fr. Vines in Les Mondes, and a like study was published by the Revue des Questions Scientifiques of Brussels. Mr. R. H. Scott, F. R. S., spoke in glowing terms of the great discovery before the Royal Meteorological Society of London, whereof he was President. Such authorities as Ferrel, Waldo, Garriott and Algue have all introduced the laws of Fr. Vines into their meteorological works.

Shortly before his death, the eminent Jesuit Meteorologist wrote his Investigaciones Relativas a la Circulacion y Traslacion Ciclonica at the request of the officials of the Chicago Exposition. He died three days after the work had been mailed, and before they had received it that had asked him to write a further explanation of the laws he had discovered. In 1895, this posthumous work was published together with the Apuntes; they made the most noteworthy and useful volume ever written on the cyclones of the Antilles. The Investigaciones has already been translated into German by P. Bergholz, the Director of the Observatory of Bremen.

On July 23, 1893, the great Jesuit died. So beloved and revered was he in Havana, that La Lucha published a special edition at 11 P. M. to announce “The Last Hour and Death of Fr. Vines.”

On August 29, 1893, Fr. Lorenzo Gangoiti took the place of Fr. Vines. He has since kept up the high reputation established for the Observatory by his renowned predecessor. His modesty and bearing are a boast to Cuba.

(To be continued)