

BULLETIN  
of the  
American Association  
of Jesuit Scientists

Eastern Section  
Founded 1922



Published at  
CHEVERUS HIGH SCHOOL  
Portland, Maine

Vol. XX

MAY-JUNE, 1943

No. 4

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EASTERN STATES DIVISION

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## THE EDITOR'S PAGE

It is always interesting to know what others are doing. The Editor's mail recently contained the following from Canada. Fr. Burke-Gaffney writes from St. Mary's College, N. S.

"There have been no changes in our courses as a consequence of the war. Two years ago N.S.T.C. and affiliated Colleges agreed that the Engineering Course could not be shortened, summer work is an essential part of the course; the students must have their "holidays" in which to find jobs providing practical experience. All science students are now required, by the Bureau of Technical Personnel, to fill out a form on starting their courses. They must state whether they wish, on graduation, to go to industry or the armed services. If the latter, what branch? Those who elect a specialized branch are liable in the summer of third and fourth years to call for specialized training."

Fr. O'Connor at Loyola College, Montreal, writes;

"Throughout Canada the war has brought changes to University curricula and is bringing still further changes next year. But these vary greatly with each University. It is touching us chiefly in lessening the enrollment in the Arts courses and in intensifying the activities of the C.O.T.C. (your O.T.C.) Precise course changes due to the war have not yet reached our Colleges, but it is not unlikely that next fall we shall be able to help in some of the navigation courses given to men already in service. This is being done now in the Universities and the scale of it will be greatly increased next year."

Did you note the fine article in the February issue of the Journal of Chemical Education, "An Exhibit of Chemical Substances Mentioned in the Bible"? It is interesting to recall in connection with it, the article, "Christ as Apothecary of the Soul", in the February 1934 issue of the same Journal. The Frontispiece of that issue is an old painting of our Lord holding the scales, and on a table in front of Him, in addition to many symbolic representations, are the Chalice and Sacred Host.

# BIOLOGY

## SOME FACTORS THAT INFLUENCE BLOOD PRESSURE

JAMES J. DEELEY, S.J.

Any method for the measurement of the human blood pressure must of necessity be indirect. The underlying principle involved looks to the balancing of air pressure against the pressure of the blood in the brachial artery and then estimating the former by means of an aneroid manometer. The instrument used for this purpose is a flat rubber bag covered by a non-distensible envelope of cotton fabric. The cavity of this bag is connected by a rubber tubing to the manometer and by another tube to a hand bulb or small pump. Thus the bag can be inflated to any desired pressure. A small valve between the pump and bag permits the escape of air and the reduction of pressure. The uninflated rubber cuff, which should be at least 12cm. in width, is wrapped snugly around the upper arm just above the elbow. The cuff is then inflated until the air pressure overcomes the arterial pressure and obliterates the arterial lumen. The pressure is then cautiously increased and then reduced until the arterial pressure overcomes the air pressure and the blood escapes beyond the cuff into the peripheral section of the artery. At this instant the pressure in the cuff is read from the manometer. Since the air pressure practically balances the systolic pressure, the manometer reading indicates the value of the latter. It is essential to the method that the reading be taken when the blood escapes beneath the cuff.

The more accurate method for determining this pressure was introduced in 1905 by the Russian physician Korotokow. In this method certain sounds heard during auscultation of the brachial artery below the cuff are used as indications of the systolic and diastolic pressures. Under ordinary circumstances if a stethoscope is placed upon the brachial artery no sound can be heard. If, however, the artery is compressed by the manometer cuff so as to completely arrest the flow of blood for a moment, a sharp light tapping sound in rhythm with the heart beat will be heard when the pressure is again released just sufficiently to permit the arterial lumen to open and to allow a jet of blood to pass beyond. As the pressure of the cuff is continuously decreased the sound undergoes a series of changes in quality and intensity. Four distinct phases of the sound are heard in succession in the normal individual . . .



1. Sudden appearance of a clear, but often faint, tapping sound, growing louder during the first 10mm. fall in pressure.
2. A murmur during the next 15mm. fall in pressure.
3. Sound becomes clearer and louder during the next 15mm. fall.
4. Throughout the next 5 to 6mm. fall the sound becomes muffled, finally disappearing.

The beginning of the first sound is taken as the systolic pressure, the fourth sound, just before its complete disappearance is taken as the index of the diastolic pressure. This latter sound coincides with the moment that the blood escapes beneath the armlet in a continuous stream.

In man this measurement is attended by certain inaccuracies, due to the resistance of the tissues of the arterial wall itself to the compressing force, sclerotic changes, simple hypertonus of the muscular coat, repeated compression and depression before the actual determination is made and definite arteriosclerosis. However, definite and approximately accurate estimations are obtained by the use of the broad armlet, which distributes the applied pressure over a wide area.

The average systolic pressure of young male adults is about 120mm.Hg.; the diastolic pressure falls to 80mm., thus making the mean pressure about 100mm. and the pulse about 40. Therefore the normal ratio of systolic, diastolic and pulse pressure is 3-2-1. Age exerts a definite influence upon the blood pressure levels in the normal individual. A steady, though not great rise in blood pressure from adolescence to old age is the rule in health. Some investigators have found a correlation between the systolic pressure and body weight. Comparing groups of heavy and normal individuals, the former were found to have a pressure on the average of 7.5mm. higher than the latter. In definitely obese individuals, not subject to any ailment, the difference is even more pronounced. In persons, who are overweight the blood pressure is definitely greater.

Digestion, emotional influences, exercise, and strange to say, posture, have a marked influence on either the systolic or diastolic pressures or both. Digestion affects a 6 to 8mm. rise, lasting for an hour, on the systolic pressure, while the diastolic pressure, if at all, is reduced as a result of the vasodilation in the digestive organs and the skin. Excitement, fear, and worry particularly effect the systolic pressure. These effects are brought about by increased cardiac action and changes in the state of the vessels, all of which is brought about by impulses effecting the nerve centers in the medulla. Recent investigation seems to point to a release of adrenaline or some allied substance as an important factor in this change. Of all the physio-

logical conditions, exercise, if of a strenuous nature, has the most marked effect on the arterial pressure. Even at the instant that the muscular exertion is contemplated the pressure commences to rise and reaches a height of 180 to 200mm. In light exercise the diastolic pressure may remain at normal level while the systolic pressure is sure to rise several millimeters. Immediately after the conclusion of exercise the pressure drops to normal or even slightly below normal. It then mounts rapidly to the previous high level, from which it gradually declines again and in a healthy person reaches the normal level within from one to four and a half minutes. Some investigators attribute this to a sudden relaxation of the abdominal muscles. It seems that the blood is drained into the venous reservoirs during exercise. When deprived of their support (abdominal muscles) the capacity of the reservoirs is increased and the flow of blood into the heart is curtailed. It is not until the increased capacity of the reservoirs is used up by the inflow of blood from the recently active muscles that an adequate flow into the heart is restored.

The diastolic pressure is higher in the standing than in the sitting position and lowest in the reclining. The change occurs whether the postural change is brought about actively or passively and is evidently a compensation for the gravity effect. It is convenient at this point to consider the arterial and venous systems together in their relation to the weight of the blood column and the effect of gravity on this. Above the level of the heart gravity opposes the energy of the cardiac contraction in the arteries but aids in the veins. Below heart level the reverse is true, the hydronic and the hydrostatic factors being summed in the arteries and opposed in the veins. In the case of the arteries the effect of gravity in the vessels above the heart is fully compensated, i.e., the pressure in the brachial artery is as high if not actually higher when the subject is standing than when he is lying down. The pressure of the arteries in the lower limbs varies greatly with the position. In the vertical head-down or in the L position the pressure in the artery is much lower than it is in the recumbent position. In the standing position, on the other hand, the pressure is much higher. Owing to this hydrostatic effect the nearer the foot the measurement of the pressure is taken the higher will be the observed pressure. The difference between the pressures in the brachial and leg arteries in these different positions is equal to the height of the blood column which would reach from one artery level to another. However, it is in the venous system that the effect of gravity is more pronounced. This is due to the lower venous pressure and the greater distensibility of the venous walls and to the fact that the height of the blood column which must be raised against gravity from feet to heart is much greater than that of the arterial blood column from the heart to the brain. If any of the factors concerned with the raising of the blood against gravity, namely,

left ventricular contraction, support of the abdominal and limb muscles, suction and force-pump action of the respiratory movements, the venopressor and capillary tonus mechanisms, fail, accumulation of the blood, venous stasis, in the dependent parts is likely to occur. Thus a man assuming the erect posture after a protracted illness overcomes the hydrostatic effect only with great difficulty. The blood then subsides in the capacious abdominal veins and capillaries and the right heart is no longer adequately fed with blood. The blood supply to the brain is thus reduced, resulting in vertigo (giddiness) or syncope (loss of consciousness). A sudden drop of about 25mm. is sufficient to cause unconsciousness. Even a healthy soldier is liable to faint, if kept standing in an immovable position for a long period. Here, however, we have the rather obvious means of restoring the person who has fainted taken care of by nature itself. It is an interesting reflection in this connection that when a person falls to the ground unconscious the horizontal position automatically annuls the effect of gravity. The head-low position and pressure upon the abdomen are obviously the most effective means of getting the blood to the heart and so restoring circulation. On the other hand, if a person were held in the upright position after consciousness had been lost death might result. Under chloroform and to a less extent under ether and other anesthetics, in shock and various unconscious states the compensatory responses are depressed and any change from the horizontal position under these conditions is therefore fraught with danger to life.

Blood pressure may be persistently above or below the normal range. These departures from the normal are termed hypertension or hypotension respectively. It is difficult to draw a sharp dividing line between the normal and the abnormal but an elevation above the average normal for a particular age of 15mm. in the systolic and 8mm. in the diastolic may be considered definitely abnormal. Hypotension may be defined as a reduction below 110mm. in the adult male and 100mm. in the adult female of any age.

Hardening of the arteries is essentially a disease of the old. It seems in part to be a specific manifestation of the unexplained but rather general loss of elasticity of many tissues in old age. Usually associated with this is a chronic elevation of the arterial blood pressure. The systolic pressure may be as high as from 250 to 300mm. By some this is looked upon as a compensatory adjustment, by which blood is forced through the narrowed vessels, and a more or less adequate circulation is maintained. The seriousness of hypertension lies partly in the danger that some blood vessel may rupture. This rarely occurs in large vessels with dangerous extensive hemorrhage. More often it involves vessels of such size that the loss of blood is not important. However, if this bleeding occurs in some vital organ, particularly a friable structure like the brain, serious damage would result. Many



brain cells may be torn and damaged by the blood escaping under high pressure. We are all familiar with the "cerebral incident" which often causes paralysis and is commonly known as a stroke. Thus we see the rationale of having people with high blood pressure lead as quiet a life as possible, especially when we recall that muscular exercise and excitement elevate the arterial blood pressure. Hypertension may also be fatal because of heart failure. The heart being unable to eject sufficient blood against the high peripheral resistance, becomes overloaded, gradually dilates and eventually fails entirely. It is a well attested fact that in chronic glomerulonephritis as well as in primary or essential hypertension increased peripheral resistance is the immediate causative factor of the general blood pressure rise. The hypertrophy of the heart in the cases enumerated is purely secondary—a physiological compensation rendered necessary by the greater resistance offered to the flow of blood through the peripheral vessels.

The less common defect of chronic low blood pressure (hypotension) becomes incapacitating when the reduced arterial pressure fails to drive blood through the tissue and organ capillaries at an adequate rate. In an adult an arterial pressure that is persistently below 110mm. and for which no cause can be found is referred to as essential or primary hypotension. The subjects of this phenomenon, beyond showing a greater susceptibility to fatigue or being subject to giddiness and headaches, suffer no ill effects; they are more likely to be free from cardiac and renal diseases, the condition for this reason being said to forecast longevity. Apart from the foregoing type, low blood pressure occurs either as a temporary or persistent phenomenon in many conditions. Some of these are hemorrhage, traumatic shock, anesthesia, tuberculosis and debilitating diseases of various kinds. It may also occur as a result of vasodilation and cardiac asthenia associated with acute fevers; from myocardial failure; and in Addison's disease or hypothyroidism.

Thus we see that blood pressure varies during mental and muscular work and shows a tendency to fall during fatigue. Cold drugs which constrict the arterial pulse may raise it, while heat drugs of the vasodilator groups tend to lower it. It is dependent upon the strength of the heart beat, the elasticity of the arteries and the tone of the muscular tissue in their walls, and reacts proportionately to the resistance offered to the blood flow by the peripheral arteries. It is also dependent upon such minor factors as respiration and resulting changes in the chest cavity, the amount of blood in the body and finally, but not so minor, the effect of gravity. It is increased during such diseases as arteriosclerosis, diseases of the kidneys and the liver, and acute fevers. It is reduced when the heart beat is weak, when the blood vessels are relaxed and when the total quantity of the blood in the vessels is appreciably reduced. It is best determined by the use of the Sphygmomanometer.

# CHEMISTRY

## SYNTHETIC RUBBER

REV. RICHARD B. SCHMITT, S.J.

"Let's take the car, we will get there much quicker and take us right to the door." This remark or something similar was the casual method in our daily routine. But *now*: "We can't take the car, the tires are worn out."

Most people speak of tires and tubes and golf-balls and a hundred other rubber materials and keep wondering, how much longer must we wait, until we have the use of the car again?

It required a war to make us realize that we are experiencing a real emergency and that an important way of our daily life depends upon rubber.

How are we really progressing in this emergency? Did we solve the problem chemically and industrially, and what are the prospects? A further development might be: what are the prospects of natural rubber from South America, Central America and Africa; then, what about guayula rubber in the south-western part of the United States, and what about reclaimed rubber? In the present crisis vital considerations.

The ideal location for growing natural rubber is in a zone on the earth's surface  $10^{\circ}$  latitude above, and  $10^{\circ}$  below the equator. In normal times 97.2% of the rubber came from the Far East, and 2.8% from South and Central America; and so we need the rubber-like material called synthetic rubber. This does not mean that the products are inferior. On the contrary, some properties are decidedly superior. But it does mean, that the manufacturer must develop new compounding materials and new techniques for the vulcanization, formulation and fabrication of these compounds into finished articles. This is the biggest job ever tackled by chemistry; and this problem has used up more man-power than any other manufactured article. Millions of tons of chemical or synthetic rubber are needed to bring the war to a victorious conclusion.

There are just two words, we may add to our vocabulary to talk about synthetic rubber: BUTADIENE (4 syllables), because this is the *raw material* from which synthetic rubber is made; and POLYMERIZATION (6 syllables) is the *chemical process* for compounding this rubber-like material. Our chemists know at least, fifteen methods of making this raw material, but only a few are practical and economical; a factor which must be kept in mind for all manufacture. Butadiene can be made from petroleum, from ethyl alcohol, from

acetylene or from the fermentation of starch. Polymerization means that by the multiplication of this basic unit the synthetic rubber is compounded; this unit contains five carbon atoms and eight hydrogen atoms, and is called a hydrocarbon, like gasoline is a hydrocarbon.

The present governmental program calls for the procurement of 68,000 tons of crude rubber and approximately 910,000 tons of synthetic rubber by 1944. (Baruch report, page 56). During recent months technical developments in the processing arts are proceeding rapidly and successfully. From the present indications, and supposing no interference, by September of this year, there will be about 76,000 tons available for civilian use. By the end of next year, the production will be 1,100,000 tons. In the past few months, the price of butadiene dropped from fifty cents per pound to eighteen cents a pound; and if all goes well, it may be manufactured at five cents per pound. This is indeed comforting to all.

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## SWIMMING POOL NOTES II.

REV. BERNARD A. FIEKERS, S.J.

Since the previous article<sup>1</sup> appeared in this Bulletin in 1940, the use of chlorine or free chlorine vehicles for swimming pool sanitation purposes has gradually become more and more difficult. This is due to the increased demand for free chlorine in war production. Some of the advances that have been made to obviate this difficulty are here discussed.

Could chlorine be made at the pool side by some electrolytic method? A type of apparatus has appeared on the market for just that purpose. It is the "Valchlorator"<sup>2</sup>. As is to be expected, a 110 volt direct current generator is needed where A.C. service has been installed, and thus its applicability would also seem to be limited by priorities when direct current cannot be led to the poolside. Still, if direct current could be made available, this apparatus might interest some of our institutions as an available chlorine supply, not only for the swimming pool, but as well for the laundry, for the chemical laboratory, for war gas decontamination and for general disinfecting purposes.

Another expedient to avoid the chlorine priorities situation is the substitution of liquid bromine for chlorine in solving the swimming pool sanitation problem. A survey of swimming pool literature<sup>3</sup> from

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<sup>1</sup> THIS BULLETIN, 18, 98-102, Dec. 1940.

<sup>2</sup> Manufactured by the Valhalla Company, 231 South LaSalle St., Chicago, Ill.

<sup>3</sup> Twining, R. J., The Chemistry of the Swimming Pool, The HORMONE, (Holy Cross College, Worcester, Mass.), 14, 57-59 and 68, (1941).



chemical sources has brought to light the fact that the same concentration of bromine by weight is required to keep the pool sterile as that ordinarily required of chlorine. It is interesting to note here, that, considering the ratio of the atomic weights of these elements, it would seem that bromine is twice as effective a bactericide per atom as chlorine.

Recent attempts to promote the bromine substitute have brought many interesting properties of bromine for this purpose to light. Bromine has been used to a considerable extent in Germany during the last twenty-five years for disinfecting purposes. In this country it has enjoyed some usage during the past ten years, but its widespread use has been hampered by the mammoth demands on it for the production of antiknock fluids in the petroleum industry. During the present emergency its production has been increased, and during the last year or two it has been used in the Midwest in approximately twenty-five pools as a disinfectant.

Coming in a highly concentrated liquid form, it has a certain advantage over tank chlorine, but is at a disadvantage when compared to the high test chlorine vehicles like true calcium hypochlorite. From the point of view of bather's comfort, greater advantages are claimed for bromine than for chlorine. Whereas chlorine irritates the mucous when the concentration surpasses approximately 0.5 parts per million (p.p.m. or milligrams per liter), bromine can be used safely and comfortably up to 2.5 p.p.m. On the average 2.0 p.p.m. bromine can be tasted in water. Greater safety is claimed for bromine than for chlorine gas. Fire prevention codes often restrict the use of chlorine in cylinders over ten pounds in pressure; on bromine, there is no restriction as it is a liquid at ordinary temperature and is packaged in sealed ampules. From the point of view of convenience, it is claimed that chlorine dispensers can be cheaply "converted" to those of the bromine type. The solubility of bromine, being much greater than that of chlorine, is a property that can be used to advantage in the design of dispensing equipment. Bromine solutions are said to be less sensitive to sunlight but far more active than chlorine on organic material. This property of photosensitivity, judging from the writer's experience with chlorine, might on being investigated prove bromine to be an economically superior disinfectant for the outdoor pool. The quotation 58½ lbs. net for \$32.00 wholesale today puts the price intermediate between that of crude chloride of lime for the lower limit, and high test true calcium hypochlorite for the upper limit based on the actually available chlorine content of these substances in the 1939 market.

Tests for bromine can be made, and it can be controlled by the orthotolidine<sup>4</sup> method just as chlorine is. But there are other analytic

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<sup>4</sup> See Perchloron Bulletin (Pennsylvania Salt Manufacturing Company, 1000 Widener Building, Philadelphia, Pa.); for general information; also page 14, for directions for making control sets and for carrying out determinations. These sets could be made up in any reasonably well equipped chemistry laboratory.



methods for bromine as well. It is not clear to the writer just what effect the use of bromine would have on the pH control of the pool. But it seems reasonable at first sight that no drastic changes on this point would have to be made.

Withall, liquid bromine cannot be handled by anyone and everyone in as foolproof a fashion as some of the high test chlorine carriers. It would seem that the use of rubber in any homemade dispensing equipment would be foolhardy; the dispenser is a glassblower's job.

Bromine as a swimming pool chemical has been promoted recently by the "Halogen Engineering Associates".<sup>5</sup>

A third investigation made by the writer dealt with the possibility of finding a suitable and economical paint to be applied to the concrete walls and floors of an outdoor tank. The Philadelphia Quartz Company, through their organ, "Silicate P's and Q's", had been singing the praises of a silicate of soda base for surfacing concrete in foundations, floors, silos and so forth. On writing them for specific information in connection with the problem at hand, they submitted a paint vehicle formula that was not specifically designed for use in swimming pools, but one which by report had been used successfully for that purpose. If used, they expected that the tank would have to be repainted each year, and they suggested the added precautions that:

- 1) the pool be thoroughly dried before the application of the paint,

- 2) that the weather should be reasonably clear when the job is started with promise of two or three days before it might be exposed to rain, and further:

- 3) that the pool remain empty for a week or more after painting in order to get the maximum permanency of a hard resistant coating which stands up well under ordinary weather. The company's directions cannot be reprinted here; but undoubtedly they are available for anyone who requests them by writing to the company.

The first installment of these notes was based on the writer's own experiments; this installment draws mostly from trade literature on the topic. The first installment seemed to be favorably received and tried out with some success elsewhere in the American Assisntancy. Probably some of the suggestions here made, have already had some experimental confirmation at some college or other in the provinces. In that event, it would seem that any report of success written in these pages would be most welcome, especially today, when the armed forces generally demand pool facilities of their prospective training colleges.

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<sup>5</sup> Halogen Engineering Associates, 437 Orleans St., Chicago, Ill. It was from this company's descriptive folder that much of the information on bromine used in these notes was culled

# MATHEMATICS

## A STUDY OF INFINITE SERIES BY GRAPHICAL METHODS

REV. EDWARD C. PHILLIPS, S.J.

A recent Publishers' announcement of a college textbook.\* carried the following diagram and explanatory text illustrating the graphical summation of infinite (geometrical) series.

"Draw the straight line AD, Figure 58, and select the unit so that the segment AB will represent a. Draw BK perpendicular to AD and so that  $BK/AB=r$ .

Draw KG parallel to AD and so that  $KG/BK=r$ . Extend AK and BG until they meet in E. It is evident that the line AC is the sum of all the terms containing even powers of r, and CE is the sum of all the terms containing the odd powers of r. AD is then the sum of the infinite decreasing geometrical progression. KH is the sum of the 3rd, 4th, 5th, 6th, and 7th terms. AF is the sum of the first eight terms." (page 131.)

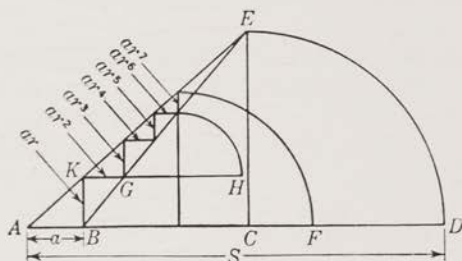


FIGURE 58

Certain assumptions are here involved: e.g. it is assumed that the two lines AK and BG meet, supposedly above the horizontal line AD to form the triangle ABE. The lines might be parallel and never meet; or they might diverge, and we would have "an increasing geometric progression" without any "sum" or limit. However, the assumed conditions assure that the lines will not be parallel and will converge, i.e. that the angle KAB will be less than  $45^\circ$ , and this is easily recognized as the sufficient graphical condition for convergence corresponding to the analytical condition that the ratio r of the progression be less than 1.

\* First Year College Mathematics, by Louis C. Plant and Frank R. Running; American Book Company, New York.

We wish to have greater freedom in order that we may have a general treatment of the graphical study of series. Hence instead of starting with the analytical expression for the series, we will start with an arbitrary triangle and construct the series by the indicated method of drawing successive transversals (including if desired the base as a transversal, as AB in Figure 58) each one being drawn perpendicular to the preceding one. Furthermore we will extend the "Triangle" so as to include not only the sides but their extensions, i.e. our "triangle" will consist of the three unlimited lines whose intersections form the three vertices. This allows us also to extend the notion of a "transversal" so as to include any segment having its extremities on two of the three lines; when the extremities lie outside the bounded triangle, the segment will be called an external transversal. Thus in Figure 1b, AB is an external transversal. The only condition we place upon the form of the triangle is that the three chosen lines meet in three distinct points in the finite part of the plane. As the position of the figure is immaterial we will, for convenience, place the first side or "base" line horizontal and will assume the ordinary conventions as to direction of line segments used in analytical geometry.

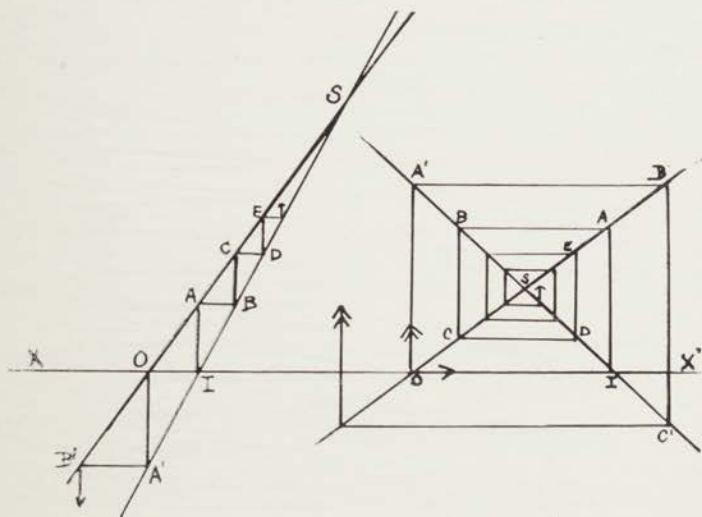


Figure 1a

Figure 1b

Hence: Let OIS (Fig. 1a) be any triangle with the base lying along the horizontal  $XX'$  axis and one vertex at the origin. Starting from the vertex O we move along the  $XX'$  axis to I, then move vertically till we meet the other side of the triangle, at A; then horizontally to B, vertically to C, and so on. We thus form two series of triangles; one, the left-hand series, OIA, ABC, etc.; all of which are similar; and the right-hand one, IAB, ECD, etc., which are likewise

similar to each other but not similar to those of the left-hand series (unless the angle AIB should happen to be equal to the angle AOI, as it is in Fig. 58).

Let the OI be taken, for convenience, as the unit of length, and let the ratio  $IA/OI=r$ , and the ratio  $AB/IA=t$ ; then from the similarity of the two sets of triangles, the series thus constructed corresponds to the following algebraic expression:

$$(I) \quad s = 1 + r + rt + r^2 t + r^2 t^2 + \dots + r^n t^{n-1} + r^n t^n + \dots$$

In order that this series may be convergent it is necessary that the steps should diminish in size and constantly approach the vertex S. We can easily see by inspection that this will be true if and only if the transversal AB is less than OI, i.e. that the product  $rt$  of the two ratios is less than unity. If we move backwards (or downwards) along this "staircase" we have an associated divergent series  $OA', A'B',$  etc.

Measuring the successive segments or transversals in any convenient scale we find, in Fig. 1a,  $IA/OI = r = 3/2$ ;  $AB/IA = t = 1/3$ , and  $AB/OI = rt = (3/2)(1/3) = 1/2$ .

If  $r = t$ , as in Fig. 58, then the series becomes:

$$(II) \quad s = 1 + r + r^2 + r^3 + \dots + r^n + \dots$$

which is the normal geometric series.\*

We will call a series of form (II) a Simple G.S., and any series of form (I) a Compound G.S.

The VALUE or SUM of a Geometric Series.

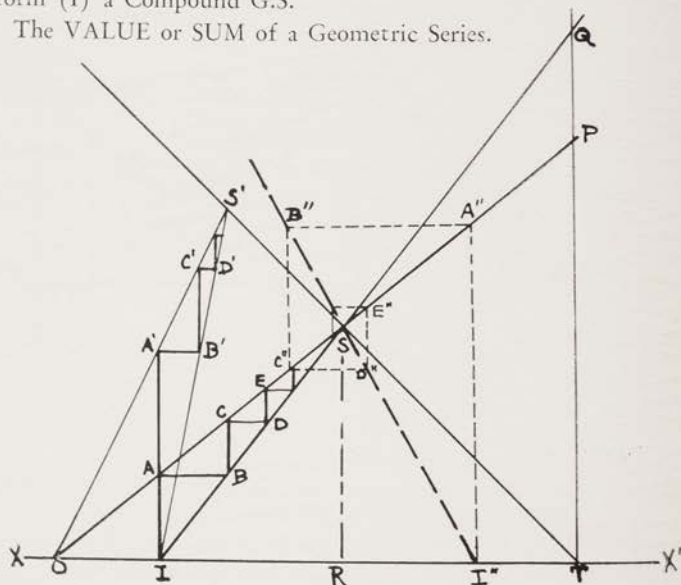


Figure 2

\*Using the trigonometric relations of the figures, we see that in (I) we can put  $r = \tan IOA$  and  $t = \tan AIB$ . In II  $r = t = \tan EAB$  as in Fig. 58.



Through the point S of Figure 2, draw a line making an angle of  $135^\circ$  with  $XX'$  and cutting it in the point T, so that angle OTS equals  $45^\circ$ . Drop the perpendicular from S to  $XX'$  meeting it in R. The sum of all the horizontal transversals (i.e. of all the odd terms of the series) is OR, and the sum of all the vertical transversals (i.e. the sum of all the even terms of the series) is RS, and since SRT is an isosceles triangle  $RT = RS$  and therefore the sum of the series is  $s = OR + RT$ .

In the particular case represented in the triangles OIS and OIS' of Fig. 2 all the quantities are positive, but as we have not restricted the form of the triangle, any one or several of the determining factors could be negative as some are in triangle OI"S, Fig. 2. The quantities involved are OI and the two ratios r and t. Combining positive and negative values of these in various ways we will obtain figures in all four of the coordinate quadrants and those involving external as well as internal transversals, thus giving us series with all positive terms, others with all negative terms and others again with mixed positive and negative terms.

On further examination of Figure 2, we see that all the series contained within any triangle with its base anchored at O and having its vertex on the  $45^\circ$  line ST (extended in both directions) will have the same sum, OT, as the series in triangle OIS. The series belonging to such a set with the same common sum or value will be called Equivalent Series. Among the equivalent series built on the same base, as OI, there will always be at least one Simple G. S. as can be seen from the following construction:

Let OI, Fig. 2 be the base of length b, and OT, of length s be the sum of the series. At T erect a perpendicular of length s-b terminating at P and the length s terminating at Q; so that PQ equals b PQ; then the tangent of the angle POT is  $(s-b)/s$  and the tangent of QIT is  $b/(s-b)$ ; therefore angles POT and QIT are complements, and hence angle AIB equals angle AOI and the series included in OIS is a Simple G. S.; moreover, from the symmetry of triangles OSI and QSP, it is clear that S lies on the bisector of the right angle OTQ. This Simple G. S. is therefore equivalent to any other G. S. with base OI and vertex on ST.

#### "Addition" and "Subtraction" of Series.

Given any two series with sums s and s', we can easily construct graphically a Simple G. S. or an arbitrary Compound G. S. having the same sum as the added sums of the two given series. Choose on the line  $XX'$  any point O for origin, lay off OT equal to  $s+s'$  and construct the  $45^\circ$  line through T; any new series anchored at O and with its vertex on the  $45^\circ$  line will have for its sum that of the other two series combined.

If we lay off on  $XX' OT''$  equal to  $s - s'$  and proceed as above we obtain a series with sum  $s'' = s - s'$ : which is equivalent to subtraction of two series.

It should be noted that the "sum" or "difference" of two series thus obtained is not a unique result; there are infinitely many series with the same sum  $s + s'$  or  $s - s'$ , as we have already indicated. We might however, place some further conditions on the new series and get a single or at least a finite number of resulting series for the "sum" or "difference"; thus we might demand that the base of the new series be equal to sum of the bases of the previous series and that the new series be a Simple G.S.; or that it be a Compound G.S. with a given value of  $t$ , or  $r$  or  $rt$ .

### "Multiplication" and "Division" of Series

Given two series with sums  $s$  and  $s'$ , we could graphically construct a line segment of length  $ss'$ , or of length  $s/s'$ , and then by the method indicated above derive the graphical series with sums equal respectively to product or quotient of the two given series. This however, would scarcely be a graphical equivalent of the usual process of arithmetic or algebraic multiplication. Hence the following process is more satisfactory and would probably give students a better idea of multiplication of series term by term.

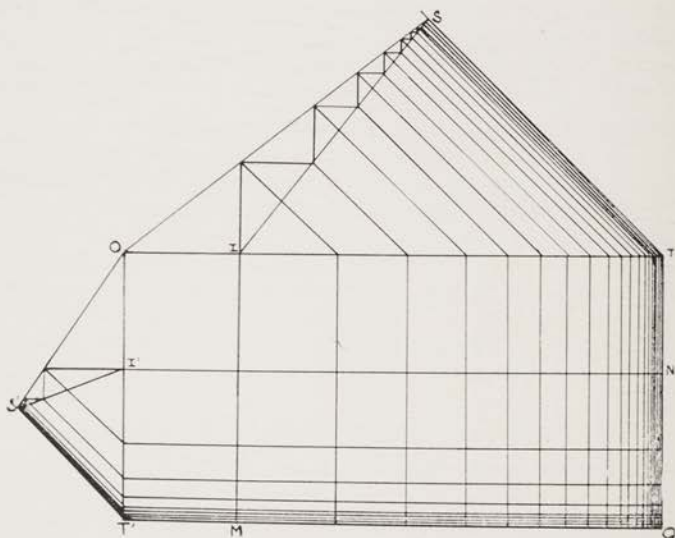


Figure 3

Take two lines at right angles to each other. Lay off, (as in Fig. 3), from their intersection O, on one line the length OT equal to  $s$  and

on the other the length  $OT'$  equal to  $s'$ . Construct the graphs  $OIS$  and  $OIS'$  of the two given series on their respective lines and project the successive terms on these lines by a set of parallels at an inclination of  $45^\circ$  to these lines. Complete the rectangle  $OT'QT$  with sides  $s$  and  $s'$ , and from the terminus of each projected term draw transversals of the rectangle as indicated in the figure. This process divides the rectangle  $OQ$  into a double infinity of smaller rectangles each of which has an area equal to the product of two terms of the two given series. Of course we cannot draw all the lines, since there is an infinite number of terms in each series; we can however, draw a sufficient number to give a striking ocular presentation of the process, and the *area* of the large rectangle is numerically equal to the sum of all the smaller rectangles and hence to the *product* of the two series.

In case one or both of the two given series have positive and negative terms we would draw our projection parallels through the termini of positive terms only thus projecting the difference (algebraic sum) of the pairs of positive and negative terms.

To "divide" one series by another: Let the sum of the divisor series be  $s'$  and that of dividend series be  $s$ . Construct a rectangle with one side equal to  $s'$  and the other to  $s/s'$ . Project the terms of the division series (by  $45^\circ$  parallels) on its side of the rectangle; then draw the diagonal of the rectangle and project the series of the divisor-side onto the other, or quotient-side of the rectangle by parallels to the diagonal. The series of terms along the quotient-side are proportional term for term to those on the divisor side; hence if we now set up the triangle determined by the first three terms of the quotient side we will have another graphical series which when multiplied by the divisor, by the method of the preceding paragraph, will give us the dividend. But as any other series equivalent to the constructed quotient-series would, when multiplied by the divisor-series, give the same product, this process of division will not enable us, except by imposing further conditions, to determine a unique quotient-series. The one we did construct was definitely determined by making it fully geometrically similar to the divisor series though larger in size.

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### SERIES WITH MULTIPLE RATIOS

The methods outlined above can be extended to more complex series, and similar methods can be devised for other forms of series. We will here indicate some of these extensions. Instead of one inclusive triangle we may use as many as we wish, restricting ourselves to those having a common vertex.

Example: Series with three ratios instead of two. (Fig. 4).

In this example the three ratios are  $r = 5/6$ ;  $s = 4/5$ ;  $t = 1/2$ ; and  $rst = 1/3$ .

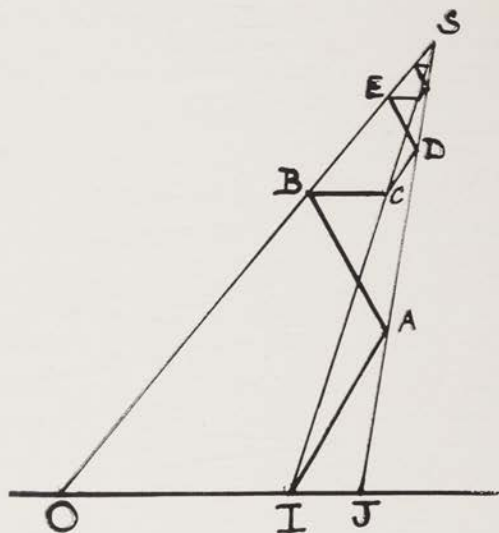


Figure 4

The series is expressed algebraically thus:

$$S = 1 + r + rs + rst + r^2st + r^2s^2t + r^2s^2t^2 + r^3s^2t^2 + \dots$$

It is easily seen that the necessary condition for convergence is that the product of the three ratios  $rst$  must be less than 1, i.e. that the segment  $BC$  is less than  $OI$ .

The greater the number of ratios the greater will be the possible combinations of algebraic signs. Moreover, when the number of ratios is odd, say  $2k - 1$ , we can no longer restrict the direction of the successive segments or "steps" to differences of  $90^\circ$ , since the (algebraic) sum of the angles which the successive steps make with each other in each complete cycle must be equal to zero or to an integral multiple of  $360^\circ$ . Only thus can it be assured that the first segment of the second cycle will have the same direction as that of the first segment of the first cycle; and so on for the other successive cycles. In the accompanying diagram, Fig. 4, the three successive deviations are  $60^\circ$ ,  $60^\circ$  and  $-120^\circ$  with sum zero.

Since, as stated above, we can always determine a Simple G. S. equivalent to any given Compound G.S., it follows that every Compound G. S. is equivalently the sum of a number of Simple G. S. all with the same simple ratio; and these can finally be added so as to form a single equivalent Simple G. S. This conclusion can also be derived algebraically from the example given above, which is factorable as follows:

$$S = (1 + r + rs) (1 + rst + r^2s^2t^2 + \dots + r^n s^n t^n + \dots)$$



## DIVERGENT SERIES

We mentioned above the concomitance of divergent series with given convergent series, the former merely being, as it were, the continuance of the convergent series in "reverse," in which the individual terms of the series increase indefinitely in size, being in fact reciprocals of the ever diminishing terms of the convergent series. We can also treat by graphical means certain classes of divergent series of which the individual terms tend towards zero as their limit. Take, e.g. the following series:

Summation of  $1/\{V(n) + V(n-1)\}$  for  $n = 1, 2, 3, \dots$

N.B. In this paragraph the symbol  $V(\ )$  stands for "the square root of".

The terms of this series decrease towards zero since the numerator is unity and the denominator increases without limit, it being understood that the positive square roots are to be taken in all cases. This series may be represented by the following construction, which has its own interest independently of the character of the series.

Construct as in Fig. 5 a square with sides of unit length and extend indefinitely one pair of parallel sides, OA and MB. With O as center and  $OB_1$  as radius lay off on OA the length  $OA_2 = V(2)$  then erect the perpendicular at  $A_2$  determining the point  $B_2$  on MB, which makes  $MB_2 = V(2)$ . With  $OB_2$  as radius and O as center lay off  $OA_3$ ; and repeat this construction indefinitely; we will then have a series of segments

$$OA_n = V(n), \text{ for } n = 1, 2, 3, \dots$$

Hence each addition to the previously constructed total segment is the difference between the square roots of two consecutive integres, namely

$$\begin{aligned} OA_n - OA_{n-1} &= A_n A_{n-1} = V(n) - V(n-1) \\ &= 1/(Vn + Vn-1), \end{aligned}$$

and hence, for  $n = 1, 2, 3, \dots$  to infinity the required series is given graphically by  $OA_1 + A_1 A_2 + A_2 A_3 + \dots + A_{n-1} A_n + \dots$

It is clear that the successive terms are growing individually smaller. From the triangle  $B_2 OA_3$  we see that  $A_2 A_3$  is equal to the tangent of the angle  $A_2 B_2 A_3$  which is one half the angle  $B_2 OA_3$  and in general  $A_n A_{n+1} = \tan A_n B_n A_{n+1}$  or  $\tan \frac{1}{2} (B_n OA_n)$ ; since the altitude of the triangle containing this angle is always unity the angle can never become zero for any finite length of the base  $OA_n$

Hence, finally, the sum of the series will never cease increasing as long as it has a finite value; i.e. the series is a divergent series of which, however, the individual terms tend to zero as their limit as the sum of the series tends to infinity.

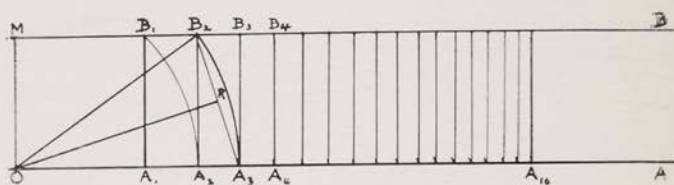


Figure 5

The graphical method can be used to build up and study "multi-valued" series, i. e. series which tend towards two or more finite limits and hence are classed among "Divergent Series," and are more specifically designated as "Oscillating Series." These, however, will not be considered at this time.

Georgetown University,  
Washington, D. C.

## NEWS ITEMS

### HOLY CROSS COLLEGE CHEMISTRY DEPARTMENT

Holy Cross was included on the list of colleges expected to train at least six hundred Naval officers, according to newspaper announcements of February 14, 1943. Naval inspectors visited the college for this purpose on February 24th. On March 7, the college appeared on the list of pre-medical training centers. This course is expected to commence on July 1st.

On January 6th, Father Fiekers addressed the Clark University Seminar group on some current research in this department, viz., aryl amine alkyl halide condensation. Professor Tansey addressed the same group on March 3rd, speaking on the reduction of aromatic nitro-compounds with sodium sulfite. The Clark University Bulletin (theses and dissertation abstracts) for 1942, carries two doctorate dissertation abstracts by members of the Holy Cross Faculty; Neutral Reduction Intermediates of Nitrobenzene, *14*, 3-8, (1942) by Father Fiekers, and the Vapor Phase Nitration of Toluene, *14*, 8-13, (1942) by former Professor Edwin T. Mitchell.

A biographical sketch of Father Theodor Wulf, S.J., (physicist), that appeared in the *HORMONE* for October 1942 has stimulated some curiosity about Father Wulf and his work, judging from correspondence received by the staff. Professor O. L. Baril's editorial in the same issue received favorable comment in the *Journal of Chemical Education*. An article in the November issue by Mr. R. M. Dee, graduate assistant, on the direct determination of oxygen in organic compounds was listed in the first number of *Chemical Abstracts* for 1943.

#### SEMINAR PROGRAM

(Spring Session 1943)

(During the Spring Session seminars will be held on Saturdays at 11)

#### PHYSICAL CHEMISTRY

March 27, 1943. REACTION RATES: ALKYL HALIDES-ARYL AMINES\* by Mr. Michael F. Kilty, Graduate Assistant. Discussion: Mr. R. A. Bruno, Sen. B.S. Chem. Director: Prof. J. J. Tansey.

April 3, 1943. THE DIPHENYL AMINE-BENZOIN SYSTEM by Mr. Michael D. Riordan, Graduate Assistant. Discussion: Mr. F. R. Carrier, Sen. B.S. Chem. Director: Prof. J. J. Tansey.

(\* Omitted from Winter Schedule).

#### ANALYTIC CHEMISTRY

April 10, 1943. CHROMATOGRAPHY AND ANALYSIS by Mr. Robert M. Dee, Graduate Assistant. Discussion: Mr. E. M. DiGeronimo, Sen. B.S. Chem. Director: Prof. G. J. Charest.

#### ORGANIC ANALYSIS

April 17, 1943. GATTERMANN REACTION: ANALYTIC ASPECTS by Mr. Charles J. McNulty, Graduate Assistant. Discussion: Mr. H. W. Dion, Sen. B.S. Chem. Director: Prof. O. L. Baril.

May 1, 1943. THE ANALYSIS OF ARYL AMINES by Mr. John P. Hardiman, Graduate Assistant. Discussion: Mr. A. E. Frost, Sen. B.S. Chem. Director: Prof. J. J. Casey.

#### ORGANIC CHEMISTRY

May 8, 1943. DIPHENYL DERIVATIVES IN THE GATTERMANN REACTION by Mr. James M. Owens, Graduate Assistant. Discussion: Mr. T. T. Galbowski, Sen. B.S. Chem. Director: Prof. O. L. Baril.

#### HISTORY OF CHEMISTRY

May 15, 1943. HISTORY OF THE CHEMISTRY AND OTHER SCIENCE DEPARTMENTS AT THE COLLEGE OF THE HOLY CROSS IN WORCESTER, MASSACHUSETTS by Mr. William P. Whelan, Jr., Graduate Assistant. Discussion: Mr. D. J. Hill, Sen. B.S. Chem. Director: Father Fiekers, S.J.

#### PHYSICAL CHEMISTRY

May 22, 1943. OPTICAL METHODS OF CHEMICAL CONTROL by Mr. Charles A. Polachi, Graduate Assistant. Discussion: Mr. J. K. Michaels, Sen. B.S. Chem. Director: Prof. J. J. Casey.

#### ST. JOSEPH'S COLLEGE PHYSICS DEPARTMENT

St. Joseph's High School of Philadelphia, is giving a course in Radio Fundamentals to its Seniors. The course is an elective and is held on Saturdays from nine till two o'clock.

The course is being made as practical as circumstances permit. The class is divided into groups of three and each group has special projects—either building or repairing radio equipment.

Fundamental theory is taught to the whole class and the experiences of each group are communicated to all the students.



## WESTON COLLEGE SEISMOLOGY DEPARTMENT

Lectures on Seismology have been delivered to the following groups within recent months by Father Linehan.

January 26. Civil Engineer Students at Worcester Polytech Institute.

February 21. New Hampshire University, Durham, N.H.

March 8. Newton Alliance Club, Newton, Mass.

March 23. St. Clements School, Brighton, Mass.

April 1. Bond Astronomical Club, Harvard University.

April 6. Sacred Heart Church Club, Quincy, Mass.

April 7. Boston Society of Civil Engineers, Boston, Mass.

May 18. Worcester Society of Civil Engineers, Worcester, Mass.

During February and March, the Benioff Vertical Seismometer that once operated at University of Vermont, was set up at Weston Observatory in a series of test runs. This instrument compared very favourably with similar instruments in operation at both Weston and Harvard, although only the short period recording is employed.

Weston ran these tests at the requests of Dr. Harlow Shapley of Harvard University, and then supervised packing the instrument and shipping it to Mexico. It is being loaned to Mexico as part of a "good will" program that is being carried on by the scientists of both countries.

The War is making itself felt at the Observatory with the shortages and new regulations that are being imposed. Although photographic supplies are not under priority, and most supplies as developers, etc., may be obtained in small amounts, still large orders of papers, as News Bromide, are very slow in being delivered. What used to take two weeks at most, now are as much as three months behind time.

Obtaining explosives for seismic survey work is also bound up with a lot of "red tape", and the permission to use same when once obtained, is becoming more and more difficult to get. One or two survey projects have been planned for the coming summer.

## ST. PETER'S COLLEGE CHEMISTRY DEPARTMENT

Every analytical balance in the department is now equipped with a damping device and also a fluorescent cold light, which increases the efficiency and the saving of time in analytical procedures.

Through fortunate circumstances, we have acquired a Kuhlmann micro balance; it is one of the original balances made by Kuhlmann who made the first micro balance for Dr. Pregl, who is the Nobel prize winner.

## CANISIUS COLLEGE CHEMISTRY DEPARTMENT

Twenty-one students majoring in Chemistry at Canisius College applied to the American Chemical Society to form a Student Affiliate Group of the ACS. Nine of the 21 students are members of the Senior class which will graduate in December, 1943, and they will form a nucleus for the remainder of the Chemistry-major students in the 3 lower classes., who may apply to join the newly organized group.

The idea of Father T. Joseph Brown of having the professors, graduates and students of Jesuit Universities and Colleges attend a dinner or luncheon during the meeting of the American Chemical Society will be continued at the Detroit meeting April 12-16, 1943.

Father George J. Shiple, S.J., Chairman of the Department of Chemistry, University of Detroit, has assured Father Brown that a luncheon will be held on Thursday, April 15th at Detroit.