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CONTENTS

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PHYSICS

CORNU'S SPIRAL AND DIFFRACTION ANALYSIS

Part II[1]

Stanley J. Bezuszka, S.J.

After the preliminary mathematical discussion given in Part I, we can now turn to the graphical treatment of the problem and analyze the solutions from this point of view. A frequent representation of the Cornu Spiral employs the vector concepts involved in considering it as the closing side of a polygon. When the sides are thus treated as proportionality factors, representing the amplitudes of the wave disturbance, and making definite angles with a fixed axis to represent the phase shifts, the spiral can be roughly plotted. If now the elements of arc are taken infinitely close to one another, the familiar smooth curve of the spiral results. But in addition to this procedure there are a few other methods in common use.

The original two definite integrals of Fresnel

$$\int_0^{\infty} \cos\frac{\pi v^2}{2}\,dv = \tfrac{1}{2} \qquad\qquad \int_0^{\infty} \sin\frac{\pi v^2}{2}\,dv = \tfrac{1}{2} \quad . \ . \ . \quad (22)$$

cannot be evaluated by standard forms in finite terms except for the case formerly mentioned, that is, when v is set equal to ∞, and then they are respectively equal to $\frac{1}{2}$. However, Fresnel, Gilbert, Cauchy and Knochenhauer employing different approaches have drawn up a table of values for the Fresnel Integrals for various other points. Accordingly, one of the simplest methods for drawing the spiral is to take the Fresnel Integrals and evaluate them for v from 0 to ∞, getting the corresponding table of values for the integrals and plotting the result on a Cartesian Coordinate system. This was the process originally used by Cornu.[2]

There is a geometric method for the construction of the Cornu Spiral which may appeal to many readers. This process utilizes the previously derived equations:

$$\tan\gamma = \frac{d\eta}{d\xi} = \tan\frac{\pi v^2}{2} \qquad \gamma = \frac{\pi v^2}{2} \quad . \ . \ . \ . \ . \quad (21)$$

$$\rho = \frac{ds}{d\gamma} = \frac{1}{\pi v} = \frac{1}{\pi s}$$

1. The Author wishes to acknowledge his sincere obligations to Dr. Reinheimmer of Boston College whose assistance and technical advice have made this part possible.

If now we take our Cartesian Coordinate system and mark off lengths in units of one-tenth, on both the abscissa and ordinate, we can construct the spiral as follows:

$\rho = r$ is the radius of curvature of the curve in the considered point.

$\gamma = g$ is the angle which the radius of curvature in the considered point makes with the ordinate.

Since for an element-length of the spiral equal to 0.1, the arc coincides with the abscissa length 0.1, we can get our first point by using the origin of the coordinate system and marking off a length of s equal to 0.1 on the abscissa (s of course is the same as v). Substitute for s equal to 0.1 in the above equations getting r equal to 3.18 and g equal to 0.9 degrees. Now using the point 0.1 as the origin, draw a line a making an angle of 0.9 degrees with the ordinate. Lay down on this line a, starting from the point 0.1, the length of 3.18 units and obtain thus the point A. A is now the center of a circle tangent to the curve in the point 0.1. Draw an element of an arc of a circle with A as center and a radius of 3.18 units, and mark off from the point 0.1 along the arc just drawn an element equal to another 0.1. This gives the next point for our construction and a total value of s equal to 0.2. With s equal to 0.2 , r is 1.59 and g is 3.6 degrees. Starting with the point 0.2 on the arc as the origin, draw a line b making an angle of 3.6 degrees with the ordinate. Lay down on this line b , starting from the point 0.2 , the length of 1.59 units and obtain thus the point B . B is now the center of another circle tangent to the curve in the point 0.2 . Draw an element of an arc of a circle with B as center and a radius of 1.59 units, and mark off again from the point 0.2 along the arc just drawn another element of 0.1. This is the next point for the construction and gives a total value of s equal to 0.3. If the above process is now repeated we can construct as many convolutions of the spiral as are desirable. The following sketch, which has been greatly exaggerated to exemplify the process, may assist the verbal description.

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2. For an evaluation of the Freshnel Integrals and calculation of intensity, the following references are useful:
   É. Verdet: "Oeuvres" Tome V, 'Leçons O'Optique Physique', Tome I Ed. 1869, p. 328 et seq.

   H. Bouasse and Z. Carrière, "Diffraction" Ed. 1923, p. 312.
   Thomas Preston, "The Theory of Light", Fourth Edition 1912, p. 290 where a table worked out by Gilbert is printed.

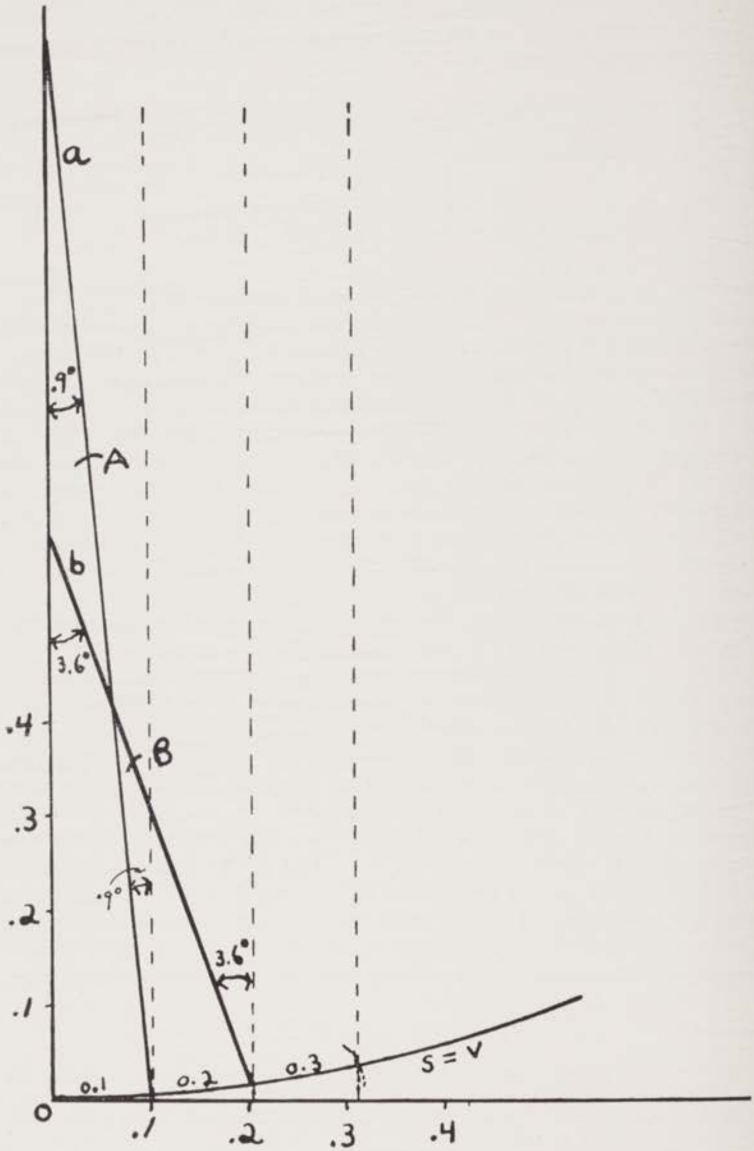

Fig. A

To use the spiral as a means of immediate calculation for the intensities, we shall take the two special cases previously discussed, namely, the single slit and the straight edge.

In general, any vector line on the spiral connecting two points represents the amplitude of the resultant vibration, and the angle it makes with the abscissa will be its corresponding phase. The straight line ZZ' when squared and multiplied by ½ gives the intensity of an entire wave when unobstructed by any object. (cf. Fig. B). It is important to note that for unobstructed light:

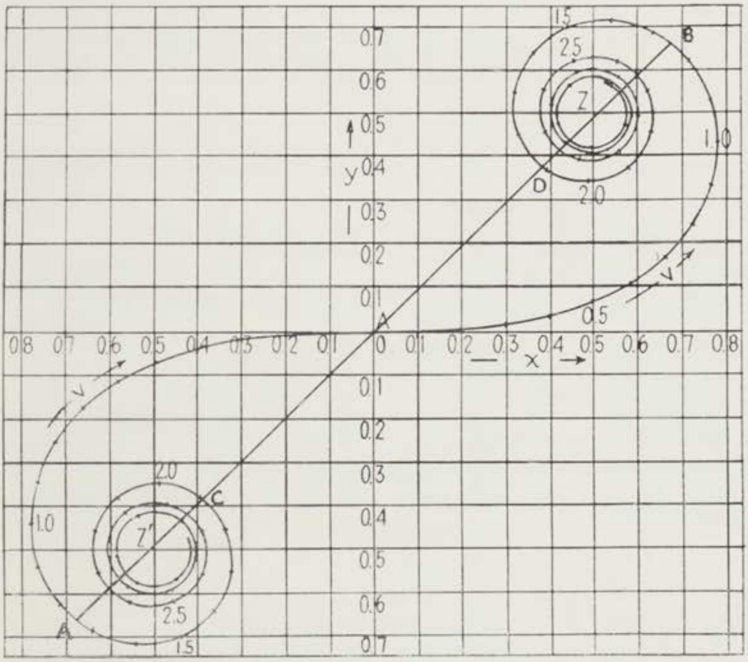

FIG. B

$$I_0 = \frac{A^2}{2(\rho_0 + \rho_1)^2}(-\infty, \infty)^2$$

$$= \frac{A^2}{2(\rho_0 + \rho_1)^2}(2) = \frac{A^2}{(\rho_0 + \rho_1)^2}$$

Then for any other intensity whatsoever, (taking the straight edge as the particular case), $I = \frac{I_0}{2}(-\infty, v')^2$

if I_0 (the 'Natural Intensity') is set equal to one, we shall get

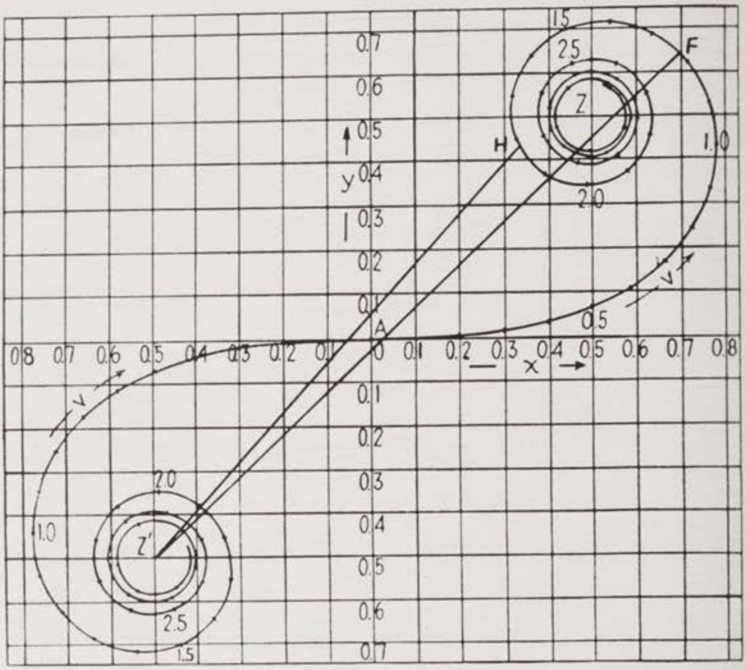

FIG. C

all other corresponding intensities in terms of the natural intensity. On the other hand, if I_0 is set equal to 100, then we get the intensity values for all other points as percentages of the natural intensity.

Now for the single slit, and calculating only the intensity distribution for the center points, namely, the maxima or minima for the point d equal to zero, we have from our preceding equations:

Maximum when $$v_1 = \sqrt{\frac{3}{2} + 4n}$$

where n is zero,

and from $$\frac{v_1 + v_2}{2} = \frac{d}{P} \qquad (19)$$

for d equal to zero

$$v_1 = -v_2 \qquad (19a)$$

Consequently, $$v_1 = -v_2 = 1.23$$

Minimum when $v_1 = \sqrt{\frac{7}{2} + 4n}$ where n is zero and using (19a)

Consequently, $v_1 = -v_2 = 1.88$

If we now mark off the points v equal to 1.23 on both the positive and negative side of the spiral and connect them, we get the straight line AB (Figure B) which when squared and multiplied by ½ gives the intensity of the center shown in figure 2 . Note also that for figure 3, if we connect the points v equal to 1.88 on both sides of the spiral, the line will correspond with the previous one, but now has a length of CD . If this amplitude is again squared and multiplied by ½, we have the intensity of the minimum for the set point d equal to zero. Thus alternating from the outside to the inside half portions of the spiral we get the successive values of maxima and minima for just the center points of the diffraction patterns of a single slit.

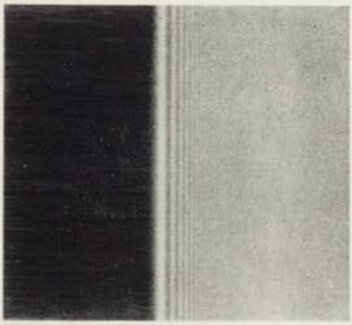

FIG. 1

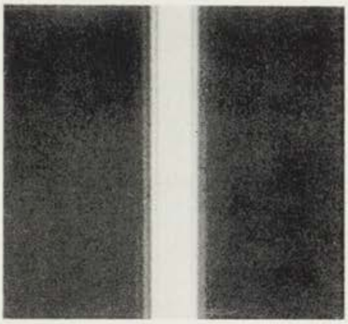

FIG. 2

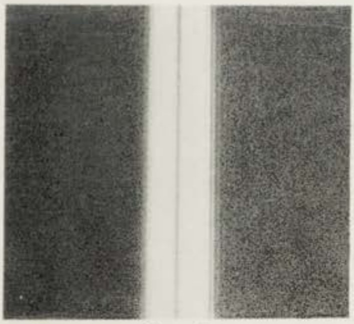

FIG. 3

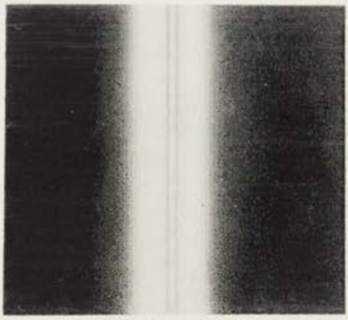

FIG. 4

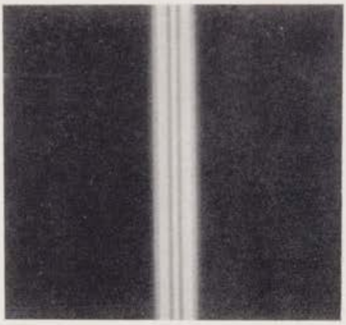

FIG. 5

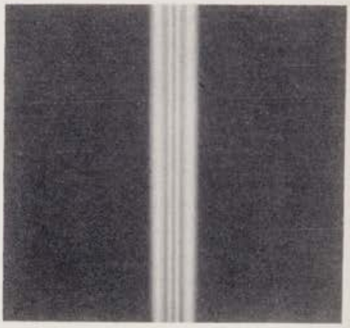

FIG. 6

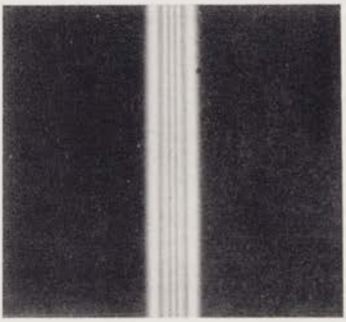

FIG. 7

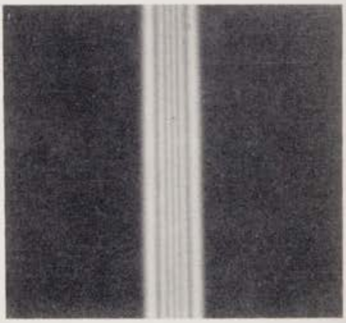

FIG. 8

For the intensity distribution of intermediate points, the method is a trifle more involved and laborious. We first solve the equations

$$v_1 - v_2 = \delta \sqrt{\frac{2}{\lambda}\left(\frac{1}{a} + \frac{1}{b}\right)} \quad . \; . \; . \; . \; . \; . \; . \; (17)$$

$$\frac{v_1 + v_2}{2} = \frac{d}{P} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (19)$$

for varying values of d, getting definite values for v_1 and v_2. By connecting these points on the spiral and calculating the intensity by the method given above, we can construct the complete intensity curve.

In the case of the straight edge, one end of the line will always begin at the point Z', and terminate on various values of v on the other portion of the spiral. The point v will also extend on the negative portion of the spiral for a short distance since this represents the intensity in the geometric shadow and falls off very rapidly. Thus the illumination distribution at the plate will alternate between maxima and minima. Values can be calculated from the theory of Part I, and the points plotted. Some of the values found for the

maxima are: $v = 1.225$, 2.345 , 3.08

for minima: $v = 1.821$, 2.739 , 3.391

Connecting the point Z^1 with the above values of v , (Figure C) then squaring the distance and multiplying by ½ , will give the corresponding intensities of the maxima and minima for the straight edge diffraction pattern in figure 1.

In taking the photographs of the diffraction patterns, monochromatic light (5461 A) was used, the set up consisting of a mercury vapor lamp and a Wratten Filter No. 77. Figures 3 and 4 are composities, that is, two negatives were used, one for the dark centers and another for the fringes. Then in the print, the center was covered and the fringes exposed, and on the same print was superimposed the dark center. This process has distorted to some extent the appearance of the slit-width and obscured a few fine fringes that begin immediately on both sides of the two dark lines as is seen in figure 5 which is an exposure of just the center portion of figure 4. In figures 6, 7 and 8 , the increased slit-width made fringes impossible to photograph, and here just the center portions with only one or two close fringes are shown.

Figure	Slit-width
2	0.225 mm and 0.480 mm
3	0.734 mm
4	0.871 mm
6	1.078 mm
7	1.206 mm
8	1.333 mm.

The original negatives are still in the possession of the Author and may be had if individual copies of the prints are desired.

THE ELECTRON MICROSCOPE

Rev. Laurence C. Langguth, S.J.

In a previous issue of the Bulletin[1], Fr. Fiekers described a new "supermicroscope" then being produced commercially in Germany. It was an electronic rather than an optical device, and claimed a two thousand percent increase in magnification over its optical predecessors. The recent appearance of advertisements announcing the development of a similar instrument by an American manufacturer[2], and

[1]Bul. A A. J. S. *16*, 3 (March 1939), pp. 121-124.

[2]RCA Manufacturing Co., Camden, N. J. They distribute an excellent illustrated booklet describing the instrument; to it acknowledgment is made for some of the material in this paper not otherwise credited.

the expanding use of the instrument especially in Europe and in the fields of biology and medicine, suggest the utility of a more extended treatment of the principles of its operation and of the limitations it encounters.

In the early history of the development of the compound microscope, it was thought that the magnification that could be obtained was limited only by the skill of the designer in grinding lenses and mounting lens combinations so as to minimize spherical and chromatic aberration[3]. This was later discovered to be only partly true; for though the magnification can be increased without limit, still it is effective in bringing out useful detail only up to a certain point. This ability to register detail is known as the resolving power of the instrument, and the limitation it meets is an inherent consequence of the wave nature of light. For according to the wave theory, any selected point on the object to be imaged will not be represented by a correspondingly inextended point in the image, but rather by a series of minute concentric circles, alternately dark and light (if monochromatic light be used), centered upon what would be the geometric image of the point. These concentric circles are diffraction patterns of the lens aperture in the plane of the image. If two point sources in the object are extremely close together, their diffraction patterns in the image plane will to some extent overlap and produce a degree of confusion. Just how much confusion may be permitted is an arbitrary decision, but it is generally agreed that two such points shall be said to be resolved when the central maximum of one of their diffraction patterns falls upon the first minimum of the other[4]

The closest approximation of two such points which can just be resolved by a microscope, is defined as the instrument's *resolving limit*. Abbe, the brilliant theoretical optician with Zeiss at Jena, derived the classical expression for this resolving limt—

$$R = \frac{1.22\,\lambda}{2\mu \quad \sin\alpha}$$

where $\lambda =$ the wavelength of light used

$\mu =$ the refractive index of the medium in which the object is immersed

$\alpha =$ the semi-angle of the cone subtended by the aperture of the lens at the object

Evidently the resolving *limit* can be decreased (or the resolving *power* increased) by decreasing the wavelength of the light, by increasing the refractive index of the external medium, or by increasing the angle of the cone of rays at the objective lens.

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[3]Carpenter in Encycl. Brit. (Werner Ed. 1904) s. v. Microscope.

[4]Meyer, The Diffraction of Light, X-rays, and Material Particles, p. 195.

Past attempts to improve the resolving power of the light microscope have been directed to each of these points, but each has its limit. The angle of the cone of rays is increased by increasing the aperture or the diameter of the objective lens, but as this is made larger spherical and chromatic aberration also increase and the correction of them becomes a very difficult matter. Moreover, with a focal distance of the order of a millimeter as in the present high power objectives, any increase in the diameter of the lens beyond a certain point is valueless, since the outer portions of the lens are so far from the object and at such a flat angle that they are practically useless in gathering light. This appears very clearly in Abbe's formula if we note that at 65° the sine function has already attained more than nine tenths of its value, so that further difficult increases in the angle will produce almost negligible increases in the sine. And at any rate the maximum value of the sine is unity.

The refractive index of the external medium is increased by the familiar biologic technique of oil immersion, where the oil is of greater index of refraction than the air which it displaces, and approximately the same as that of the glass with which it lies in contact. But this too has its limits, since there is no fluid used for the purpose whose index of refraction comes close to being twice that of air. Thickened cedar oil is the common immersion fluid, with index of about 1.52, a figure which is nearly the same as the index of crown glass.

The product of the two magnitudes just discussed, $\mu \sin \alpha$, is called the Numerical Aperture (N. A.) and is a measure of the resolving power of the objective lens system. The practical limit of the Numerical Aperture is generally taken to be about 1.4, which is obtained with a semi-angle of 67° and an index of refraction of 1.52[5]. However, by using a liquid of high index such as monobromonaphthalene, and special glass of similar index, it is possible to attain N. A. 1.65[6]. But even supposing that the sine of the semi-angle could be made unity (which would require a lens of infinite diameter!) and supposing that an immersion medium could be obtained of index 2, then still Abbe's formula would give for the resolving limit

$$R = \frac{1.22\,\lambda}{4} \quad \text{or about } 0.3\lambda$$

Very evidently, it is the wavelength of light which is the inexorable limiting factor.

The attempt to increase the resolving power of the microscope in this direction is also familiar, in the use of a blue filter on the illuminant, to cut off as much as practicable of the longer radiations at the red end of the spectrum and to concentrate on the shorter ones at the

[5]Gage, The Microscope (14th Ed. 1925), p. 296.

[6]Meyer, Op. cit., p. 208.

blue end. Further advances of course are made by using ultraviolet light, with photographic plates and quartz lenses. But in the visible microscope, with all possible refinements, the resolving limit cannot be brought much below 0.0002 mm. or 2000 A°[7]; and the magnification which can profitably be employed will not much exceed 1500.

The electron microscope makes its very spectacular advance by employing the very much shorter wavelengths associated with the electrons in a beam of cathode rays. Cathode rays were discovered during the latter half of the last century, and just at the close of the century were identified as streams of electrons, by reason of their deflection in electric and magnetic fields. That the electrons possessed a wave aspect as well as a particle aspect was predicted by de Broglie in 1924 and experimentally verified in 1927 by Davisson and Germer. Meanwhile, the real beginnings of electron optics had come in 1926, with the discovery by Busch that the electron stream could be not merely *deflected* but *focussed* by electric or magnetic fields that were symmetrical about the axis of the stream. And the pioneer work on the electron microscope was done in 1932 by Knoll and Ruska.[8]

The wavelength of an electron in motion is a function of the accelerating potential applied to it. This wavelength decreases as the voltage increases, being an inverse function of the square root of the potential, in accordance with de Broglie's formula—

$$\lambda = \frac{h}{m\,v} = \sqrt{\frac{150}{E}} \times 10^{-7} \text{ mm.}$$

where h = Planck's constant
m = the mass of the particle
v = its velocity
E = the accelerating potential in volts

Assuming a potential of 50,000 volts, which is a value commonly used in the electron microscope, we find that the wavelength of the electron is 0.003×10^{-7} mm. or 0.003 Å.

This value we shall substitute in Abbe's formula to approximate the order of resolving power we may expect. The value of μ will be unity, since the object is in a space free of the magnetic focussing field. And the term $\sin\alpha$ may be replaced by its approximate equivalent $D/2f$, where D is the aperture diameter of the electronic equivalent of the objective lens, and f is its focal distance. The apertures thus far used have been extremely small, as we shall see more in detail

[7]The Angstrom (Å) is a unit of length equivalent to 10^{-7}mm. or the ten-millionth part of a millimeter. The resolving limit of the human eye is generally taken to be 0.01 mm., hence 100,000 Å.

[8]Martin, in Nature, *142*, (Dec. 17, 1938), pp. 1062-1065.

later; so assuming that $D = 0.1$ mm. and that $f = 2.5$ mm. (N. A. 0.02), Abbe's formula may be rewritten to give the approximate relation

$$R = \frac{1.22\lambda}{2\,D/2f} = \frac{1.22 \times 0.003 \times 10^{-7}}{2 \times 0.02} = 0.0915 \times 10^{-7}\text{mm.}$$

or 0.0915 Å. This is to be compared with the figure of about 2000 Å as the best resolving limit of the light microscope, to appreciate the tremendous advance possible with the electron instrument.

Needless to say, this figure has not yet been attained in practice. The best that has been attained so far (1940) is a resolving power of 30 Å, and particles of only 10 Å diameter are recognizable[9]. What are the causes that act to prevent the attainment of higher values in resolving power?

Perhaps the most serious cause is the spherical aberration of the electron lens, a defect exactly analogous to the similarly named "defect" or property of the usual light lens. It consists in the fact that rays passing through the lens near its outer portion are deflected more sharply, and hence brought to a focus at a point nearer to the lens than rays passing through its center. No one has yet succeeded in devising an electron lens free of spherical aberration, so the only way of decreasing its defocussing effect is to decrease the aperture of the lens. The wavelength of the electron being so much shorter than that of visible light, it is possible to make the aperture much smaller than that of a light microscope before incurring serious loss of resolving power by diffraction. As a matter of fact, electron microscopes now commonly have a numerical aperture of the order of 0.01 or 0.001, contrasted with a maximum of about 1.4 for the light microscope; but it is evident that if this aperture could be increased the resolving limit would be much extended. Rebsch, however, has investigated the problem theoretically[10], and claims to have proved that the resolving limit of the electron microscope will always be limited in this way to a value about ten or one hundred times the wavelength, instead of going even below the wavelength, as might be inferred from Abbe's optical formula.

Moreover, since the wavelength of the electron is a function of its accelerating potential, any fluctuation in that potential will be reflected in the character of the cathode beam; and the electrons instead of being all of one velocity and one wavelength (hence monochromatic, by analogy with visible light), will be of differing wavelengths distributed through a given spectrum. Now electron lenses,

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[9]Ardenne, in Zeits. f. Physik, *115* (1940), pp. 339-368 (as abstracted in Science Abstracts, Sect. A, *43* (July, 1940), Abs. No. 2257).

[10]Ann. d. Physik, *31* (March, 1938), pp. 551-560 (as abstracted in Science Abstracts, Sect. A, *41* (June, 1938), Abs. No. 2605).

whether electromagnetic or electrostatic, are subject to chromatic aberration just as an ordinary light lens, and again, no one has as yet been able to design an achromatic one. Therefore the way to remedy the trouble is at its source, by maintaining the accelerating voltage as nearly constant as possible. This is not too difficult. High potentials of the order of 50 kv. are most conveniently generated from alternating current, transformed, rectified, and filtered to remove the residual a. c. "ripple" voltage. Two Canadian experimenters have constructed a microscope, in which they have been able to reduce their ripple voltage to less than one volt in fifty thousand, when the current drawn is less than 0.1 milliampere[11]. In addition, unpredictable fluctuations in line voltage cause corresponding fluctuations in the rectified potential, which in general will be larger than such a ripple voltage figure, but these too can be reduced in great part by standard voltage-stabilizing circuits.

The polychromatism of the source, then, can be reduced pretty much below the threshold of disturbance, but more bothersome is the fact that any existent steady velocity distribution in the beam is enhanced by its passage through the specimen to be examined. Present-day microtomes cannot cut slices much thinner than about 10^{-3} mm. If such a specimen have atomic properties similar to those of aluminum, then the velocity distribution in the electron beam after it passes through the specimen will be about 580 volts,[11] which substituted in de Broglie's formula will indicate a 1% variation in the wavelength of the electrons. This polychromatism seems to be unavoidable, except by making the specimen sections very much thinner, and hence seriously limits the resolving power of the instrument in some applications.

There is a further cause of limitation of resolving power, involving the mechanism of collision—the collision of electrons in the cathode rays with the molecules, atoms and electrons of the specimen. The electron microscope operates not by reflected but by transmitted light, and discrimination between the parts of the object depends not on their relative *absorbing* power, whether natural or artificial due to impregnating dyes and stains, but on their differing *indices of refraction*. In other words, the object lying in the path of the cathode beam scatters the electrons. Those parts of it which cause most scattering will appear darkest in the image, because they will have allowed few electrons to pass on sufficiently straight to enter the narrow aperture of the objective lens. But the support upon which the specimen is mounted (not glass, for that would stop all the electrons, but a

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[11]Prebus and Hillier, Canadian Jour. of Research, Sect. A, *17* (April, 1939), pp. 49-63

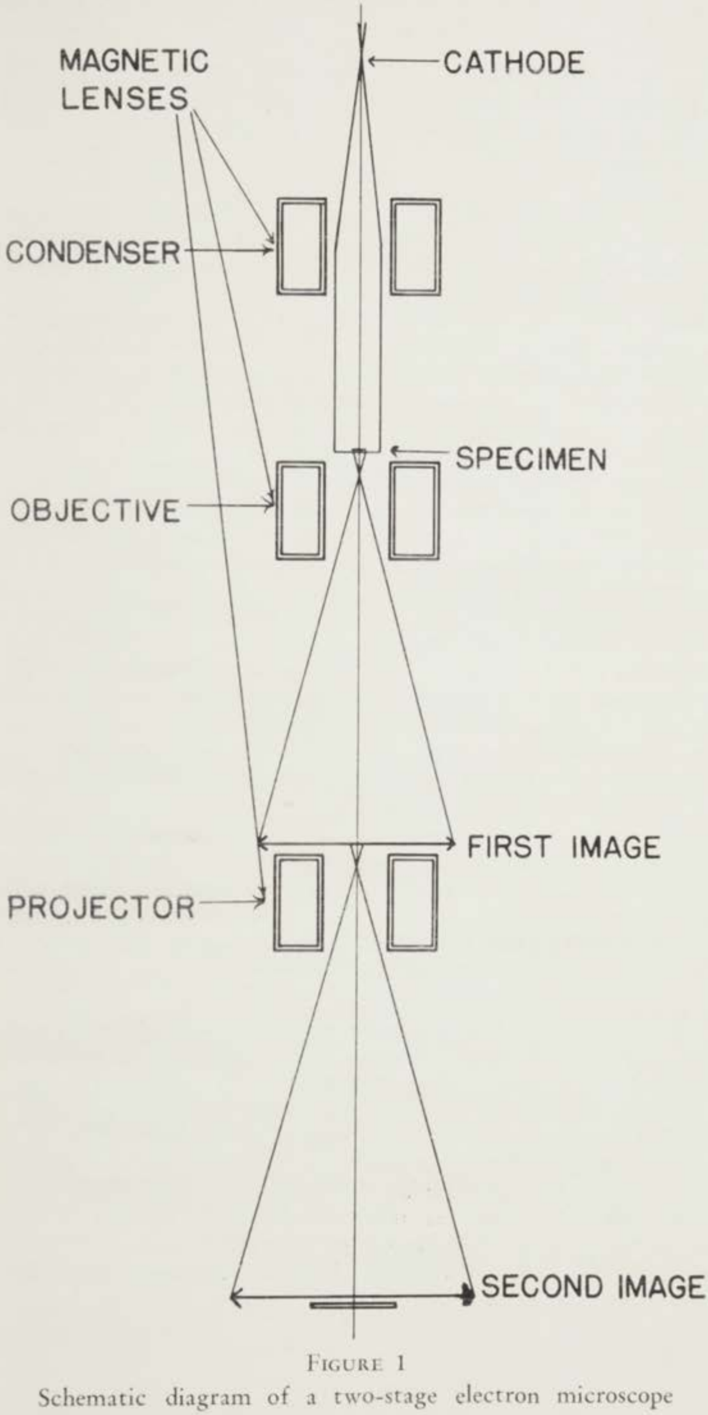

FIGURE 1
Schematic diagram of a two-stage electron microscope

thin film of nitrocellulose) itself has some scattering power. Hence the possibility of detecting an object or of separating two points close together depends on the relative scattering power of the specimen as distinct from the support[7].

These limitations imposed upon the electron microscope in principle are aggravated in practice by the difficulties of accurate design and rigid mechanical construction. Most of the microscopes described, and that includes the one offered by the American manufacturer, are modeled closely on the German instrument already described by Fr. Fiekers. The arrangement of parts is shown schematically in Figure 1. The electrons stream from a hot cathode at the top, pass downward through suitable accelerating and collimating anodes and through the condensing lens to the specimen chamber. Immediately below the specimen chamber is the objective lens, which focusses the electrons in the first image plane about a half-meter below. Here is produced a real image of the object only slightly magnified (of the order of 100 times). A fluorescent screen may be manipulated into position in this plane, for observation of the specimen and preliminary adjustments, and for selecting that portion of the image which it is desired to magnify further. When the fluorescent screen is swung out of the way, the electrons pass down through the projection lens, whose magnifying power multiplies the power of the objective lens, producing a second real image upon a second fluorescent screen or upon a photographic plate. The overall length of the electron path is about two meters; all of this space must of course be maintained at high vacuum, and the parts must be quite free of mechanical vibration.

The lenses most commonly used are electromagnetic rather than electrostatic, consisting of solenoids completely enclosed in a cylindrico-annular shell of soft iron, which is slotted completely around a circumference of its inner surface. The electron stream passes through the coil parallel with the axis of the cylinder. (The generalized form of such a short magnetic lens is shown in Figure 2).

Now an optical lens is a block of homogeneous substance, whose index of refraction differs abruptly at its surface from the index of refraction of the surrounding medium, and its refractive power can be adjusted directly by changing the contour of that boundary surface. An electron lens, on the other hand, consists of a field of continuously varying intensity, wherein the refraction does not take place abruptly at any boundary surface but is effected continuously throughout the field. Moreover, the refractive power as represented by the lines of force of the field cannot be changed directly, but only indirectly by changing the winding of the coil and the contour of the electrode surface. It is a very difficult matter, first to design the

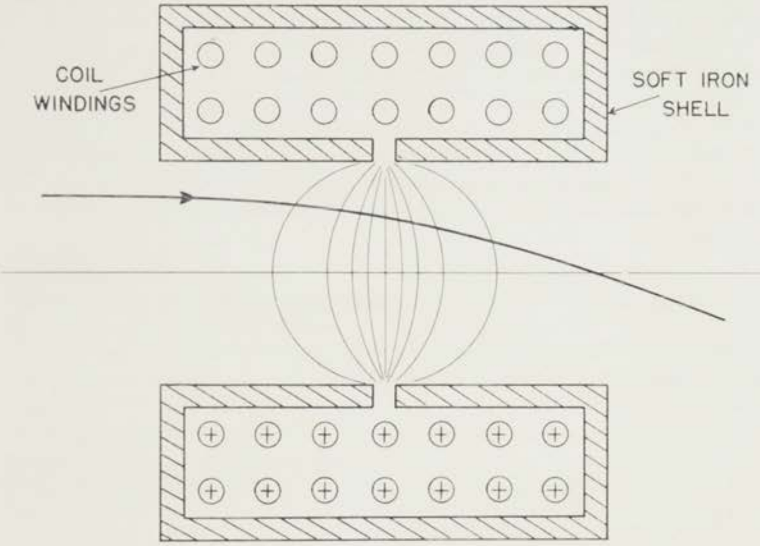

FIGURE 2

Generalized cross section of a short magnetic lens, showing the path of an electron through the magnetic field.

proper contour[12], and then to manufacture according to specifications a lens which will produce the desired arrangement of the field. Very slight inaccuracies in machining and assembling the parts, and very slight irregularities in the permeability of the iron shells, will cause serious distortions in an electron path of two meters. In the same considerable distance, the magnetic field of the earth may have a marked effect upon the electron beam, unless there is adequate shielding.

And electron lenses differ in this respect also from light lenses, that their refractive power and hence their focal length is variable, depending both on the accelerating potential applied to the electron stream, and on the current flowing through the coil of the lens (or the applied potential, if electrostatic lenses are used). A lens which may be perfectly symmetrical for one accelerating potential and for one current value, may cause bad distortion when these quantities are appreciably changed, as they must be changed to produce different magnifications. For in the electron microscope, as is evident, to change

[12]Ramberg and Morton in Jour. Appl. Phys., *10* (July, 1939), p. 467 describe the electrolytic tank, a device for experimentally investigating the field produced by any given electrode configuration.

magnifying power it is not necessary to substitute different lens systems, just as to focus it is not necessary to change the length of the lens tube or its distance from the object. All the parts remain rigidly fixed, and magnification and focussing are both effected by purely electrical controls. The magnification of the American instrument, for example, may be varied in this way between 1500 and 25,000 (with sufficiently fine definition, it is claimed, to require further optical enlargement to obtain full useful magnification. The manufacturers display a photograph of carbon black at 72,000 diameters).

As pointed out by Fr. Fiekers in the previous paper on the subject, new techniques will have to be developed to cope with the new conditions of specimen imposed by electron microscopy. A distinct inconvenience in the manipulation of the instrument, for example, is the requirement of high vacuum. But the American manufacturer uses an ingenious air-lock device to permit isolating the specimen-chamber and the plate chamber. Both specimen and plate may be removed and replaced without breaking the vacuum in the main column; and the time required for the change is, with a good pump, about one minute[13].

Other investigators are experimenting with radically different types of electron microscopes, where the object is mounted in air and irradiated by X-rays. These cast a shadow of the specimen upon a metallic window at the end of the vacuum column, producing thereon an electron image of the specimen. This image, serving both as the source of electrons for the lenses, and as the real image to be magnified, is enlarged by projection upon the fluorescent screen or photographic plate. The resolving power should be about as good, since X-rays have wavelengths of about the same order as electrons, and it should prove possible thus to examine living objects, which are at present killed by the electron bombardment.

Biological tissues are sometimes difficult to examine in the present microscope, since they are very likely to be damaged by heating, or so highly charged that electrostatic repulsive forces may disintegrate their structure. Martron, then working in Brussels, developed a technique to escape these difficulties, and it is illustrative of the methods that are sure to be devised. He impregnates the tissues with osmium, and after brief exposure to the electrons the tissue is completely burned away, leaving simply a framework of the osmium as a perfect model of the tissue structure. This is mounted upon a copper net or thin aluminum foil so as to carry away the heat and charge[14].

The technique of staining is useless, but also unnecessary. We have already noted that discrimination between the parts of an ob-

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[13]Marton in Phys. Rev. 58 (July 1, 1940), pp. 57-60.

ject depended upon the degree of scattering they produced, and that the aperture of the objective lens in the electron microscope is so small that an electron deflected only very slightly will be prevented from entering it. Hence very minute differences in the index of refraction of parts of a specimen produce sharply differentiated lights and shadows in the image. Investigators report that specimens which require dyeing under light, give ample contrast without dyes in the electron microscope.

Indeed, the instrument though very new seems already a useful and practical tool, destined to be continually improved as it is more widely used. We may well agree with Martin when he writes[5], ". . . the subject is still in its infancy and at a stage corresponding to the period in the history of the optical microscope before the achromatic lens was invented." Or perhaps we have just now passed through that period, since there are indications that a plane electron mirror may provide the correction for the chromatic aberration of electron lenses[12].

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[12]Marton in Rev. d'Optique, *14* (April, 1935), pp. 129-145 (as abstracted in Science Abstracts, Sect. A, *38* (August, 1935), Abs. No. 3643).

THE TRANSITRON OSCILLATOR

JOHN F. FITZGERALD, S.J.

In the laboratory and lecture hall a flexible and reliable source of wide range electric oscillations is a great help, if not almost a necessity. Many are the uses to which such an oscillator can be put illustrating, in conjunction with a cathode ray oscillograph, phenomena which otherwise remain facts attested to by the professor. The present paper describes a suitable oscillator, the Transitron[1], and presents one of its many applications, the demonstration of radio transmission.

To understand the operation of the transitron oscillator it will be well to recall the fact that a tuned circuit, consisting of inductance and capacitance in parallel, can be maintained in oscillation if a device capable of supplying energy at the proper rate is connected to it. Such a device is the transitron circuit which provides the required power by virtue of its negative resistance characteristic.

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1. Cledo Brunetti. A Practical Negative Resistance Oscillator. Rev. Sc. Instr 10, 85—(1939).
   The Clarification of Average Negative Resistance with Extensions of Its Use Proc. I.R.E. 25, 1595—(1937).
   The Transitron Oscillator. Proc. I.R.E. 27, 88—(1939).
Terman, Radio Engineering, p. 153. (McGraw-Hill, 1937).

For an ordinary resistance the ratio $\frac{de}{di}$ is positive whereas for a negative resistance this same quantity is negative i. e. an *increase in voltage* produces a *decrease in current,* a *decrease in voltage* produces an *increase in current.* When a negative resistance supplies more energy to the circuit in which it is incorporated than it absorbs, the device manifesting this negative resistance characteristic can supply useful power. In the case of a vacuum tube the resistance of an electrode circuit can be obtained from the slope of a static current-voltage curve. If, for example, plate current is plotted over plate voltage, the reciprocal of the slope of this curve gives the a. c. plate resistance of the tube. Similarly, for any other element of a vacuum tube, a current-voltage curve enables one to determine the a. c. resistance of that element; e. g. the screen current-screen voltage curve for the screen grid circuit. If the slope is positive, the current increases with increasing voltage and the resistance is positive; if the slope is negative, the current decreases with increasing voltage and the resistance is said to be negative within the limits of this negative slope. Electronic textbooks and vacuum tube manuals will commonly show such negative slope curves when treating of the plate characteristic of tetrode tubes or when discussing a common circuit using this negative resistance characteristic—the Dynatron oscillator.

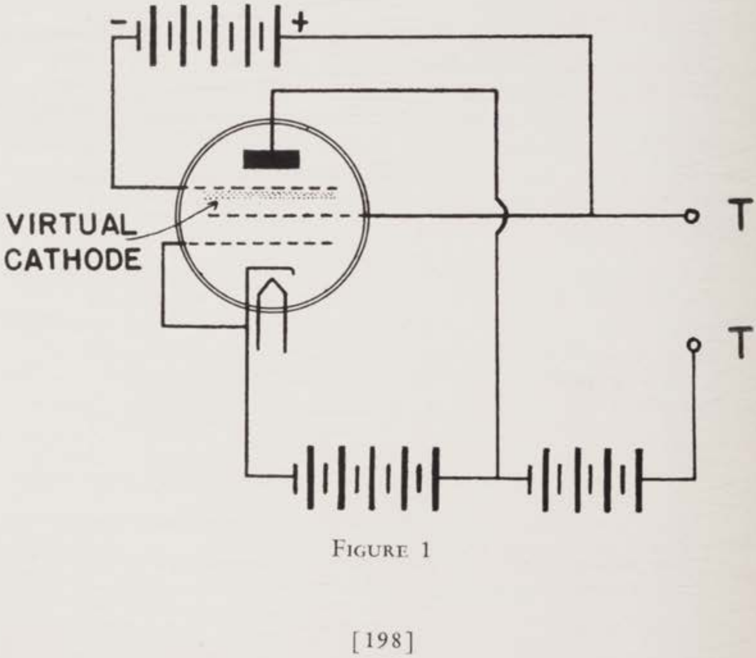

FIGURE 1

In the circuit under discussion the screen grid and plate of a standard pentode[2] are maintained at a positive potential while the suppressor grid is held at a negative potential sufficient to reflect back to the screen some of the electrons which have passed through the spaces of the screen grid. Since some of the electrons received by the screen come from the space charge formed between the suppressor and the screen by the retarding field due to the negative suppressor and the low plate voltage, this space charge may be regarded as a virtual cathode or source of electrons.

Considering for the moment the virtual cathode, suppressor grid and plate as a simple triode, it is seen that the current to the plate increases with decreasing (less negative) suppressor potential. Now let the suppressor grid be connected to the screen grid through a biasing battery (fig. 1) and let the potential of the screen be made more positive. This will make the potential of the suppressor less negative with respect to the plate and will, therefore, increase the current to the plate. But, since in the transitron circuit the screen grid current consists largely of electrons returned from the virtual cathode, an increase in the plate current, leaving fewer electrons to be returned, decreases the total screen current. Similarly, a decrease

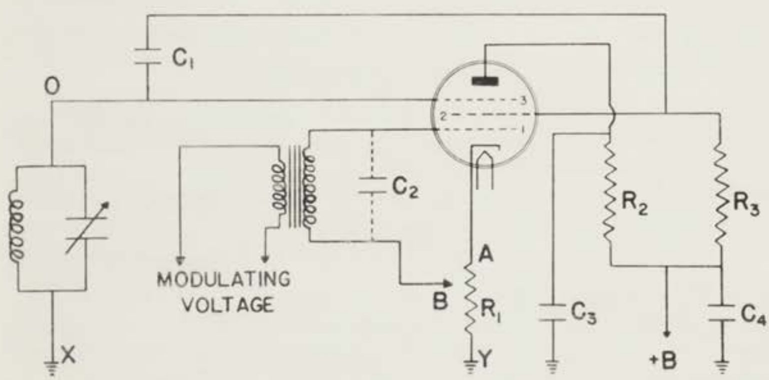

FIGURE 2

Lecture Table Radio Transmitter

R_1 = 2,500 ohms	C_1 = 0.01 microfarad
R_2 = 70,000 ohms	C_2 = 0.02 microfarad
R_3 = 52,000 ohms	C_3 = 0.1 microfarad
+ B = 250 volts	C_4 = 0.1 microfarad

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2. Any ordinary 3 grid tube should prove satisfactory. We have used the following types: '57, '77, '78, 6C6, 6D6.

in the positive screen potential increases the negative potential of the suppressor thus increasing the total screen current. In other words the screen grid circuit of the transitron oscillator exhibits a negative resistance characteristic i. e. a negative volt-ampere characteristic and is capable of supplying energy to a tuned circuit inserted at T T (fig. 1).

It will be noted that the circuit used for the lecture table demonstration (fig. 2) differs from the one previously discussed in that grids #2 and #3 are connected by a condenser and the tuned circuit is placed in the #3 grid return instead of in the screen grid circuit. The capacitive connection of suppressor and screen is permitted since we are concerned only with a variational negative resistance i. e. a negative resistance to a. c. Also, because condenser C_1 maintains the two grids at the same a. c. potential, the tuned tank may be placed in either circuit. The suppressor arrangement has the advantage of permitting the grounding of the rotor plates of the tuning condenser. If it is decided to place the tuned tank in the screen grid circuit, a high resistance (e. g. 10^5 ohms or more) should be used to connect #3 grid to ground and condenser C_1 should have, at the frequency used, a much lower impedance than this resistance. Any combination of inductance and capacitance which makes the tuned circuit ratio $\frac{L}{RC}$ greater than the minimum negative resistance obtainable with the electrode voltages used should prove satisfactory.

We were led to determine upon the transitron as the generator of oscillations by the claims made for this circuit. In the first article cited above[1] we find this claim: "In all properties such as wave form, stability, ease of operation and maintenance it (transitron) surpasses the ordinary types of oscillators." More specifically for the transitron is claimed:

1. Quasi-sinusoidal wave form. For normal values of R (with a good tuning condenser R equals the resistance of the tank inductance) one can generally expect a voltage output of negligible harmonic content if $\frac{L}{C}$ is less than 10^6.

2. Frequency stability. The frequency will not shift or change greatly with nominal changes in tube element voltages. If R is small, the frequency stability is comparable to that of a crystal oscillator without temperature control.

3. Nearly constant amplitude of oscillations over a good range of frequencies.

4. Large freedom in the selection of d. c. tube voltages—an almost infinite number of combinations that will work satisfactorily.

An oscillator of excellent wave form is possible using 0, 2, 4 volts for the suppressor, plate and screen potentials respectively (control grid tied to cathode).

All of the above claims have been confirmed as far as possible. Within the range of frequencies capable of observation on our cathode ray oscillograph (range of timing axis—20-15,000 cycles per second) the form is to all appearances a sine wave of good frequency stability. For the r. f. range, beating with a Broadcasting Station carrier produces a beat note which seems to maintain a constant pitch, a direct confirmation of the frequency stability and indirectly of the wave form. Approximately constant amplitude of oscillation has been obtained over the frequency range 580 K. C.—1,230 K. C. The great freedom in the selection of d. c. tube voltages is best illustrated by noting that while fig. 2 calls for a supply voltage of 250 volts, the circuit is still oscillating when the voltage has been reduced to 20 volts. Also, while the screen is normally the most positive electrode, we have obtained r. f. oscillations when the screen was at 135 volts and the plate at 165 volts and when the screen was at 125 volts and the plate at 128 volts.

Having obtained an excellent and reliable source of r. f. oscillations, there remains the impressing of the signal modulation. In the discussion and description above no mention has been made of the control grid, since we have been concerned so far with the generation of undamped, unmodulated oscillations. Having saved, as it were, one grid, we are now able to utilize its valve action to modify the amplitude of oscillation of the r. f. carrier wave. As indicated, an audio transformer connecting the modulating source to the control grid circuit makes amplitude modulation conveniently possible. Condenser C_2 is not essential and may be eliminated if it is so desired. The usual cathode resistor by-passing condenser is not shown since it was not necessary in our circuit for the purposes of the demonstration. When modulating the r. f. the control grid return B (fig. 2) should be located below that value which permits the maximum carrier amplitude for the tube voltages used. For the source of modulating voltage we have used with good results 60 cycle a. c. from a toy step-down transformer supplied from another step-down transformer or from a General Radio Variac auto transformer; a battery operated tuning fork (1,000 cycles per second); a 1.5 volt double carbon button microphone.

With a few feet of wire connected at O (fig. 2) for an antenna, the presence of the undamped, unmodulated locally generated oscillations can be detected, heterodyning with a commercial carrier, by an ordinary radio receiver. A cathode ray oscillograph, connected directly across the resonant circuit at O and X, can also be used. It is to be noted that the oscillograph load considerably decreases the energy available for transmission. If there is a 'quiet spot' on the radio receiver dial, tune both oscillator and receiver to this frequency for the reception of the local program. Lacking this 'quiet spot', tune the receiver to one of the weaker stations and with the receiver volume decreased place the oscillator antenna so that the field of the demonstration transmitter is strong enough to be received. If a key or suitable switch is inserted at X, code signals may be generated and received by heterodyning with a commercial carrier. Another method of obtaining code signals is to insert at Y a resistance (15,000-20,000 ohms) sufficient to stop oscillations and shunt this added resistance by the key.

The transitron circuit described in this article is a reliable source of electric oscillations. The frequency range is very great—a. f. and r. f. being obtainable with one combination of potentials. No delicate adjustments are necessary. For the lecture table transmitter only standard parts found lying about any laboratory or easily removed from a discarded radio receiver are required. The circuit constants, supplied here merely for convenience, are by no means critical values, representing only one combination which we have used. If a cathode ray oscillograph and variable test resistances are available, one can very readily adapt the circuit to almost any combination of fixed value parts on hand.

MATHEMATICS

THE PROPOSALS OF FATHER CHRISTOPHER CLAVIUS, S.J., FOR IMPROVING THE TEACHING OF MATHEMATICS

Rev. Edward C. Phillips, S.J.

(NOTE. The following papers are a translation from the Latin autographs of Father Clavius preserved in the Fondo Gesuitico of the Manuscript Department of the R. Biblioteca Nazionale Centrale Vittorio Emmanuele II in Rome. This Italian central National Library is housed in the old buildings of the Collegio Romano (now the Gregorian University), which were confiscated by the newly established Government of United Italy, along with the books and manuscripts of the College, about 70 years ago.

Clavius, as is well known, was the leading Jesuit Mathematician of his age and held the post of Professor of Mathematics in the Collegio Romano for about forty-seven years, (1565-1612) the latter portion of which was devoted almost exclusively to the work of writing and publishing numerous works on Mathematics and allied subjects. The exact date of these memoranda is not indicated, but they were probably prepared about 1575.

The translation was made at Woodstock College by Mr. Edwin Cuffe, S.J., with technical advice from Mr. Edward H. Nash, S.J., from the text as published in that volume of the *Monumenta Historica Societatis Jesu* which is entitled *Monumenta Paedagogica Societatis Jesu quae primam rationem studiorum anno* 1586 *praecessere;* pages 471-476. Fr. Clavius' contributions form two out of four documents on this subject.

For a better understanding of some portions of these documents, it should be remembered that at that period "Mathematics" was a term including astronomy and much of what would now be taught in physics.)

Document No. 34.

A METHOD OF PROMOTING MATHEMATICAL STUDIES IN THE SCHOOLS OF THE SOCIETY

In the first place the teacher chosen must be a man of learning and of more than ordinary authority: if either of these qualities be lacking, it seems (from our own experience) that the scholars cannot be drawn to the study of mathematics. To the end that the teacher have greater influence with the scholars, that mathematical studies be held in greater esteem, that the scholars may understand the necessity and usefulness of these studies, the mathematics teacher should be invited to the more important "acts", wherein doctorate degrees are given and public disputations held, so that, if he be capable, he may also propose difficulties and assist the disputants. For this way the scholars, seeing the mathematics professor attending, with the other teachers, acts of this kind, and sometimes even proposing arguments, will easily come to see the truth of the case, that philosophy and mathematical sciences are not two separate things; especially since up till now scholars seem almost to have despised these sciences, following this one line of thought,—they consider these sciences as held of no value, and as being useless, because the mathematics professor is never invited to public acts with the other professors.

There also seems to be need that the teacher have some inclination and liking for teaching mathematics, also that his efforts be not scattered in other work; otherwise he will hardly be a help to his scholars. However, that the Society may be able always to have capable teachers of mathematics, a number of men fit and able to undertake such positions ought to be chosen and organized in a private academy for the study of the branches of mathematics; otherwise it doesn't seem possible for these studies to survive, (much less advance,) in the Society; but since they bring considerable renown to the Society, and since, time and again, talk about them comes up in conversations and gatherings of men of parts, at which gatherings it is taken for granted that Jesuits are learned in mathematics, it happens that Ours present are constrained to silence, to their own confusion. This same we have heard from those whose own experience this has been. For the present I omit mentioning that natural philosophy, without mathematics, is imperfect and maim.

So much about the teacher of mathematics: now a few words on the scholars.

In the second place, therefore, the scholars must recognize that these sciences are of use, and necessary for the correct understanding of the rest of philosophy, and at the same time a complement of all the other arts if one is to acquire *eruditio perfecta*; to go further, scholars should understand that there is such mutual relation between mathematics and natural philosophy, that, unless they support one another, neither can safeguard its own position. This being true, the first need is that scholars of natural philosophy attend classes in mathematics; this custom, till now, has always been kept in the Society. For if mathematics is studied at some other time, scholars of philosophy would come to think, (and can we blame them?) that mathematics is in no way needed for natural philosophy, and very few of them would ever be eager to learn this science: but because it is evident to anyone understanding the subject that without mathematics there is no right understanding of natural philosophy, especially of those sections treating of the number and movements of the stars, of the plurality of intelligences, of the effects of the stars, (which are the results of different conjunctions, oppositions, and other relations between each other,) and again where treating of the infinite division of the continuum, of the ebb and flow of the sea, of winds, comets, rainbows, sun and moon-halos, and other meteorological phenomena, of the relation of motions, qualities, actions, passions, reactions, etc., concerning which the mathematicians have much to say. We could cite infinite instances from Aristotle, Plato, and their great commentators, which are completely unintelligible without a fair knowledge of mathematics. Besides all this many a professor of philosophy, because of his ignorance of mathematics, has made no end of mistakes, and most serious mistakes, and (what is worse) has published them. It would not be hard to give instances.

Similarly our philosophy teachers should know mathematics, at least in the essentials, to prevent themselves from meeting, in like difficulties, shame and exposing to great harm that good name for learning which the Society possesses.

It goes without saying that teachers will win great influence over scholars, when the scholars recognize that their masters are treating capably those sections in Aristotle and other philosophers which refer to mathematics. In this way too scholars will better appreciate the need of mathematical studies. To this end it will be a great help if philosophy teachers ignore those questions which are of little help in understanding the philosophy of nature, and for the most part induce a poor opinion of mathematics among scholars. I mean, for instance, those questions that enunciate: mathematics is not a science; it has no demonstrations; it abstracts from *ens* and *bonum*, etc.; for experience will tell you that all this is of no good but of considerable harm to scholars, especially because teachers, (as we have been told more than once,) cannot teach it without ridiculing mathematics.

It would also be helpful if in individual conversations teachers were to encourage scholars to learn mathematics, impressing on them its necessity, and not doing the opposite, as many have in the past. In this way there will be removed all that disagreement that is observed among us by externs when such opinions are heard in our schools.

Moreover the scholastics will be greatly encouraged to study mathematics, if once a month all the philosophers are assembled in one place, where one scholar will read a short appreciation of mathematics, and then with one or two others will explain some problem from astronomy or geometry such as would be pleasant to hear and of use for the humanities. Such problems can be found in abundance. Or let him explain some mathematical text from Aristotle or Plato, in whose works such texts are numerous. Or even let him propose new and original demonstrations of some of the propositions of Euclid. And at these academies let praise be given to those who best solve the proposed problem, or are guilty of the fewest false syllogisms (which are common enough) in the invention of new demonstrations. The result of this would be that the scholastics would become eager for mathematical studies, when they see such honors before them, and would at the same time come to understand the eminence of the science, and through the academy would make greater progress in it.

Furthermore, toward the end of the philosophical course, those who wish to take honors of master or doctor should be examined in mathematics, as is usual in some other special courses. At this examination let there be present with the other professors of philosophy the mathematics professor.

(The MS. carries the following notation in the hand of Fr. Brunelli:

"P. Christophori Clavii manu scripta diligenter asservanda.")

No. 35

ON TEACHING MATHEMATICS

It was proposed last year that, for the advancement of mathematical studies (which were being almost neglected,) those who were to teach mathematics should be excused from teaching grammar, that they might, during the first year after finishing philosophy, study mathematics more thoroughly at home, and then teach publicly one or two years. This plan was approved, and has even to some extent been put in practise, and promises to be of the greatest use in encouraging mathematics and also in promoting the full equipment in other studies. Finally it has stood the proof,—this is admitted. The plan seems to have this one drawback: that since the most talented men,—and in this class are included those who for the greater service of God and the good of the Society should be selected for the posts in question,—usually finish philosophy when still quite young, the oldest among them generally being hardly twenty-four, it seems to be neither for their own good to have them (during the year in which they teach,) use the time left over from their teaching (which takes an hour or two) apparently at their own discretion; nor for the good of the school, which suffers in having teachers who are practically boys; nor for the good name of the Society, to be always using such men in teaching important subjects.

Consequently what would perhaps be best would be for those who are chosen for mathematics to spend a full year in the house of studies after finishing philosophy, learning what they would have taught immediately after, as has already been determined, but rather to make at once their theology, and finally to teach mathematics for the same length of time that they would have done according to the previous plan. This because mature men, priests and theologians would give honor rather than dishonor to the chair of mathematics, and also could the more safely be trusted to themselves in spending their spare time. This arrangement would have the following important advantages:

First, that these scholars would be able to attend philosophical disputations, public or private, and to take part in discussion, being themselves thus stimulated and helped, and also themselves helping others. But if they teach before they are theologians, it is scarcely possible that, although of unusual brilliance, they should propose anything suitable at a disputation, as they are merely philosophers. And if they question or argue with philosophy professors they may become a nuisance.

Second: as regards the time left over from teaching, by the very fact that they have made theology, they would much more usefully spend this time in repeating philosophy and in getting an understanding of the text of Aristotle, than if they had spent the same time previously studying the same matter most diligently. For years bring judgment, and the study of theology has a remarkable power of sharpening and stimulating the mind. Moreover it seems more proper to the Society for the glory of God that our scholastics make the greatest possible advance in studies, especially where this can be done with no greater expenditure of time and effort.

Third: it seems that the plan will be very helpful because these scholars can during this time conduct philosophy repetitions at home, and by that relieve the teachers who otherwise would now have heavier burdens than before.

Fourth: also because this plan is much more fitted to the study of mathematics. For, as the scholars has already been taught the first six books [of Euclid] in class, they can begin studying at the seventh book and go through to the twelfth inclusively; and then add the elements of Spherics, of Theodosius, and some of the known works of Apollonius. This could be done easily in one year, if they attend two lectures a day, which is what ought to be done. Then, during the four years of theology, just as the future grammar teachers attend domestic academies on half-holidays, so these mathematicians, for an hour after lunch when class is over, at the same time as the others, should have lessons in the theory of planets, in gnomonics, on the astrolabe, something from Archimedes and algebra, with all the matter arranged in a cycle, and in this way come to their teaching with a broader training. All this will be useful equipment for other studies as well.

Those especially ought to be chosen who, other things being equal, are outstanding in talent, industry, natural liking for mathematics and in teaching ability, but not those who surpass others in agreeableness (gratia). The judgment as to these qualifications should be diligently sought of those who are their directors in these branches. For it sometimes happens that some, either because they are not inclined in this direction, or are naturally unfitted for this study, advance well enough in other subjects, but are ill-suited for mathematics.

Against this plan one difficulty is brought from the fact that while we need mathematics professors, for whose preparation all this is to be done, yet we do not see where we are to get men to teach in the meantime until those who will become teachers of mathematics will have finished their own studies. But this is no real difficulty, for this year there are among the men finishing theology some who have never taught, and who willingly would study mathematics that others may teach, taking their places while they study. To prevent the useless wasting of time by one while another is teaching, the students could find their study in the *ordo studiorum* and the Constitutions.

(Notation on the MS. by Fr. Brunelli: "asservanda.")

THE PARTICULAR INTEGRAL OF LINEAR PARTIAL DIFFERENTIAL EQUATIONS

REV. THOMAS D. BARRY, S.J.

A linear partial differential equation is one containing partial derivatives and of the first degree in the dependent variable and all its derivatives. Its order is the order of the highest derivative present. We shall consider the case where the coefficients are constants. Thus a second order equation would take the form (after division by the coefficient of the first term):

$$F(D,D')z = (D^2 + k_0DD' + k_1D'^2 + k_2D + k_3D' + k_4)z = \varphi(x,y),$$

where Dz and $D'z$ are partial derivatives with respect to x and y respectively,

D^2z and D'^2z are second partial derivatives with respect to x and y respectively,

$DD'z$ is the second partial derivative with respect to both x and y.

The solution of a linear differential equation of order higher than the first consists of two parts: 1) the complementary function, which is the solution of the differential equation with the right-hand side made temporarily zero, and 2) the particular integral, which is a value of the dependent variable which, substituted in the left-hand side, will give the $\varphi(x,y)$ on the right. This paper is concerned with a brief method of finding a particular integral and a simple way of developing that method. We consider only the case where $F(D,D')$ can be broken up into factors linear in D and D′ with real coefficients.

Cohen, in his "Differential Equations" (new edition, p. 280, old edition, p. 243), says, treating of the particular integral: "General methods for finding the particular integral . . . may be deduced along lines entirely analogous to those for linear ordinary differential equations with constant coefficients." In his examples, he uses the method of undetermined coefficients. The main difficulty with that method is in finding the proper form for the trial integral. Murray (page 176) uses his pet method of inverse operators to solve $(D\text{-}mD')u=\varphi(x,y)$. The main difficulty with that method is that it can become very complicated in its application. In the present case, he uses nearly a whole page turning the operators upside down and inside out to evolve the following rule:

1) In $\varphi(x,y)$, replace y by y-mx, giving $\varphi(x,y\text{-}mx)$,
2) Integrate this φ with respect to x,
3) Replace y by $y+mx$,

. . . . and then in the examples uses the inverse operators. It was an attempt to avoid the complicated development of this method that gave rise to the material in this paper.

Since in our development Lagrange's method of solving partial differential equations of the first order and degree will be used several times, a brief outline of the method will not be amiss. An equation of the form $Pp+Qq=R$ (where p and q are the partial derivatives of z with respect to x and y, corresponding to our Dz and D′z, and P,Q,R are functions of x,y,z, including constants) may be put in the form: $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$. From any two pairs of these terms are found two solutions, $u_1=c_1$ and $u_2=c_2$. Then $u_2=\varphi(u_1)$ is a solution of $Pp+Qq=R$. For our use later on, it should be noted that, since $c_2=u_2$, $c_2=\varphi(u_1)$ also holds.

The possible linear factors of $F(D,D')$ are $D-mD'-b$, $D-mD'$, $D-b$, D, $D'-b$, and D'. These will be considered singly.

A. When a factor of $F(D,D')$ is $D-mD'-b$, the most general form.

When F is factored, the equation will take this form:

$$(D-mD'-b)\,(\text{product of all the other factors})\,z=\varphi(x,y).$$

If we let (product of all the other factors) $z=u$, we have:

$$(D-mD'-b)\,u=\varphi(x,y).$$

This is a linear partial differential equation of the first order and degree, so it can be solved by Lagrange's method.

$$\frac{dx}{1}=\frac{dy}{-m}=\frac{du}{bu+\varphi(x,y)}.$$

From the first two terms, we have $y+mx=c$, or $y=c-mx$, our first solution. Substitute this value of y in $\varphi(x,y)$ and equate the third term to the first. This equation can then be put in the form:

$$\frac{du}{dx}-bu=\varphi(x,c-mx).$$

This is an ordinary linear equation of the first order and degree. An integrating factor is e^{-bx}, giving the solution:

$$ue^{-bx}=\int e^{-bx}\varphi(x,c-mx)\,dx\ (+c_2),$$

$$\text{or}\quad u=e^{bx}\int e^{-bx}\varphi(x,c-mx)\,dx\ (+e^{bx}c_2), \qquad \text{(Eq. 1)}$$

where c is to be replaced by $y+mx$ after integration, giving the second solution of Lagrange's equations. If this equation is solved for $e^{bx}c_2$, we have, as shown earlier in this paper,

$$e^{bx}c_2 = e^{bx}\varphi(u_1) = e^{bx}\varphi(y+mx).$$

But this is precisely the term in the complementary function which arises from the factor $D-mD'-b$, and so it, and therefore, c_2, may be omitted, leaving for our solution of $(D-mD'-b)u=\varphi(x,y)$ Eq.1 without the last parenthesis. Since, in future solutions by Lagrange's method in this paper, the constant of integration for the corresponding integrals will always be similarly part of the respective complementary function, it will be regularly omitted.

Examination of the above process shows the following essential steps, which are to be taken as our general rule of operation:

1) In $\varphi(x,y)$ replace y by $c-mx$, and multiply φ by e^{-bx}.
2) Integrate the result with respect to x.
3) In the result of this integration, replace c by $y+mx$, multiply by e^{bx}. An example of the use of this rule will be given later.

The factors $D-b$, $D-mD'$, and D are all special cases of the factor $D-mD'-b$, and the procedure for each case may be deduced from the general rule, as follows.

<u>B. When a factor of $F(D,D')$ is $D-b$.</u> In this case, $m=0$. Therefore the substitutions in steps 1 and 3 are c for y and y for c, respectively.

<u>C. When a factor of $F(D,D')$ is $D-mD'$.</u> In this case, $b=0$, whence e^{-bx} and e^{bx} both equal unity. Therefore the general rule reduces to:

1) Replace y by $c-mx$.
2) Integrate with respect to x.
3) Replace c by $y+mx$.

As this is the case the development of which by Murray led to all this, it will here be developed separately for comparison with his method.

$$(D-mD')u=\varphi(x,y)$$
$$\frac{dx}{1}=\frac{dy}{-m}=\frac{du}{\varphi(x,y)}$$

From the first two terms, $y=c-mx$. Substitute this in the third term, equate that term to the first, and clear of fractions, giving:

$$du=\varphi(x,c-mx)\,dx,$$

whence $u=\int\varphi(x,c-mx)\,dx$ (the constant being omitted as before). In the result of this integration, make the reverse substitution for c. Examination of this process shows that the three essential operations are those enumerated above. This is the same result as that obtained by Murray, except that we have c where he has y, (which he considers constant). It has the advantage of simplicity of development and also the removal of ambiguity as to the status of y at any time, i.e., whether y means y, $y-mx$ or $y+mx$.

<u>D. When a factor of $F(D,D')$ is D.</u> In this case, $m=b=0$. Therefore, $e^{-bx}=e^{bx}=1$, $c-mx=c$, and $y-mx=y$. The general rule reduces to:

1) Replace y by c.
2) Integrate with respect to x.
3) Replace c by y.

And this reduces in practice to the following rule: integrate $\varphi(x,y)$ with respect to x, considering y as constant. This also follows from the following consideration: $Du=\varphi(x,y)$ means that $\varphi(x,y)$ is the result of differentiating some function, say $f(x,y)$, with respect to x. Therefore the inverse operation is integration with respect to x, i.e., $u=\int\varphi(x,y)\,dx=f(x,y)$.

The two remaining types of factors, $D'-b$ and D', require special treatment.

E. When a factor of $F(D,D')$ is $D'-b$.

$$(D'-b)u=\varphi(x,y)$$
$$\frac{dx}{0}=\frac{dy}{1}=\frac{du}{bu+\varphi(x,y)}$$

From the first term, $dx=0$, $\therefore$ $x=c$. Substitute this in the third term, equate that term to the second. This equation can then be put in the form:

$$\frac{du}{dy}-bu=\varphi(c,y).$$

An integrating factor of this is e^{-by}, giving the solution:

$$ue^{-by}=\int e^{-by}\varphi(c,y)\,dy$$
$$\text{or}\quad u=e^{by}\int e^{-by}\varphi(c,y)\,dy.$$

Therefore the rule of operation for a factor $D'-b$ is:

1) Replace x by c, and multiply φ by e^{-by}.
2) Integrate with respect to y.
3) Replace c by x, and multiply by e^{by}.

F. When a factor of $F(D,D')$ is D'. This is the special case of the above where $b=0$, whence $e^{-by}=e^{by}=1$. Therefore the rule reduces to:

1) Replace x by c.
2) Integrate φ with respect to y.
3) Replace c by x.

This reduces further to: integrate $\varphi(x,y)$ with respect to y, considering x as constant. This is also evident from the following consideration. $D'u=\varphi(x,y)$ means that $\varphi(x,y)$ is the result of differentiating some function, say $f(x,y)$ with respect to y. Therefore the inverse operation is integration with respect to y, i.e.,

$$u=\int\varphi(x,y)\,dy=f(x,y).$$

The rules developed above are rules of operation and are to be used successively for each factor of $F(D,D')$. WARNING. Do not incorporate any constants into any of the c's, otherwise the reverse substitutions, $c=y+mx$, etc., will give erroneous results. In the following examples, the "Types" refer to the sub-headings above, the numbers at the beginning of lines refer to the steps in the corresponding rules.

Ex. 1. $(D^3+D^2D'+DD'-D)z=e^{2x+3y}$.

$D(D+1)(D+D'-1)z=e^{2x+3y}$.

Let $(D+1)(D+D'-1)z=u$.

Then $Du=e^{2x+3y}$.

(Type D) Step 2) $u=(D+1)v=\frac{1}{2}e^{2x+3y}$,

where $v=(D+D'-1)z$.

(Type B) $m=0$, $c-mx=c$, $y+mx=y$. $b=-1$, $e^{-bx}=e^x$, $e^{bx}=e^{-x}$.

Step 1) $1/2\ e^xe^{2x+3c}=1/2\ e^{3x+3c}$

2) $1/6\ e^{3x+3c}$

3) $v=(D+D'-1)z=1/6\ e^{-x}e^{3x+3y}=1/6\ e^{2x+3y}$

(Type A) $m=-1$, $c-mx=c+x$, $y+mx=y-x$; $b=1$, $e^{-bx}=e^{-x}$, $e^{bx}=e^x$.

1) $1/6\ e^{-x}e^{2x+3c+3x}=1/6\ e^{4x+3c}$

2) $1/24\ e^{4x+3c}$

3) $z=1/24\ e^xe^{4x+3y-3x}=1/24\ e^{2x+3y}$. Ans.

The rules work even if $\varphi(x,y)$ is a part of the complementary function. In this case the factor which gave rise to that term in the complementary function should be kept to the last, as in the following example.

Ex. 2. $(D^2+DD'-2D'^2)z=(D-D')(D+2D')z=-2\sin(x+y)$.
Here both factors are of Type C. The complementary function is $\varphi_1(y+x)+\varphi_2(y-2x)$. $\sin(x+y)$ is part of $\varphi(y+x)$ which arose from the factor $D-D'$, so we shall start with:

$(D+2D')u=-2\sin(x+y)$

$m=-2$, $c-mx=c+2x$, $y+mx=y-2x$.

1) $-2\sin(3x+c)$

2) $2/3\cos(3x+c)$

3) $u=(D-D')z=2/3\cos(x+y)$

$m=1$, $c-mx=c-x$, $y+mx=y+x$.

1) $2/3\cos c$

2) $2/3\ x\cos c$

3) $z=2/3\ x\cos(x+y)$. Ans.

Ex. 3. $(D'^2+3D')z=D'(D'+3)z=x^2y$.
Since the solution for the factor D' involves only an integration, it will be simpler to start with the other.

$(D'+3)u=x^2y$

(Type E) $b=-3$, $e^{-by}=e^{3y}$, $e^{by}=e^{-3y}$.

1) c^2ye^{3y}

2) $1/9\ c^2 e^{3y}(3y-1)$ (Peirce, no. 402)

3) $u = D'z = 1/9\ x^2(3y-1) = 1/3\ x^2y - 1/9\ x^2$

(Type F) Step 2) $z = 1/6\ x^2y^2 - 1/9\ x^2y$. Ans.

If this equation is solved using the D' factor first, we get an extra term, $-1/27$. But this may be considered as the coefficient of x^0, and hence part of the term of the complementary function arising from D', which is $\varphi(x)$.

The above rules were formed on the assumption that the original equation had been divided by the coefficient of D^2. If that has not been done, the coefficient of D in one or more of the factors of $F(D,D')$ will be that coefficient of D^2 or one of its factors, which we shall call k. Then one or more of the factors of $F(D,D')$ in types A to D will be of the form $kD - mD' - b$, $kD - mD'$, etc. In the corresponding rules, b and m must changed to b/k and m/k in steps 1 and 3, and in step 2 the integral must be divided by k. Similar modifications must be made for types E and F, if D' has a coefficient.

Alternative forms for the rules may also be found, based on the fact that the integral from $\frac{dx}{1} = \frac{dy}{-m}$ may also be put in the form: $x = c - y/m$. These involve substitution for x instead of y in $\varphi(x,y)$ in the first step, and, in the second, integration of $-\varphi/m$ with respect to y.

Some Special Cases.

G. When the factor of $F(D,D')$ is $D - mD'$ and the term on the right-hand side is a function of x only. $(D - mD')u = \varphi(x)$.

$$\frac{dx}{1} = \frac{dy}{-m} = \frac{du}{\varphi(x)}$$

From the first two terms, $y = c - mx$, but there is no y in the function to be replaced. So from the first and third terms:

$$du = \varphi(x)\,dx.$$
$$u = \int\varphi(x)\,dx.$$

Therefore, for each factor of the form $D - mD'$, integrate with respect to x. This follows also from the rule under C. Since the function does not contain y, the first and third steps are inoperative, leaving only the second.

H. When the factor of $F(D,D')$ is $D - mD'$ and the term on the right-hand side is a function of y only. This may be treated in two ways.

First. Apply the rule for $D - mD'$.

Second. $(D - mD')u = \varphi(y)$.

$$\frac{dx}{1} = \frac{dy}{-m} = \frac{du}{\varphi(y)}$$

The equation from the first two terms may be solved: $x = c - y/m$. But there is no x in the function to be replaced. A solution from the last two terms is:

$$u = -\frac{1}{m}\int \varphi(y)\,dy$$

giving the rule: for each term of the form $D - mD'$, integrate with respect to y, dividing in each case by the coefficient of D'.

Ex. 4. $(D+2D')(D+D')z = x + y$. (Murray, p. 178, ex. 1)

First solution, using the second method under H for y.

$$(D+D')z = 1/2\ x^2 + 1/4\ y^2$$

$$z = 1/6\ x^3 + 1/12\ y^3. \quad \text{Ans.}$$

Second solution, using the first method under H, for y only.

$$(D+2D')u = y.$$

$m = -2$, $c - mx = c + 2x$, $y + mx = y - 2x$.

1) $c + 2x$

2) $cx + x^2$

3) $u = (D+D')z = (y-2x)x + x^2 = xy - x^2$.

$m = -1$, $c - mx = c + x$, $y + mx = y - x$.

1) $(c+x)x - x^2 = cx$

2) $1/2\ cx^2$

3) $1/2\ (x^2y - x^3)$.

Adding this to the solution for x as found in the first solution:

$$z = \frac{x^3}{6} + \frac{x^2y}{2} - \frac{x^3}{2} = \frac{x^2y}{2} - \frac{x^3}{3}.$$

This is the answer as given by Murray. However, both satisfy the equation.

I. When the term on the right-hand side is a constant. A constant may be considered a function of either x or y. Therefore the rules under either G or H may be applied.

Ex. 5. $(D-2D')(D-3D')z = 2$.

First solution, as Type G.

$$(D-3D')z = 2x.$$

$$z = x^2. \quad \text{Ans.}$$

Second solution, as Type H.

$$(D-3D')z = -y.$$

$$z = 1/6\ y^2. \quad \text{Ans.}$$

CHEMISTRY

CHEMICAL LITERATURE FROM EXTRAORDINARY SOURCES

Rev. Bernard A. Fiekers, S.J.

The Journal of Chemical Education has recently run a series of articles on "Bibliography for General Chemistry from Several Periodicals". (H.N.Alyea: ibid., *13*, 76-81, 540-544, (1936), *16*, 435-440, (1939). The articles therein listed are very modern, popular and of easy approach for the freshman. The periodicals are generally obtainable in the current periodical section of the college library. Thus a good source of side reading assignments is assured.

In making assignments for the history of chemistry course, or in preparing matter for the departmental publication, one is often urged to include items from old files of journals that only a few of the older libraries in the country possess. Photoprint or bibliofilm service is either too costly or too tardy. The writer offers a partial solution to the problem by indicating some easily accessible matter of historical interest.

"Read over the Shoulders of Giants" is the title of an advertisement appearing in consecutive issues of the SCIENCE NEWS LETTER for 1934: (SNL., *25*, 125, No. 672, 2-24-34; p. 141, No. 673, 3-3-34 and p. 157, No. 674, 3-10 (1934). This lists about 250 "classics of science" that had appeared in the SNL over the period 1922 to 1934; and of these about 75 items are of chemical interest.

These "classics" run the alphabetical gamut of "science" from anthropology to psychology. Original papers of Berzelius, Boyle, Davy, Lavoisier, Priestley, Paracelsus, Wöhler, among a host of other names grace the list. The papers "are accurate copies or translations into English of the original writings of scores of famous scientists". "One or more pictures illustrate most classics of science."

The writer does not doubt but what many of our libraries have back-files of this set stored away somewhere unbound waiting to be sold to the "junkie". One often finds these files offered on the dealers' lists at extremely reasonable prices. A library shelf is large enough for them, and the older issues are replete with suggestions for economical binding.

Is there a library that does not possess a set of the "Harvard Classics"? To volume 30 of these, Michael Faraday contributes 12 lectures: 6 on the forces of matter, and 6 on the chemical history of a candle. Contributions of Harvey, Jenner, Lister and Pasteur are to be found in volume 38. Possibly the local library possesses as well other sets that are similar to the "Eliot Opus", these in turn containing similar scientific originals.

Another source of original papers in foreign languages is to be found in certain scientific readers like Dolt's Chemical French, or the new Fotos and Eray set of German scientific readers.

Our own "Jesuit Relations" contain "classics of science" as a glance through the two volume index for common chemical items will reveal. The "Everyman Series" is said to contain the item: "Boyle's Sceptical Chymist". Even the Hadelmann Julius offerings might have something to contribute.

Other sources might occur to the reader as his interest in the problem inspires. The reference works on the literature of chemistry list the usual sources. The sources here suggested are in the spirit of Patterson, Reid, Mellor and Soule, though they are probably not mentioned by them.

NEWS ITEMS

PONTIFICAL ACADEMY OF SCIENCES

In an Apostolic Letter, dated 25 November, 1940 (cf. *Acta Apostolicae Sedis*, Vol. XXXII, no. 13, Dec. 16, 1940, page 548) His Holiness Pope Pius XII, confers on the ordinary Members of the Pontifical Academy of Sciences the title of "Excellence". The Pope in his letter recalls that his predecessor, Pius XI, had founded this Academy with a twofold purpose"Ad Apostolicae Sedis decus augendum severiorumque disciplinarum scientiam honorandum." In recognition of the exacting duties ("gravissimum sane munus") thus entrusted to the members of the Academy, as an evidence of his special benevolence towards them and of their merit in devoting themselves to the advancement of science the present Pope has been moved to honor them with this title by which they are to be addressed in social intercourse and to have prefixed to their names in the Acta of the Academy. Members, however, who are Religious are not to use, enjoy, or exact from others the use of, this title within the sphere ("finibus") of their religious Institute. Nor does the title confer any rights to ecclesiastical "precedence".

JESUIT SCIENCE BULLETIN

Index to Volume XVIII — 1940-1941

SUBJECT INDEX

AUTHOR INDEX

(Numbers refer to pages)