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SCIENCE and PHILOSOPHY

THE NUPTIALS OF MATHEMATICS AND PHYSICAL QUALITIES

ROBERT B. EITEN, S.J.

The "mysterious bond of union" in nuptials is well and forcefully described by William Habington, a 17th century English poet, in the following lines:

> Tis no dull Sublunary flame
> Burnes in her heart and mine,
> But something more, than hath a name.
> So subtle and divine,[1]
> We know not why, or how it came.

If there is to be a "Nuptials of Mathematics and the Physical Qualities" there also must be a "bond of union" uniting them into a new science, and that *bond* at present, I believe, seems rather *mysterious* or *tenuous*, to say the least. Abstract quantity taken either strictly or analogically,—the material object of mathematics, and qualities seem at first sight to be only *similar* insofar as they both are in *a category of being* and hence, it would seem that Metaphysics, the science of being *qua* being, would only have a right to discuss them. If the qualitative accidents can be "mathematized," then we must show that there is some other *similarity* or common note, besides being, between mathematical quantity and the qualities. This shall be one of the burdens which the present article is to take up.

Sometime ago I wrote an article which was entitled: A RATIONAL BASIS FOR MATHEMATICAL PHYSICS.[2] There I tried to lay down in an *outline* fashion the ultimate and proximate principles of this science. Lack of space forbade any further study of difficulties and technicalities as the present one. It is just possible that I there gave the impression that experimental physics deals only with external motion, and not also with energy, although I definitely stated the latter.[3] There I stated that the *ultimate* metaphysical foundations of

1. *Castara*, The Harmony of Love.

2. See THOUGHT, Vol. XIII, No. 50, Sept. 1938, pp. 416-432. An Abstract (about 800 words) of this article together with an abstract of Dr. Hoenen's article: De Philosophia scholastica cognitionis geometricae, are to be found in the *Bulletin of the American Association of Jesuit Scientists*, Vol. XVI, No. 4, May, 1939, pp. 146-150 under the title of "The Philosophy of Mathematics." The author of the latter is Edward C. Phillips, S.J.

3. p. 420.

this science rested upon this famous principle of St. Thomas: "The more abstract and fundamental the subject-matter of any science is, the greater on the other hand is the applicability of its principles to other sciences.[4] The *proximate* metaphysical principle of this great branch of knowledge was shown to be "the similarity in the divisibility of both abstract quantity and external motion."[5]

While the former ultimate principle will hold its ground in explaining "The Nuptials of Mathematics and Physical Qualities," it is not too clear how the latter *proximate* principle will, since qualitative accidents do not involve, it would seem, in the *technical* sense of the word, external motion. As there, so here too, Thomas Aquinas will be our guide.[6]

The radical solution of our problem, to put it briefly, lies in the distinction between *extensive* (dimensional) and *intensive* (virtual) quantity. This latter quantity can be applied to physical qualities. The development of this article will be largely devoted to explaining this distinction. After a brief explanation of the various schools of thought on this problem, the first part of this article will be devoted to the metaphysical foundations which make possible the mathematization of these accidents, while the second part will interpret from the viewpoint of the mathematical philosopher "the meaning of the mathematical description of qualities."

The Various Schools of Thought

The last three centuries have seen three important philosophical schools give different solutions to the nature of physical qualities. The Mechanistic school in general denies the existence of physical qualities, or at least holds that these qualities admit of no change in the objective world.[7] Thus for this school all physical qualities and phenomena can be reduced to local motion.[8] However, this school does not and

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4. *In Lib. Boethii De Trin.* Q. 5, Art. 3, ad 6.

5. p. 430 of my article. This is based on the following quotation of St. Thomas: "Although motion of its very nature does not belong to the category of quantity, yet nevertheless, in other respects IT HAS SOMETHING IN COMMON WITH IT, INSOFAR AS ALL DIVISION FOUND IN MOTION IS EITHER DERIVED FROM THE DIVISION OF SPACE OR THE DIVISION OF ANYTHING WITH A CAPACITY FOR MOVEMENT. Thus while a pure mathematician does not discuss motion in his field, yet the principles of mathematics can be *applied* where motion is involved." (*In Lib. Boethii De Trin.*, Q. 5, Art. 3, ad 5).

6. In the latter part of this article I am singularly indebted to Dr. Hoenen's S.J., interpretation and further elaboration of St. Thomas. I might mention that Dr. Hoenen has done some very original work in this field. Cfr. P. Hoenen, S.J., *Cosmologia*, 2nd Ed., Rome, pp. 185-204.

7. Hoenen, op. cit., p. 137.

8. Boyer, S.J., *Cursus Philosophiae*, Vol. 1, p. 437.

cannot explain the forces of inertia (*impetus*), elasticity, or the tensions and strains in the gravitational and electromagnetic fields,—all clearcut examples of *changeable* (alterable) physical qualities. It also fails to explain cohesion and chemical affinities. The Dynamists deny, as a fundamental dogma, that beings are extended. According to them material beings are subsistent forces. Insofar as they hold that there is no other motion, except local motion, they are in agreement with the mechanists. The Idealists, the exponents of the second school, "do not deny that qualities are irreducible to quantity, but reasoning according to their principles they state that quantity and qualities are nothing else than different representations of the thinking mind."[9]

The Scholastics,, who represent the third school, hold that physical qualities exist in the physical world. This position seems to be the only one that can hold its own on negative and positive grounds. On negative grounds it stands by reason of the weaknesses of the other two systems; and on positive grounds it gives us a *true* and *complete* picture of reality in accordance with the testimony of our senses, which are after all, true sources of knowledge.

According to the scholastics, a quality is a certain accidental modification of a substance.[10] Thus it is a further determination of a fully constituted essence. Let us take for instance man. Besides his essence, which is his body and soul, he has many things which modify and further determine them. Thus his body is further determined by color, resistability, shape, and other properties which are physical qualities. The constant changing intellectual and volitional processes of his soul are good example of spiritual (non-material) qualities. These latter, however, do not concern us here.

Our greatest difficulty at present, is to determine whether such properties as taste, sound, color, etc., are both *formally and efficiently*, or *only efficiently* in the object. However, it does seem that the *kind* of sense reaction we get is determined by the *kind* of object presented with the result that something more is required than mere efficient causality. Besides this, are we sure that such things as color, sound, etc., are really qualities? Here it might be mentioned that scholastic philosophers have written on these accidents, especially on *what* things are really certain examples of qualities and on *how* some qualities are in their subject, with less success than they have on many other problems presented to them. In any case it is a question in which philosophers must keep in touch with the advancements in physics, chemistry, and other sciences. I believe that Prof. R. J. Henle, S.J., has wisely taken the proper attitude when he writes:

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9. Boyer, op. cit., Vol. 1, p. 437.

10. Mercier, Métaphysique Générale, p. 312, defines it as follows: Quality is an accident which modifies the substance itself while quantity is an accident which intrinsically gives the substance its extension."

. . . philosophy must be alive to all the movements in the world of thought; . . . it must be ever busy reinterpreting itself to fit the scientific mentality of the time; . . . it must apply its principles to physical science, borrow therefrom examples, data, and even, in some fields, explanations. The two fields of philosophy and science, though distinct, can enrich and fecundate one another. A precious deal of nonsense will be left unsaid if philosophers and scientists understand each other."[11]

To The Metaphysical Foundations

St. Thomas distinguishes two kinds of quantity, extensive (*quantitas molis vel dimensiva*) and intensive (*quantitas virtualis vel virtutis*). The former is radicated in bodies alone while the latter is a property flowing from the substantial and accidental forms of things. An example of the latter is the quantitative measurement involved in determining the heat of an object. Heat is either *more* or *less*, and therefore, it admits of *degrees*. And while "the more or less" in this case is intensive and can be perceived without being measured or reduced to extensive quantity, still the minds of men desire an *objective standard* freed from all *subjectivity*, and this has been accomplished by subjecting these experiences to a definite *objective* scale or measure. Thus in the light of this, the ordinary thermometer should have a meaning to us. It should be a good example too of a method whereby qualities can be measured.

Ultimately and metaphysically *intensive* quantity is proportioned to the perfection of the nature (or form) of a thing. Proximately it is judged by the quantitative effects which it produces (*in effectibus formae*). Now since existence and action are *in order* the first effects of a form, it follows that intensive quantity is proportioned both to the existence and action of a thing insofar as they are either *more perfect* or last longer, or have a greater capacity for action.[12]

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11. THOUGHT, Sept. 1938, "The New Scholasticism," p. 472. It must be further mentioned here that it is beyond the scope of this article to discuss these two difficult problems connected with qualities. Hence I use the ordinary example given by the scholastics of qualities later on in this article. I am prescinding from whether or not they are good example of qualities.

12. Cfr. S. *Th.* I, Q. 42, a. 1, ad 1: "There are two kinds of quantity, one which is called *molar* or *dimensional* quantity, and is found in bodies alone; . . . the other is quantity of force (*virtutis*). This latter is determined by the *perfection* of some definite nature or *form*. Further, this latter quantity is judged according as a thing is *more or less heated*; or insofar as it is more or less perfectly in such a state of heat. Such a virtual quantity is determined first of all in its radical foundation, that is, in the very perfection of the form or nature. And so it is called a *special quantity*, just as it is called a great (amount of) heat on account of its *intensity and perfection*. Secondly virtual quantity is judged *in the effects of the form*. The first effect of the form is the existence since every being has its existence through its form. On the other hand the second effect is the being's action since every being

St. Thomas further considers a type of quality which he terms *magnitudo et quantitas per accidens*. He divides this quality in a two-fold way. The first is that which is proportional to the size of an object. Thus a white plank which has 10 sq. ft. of surface will have *twice as much whiteness* as a corresponding plank of only half the same surface. The second division is that which is measured or judged by the magnitude of the results or effects produced. It is the same intensive quantity which we spoke of before.[13]

For ordinary purposes these various divisions, with the exception of the quality which is proportional to the size of the subject in which it is, can be reduced to one—intensive quantity. For we judge or estimate the potential capacity or producing power of things by the quantitative effects which they produce, and likewise too, by knowing the quantitative effects of a thing, we can easily know its intrinsic capacity or perfection.

We now pass on to see how mathematics may be applied to these two qualities, if we may so call them,—quantity *per accidens* and *intensive* quantity.

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acts by reason of its form. Therefore virtual quantity is reckoned by both the existence and the action of a being. By the existence, insofar as those things which have a more perfect nature, have also a greater length of life. And by the being's action insofar as those things which have a more perfect nature, have a greater capacity for action." See also S. *Th.*, I, II, Q. 52, a. 1, et 2; II, II, Q. 24, a. 4, ad 1; ibid. a. 5, ad 2; S. *c. G.* I, c. 43; *de Ver.* Q. 29, a. 3; I *Dist.* 17, Q. 2, a. 1; I *Dist.* 19, Q. 1, a. 1, ad 1: ibid. Q. 4, a. 1, ad 1. I am nearly tempted here to speak of intensive quantity insofar as it involves concepts of theology as grace, faith, etc. Well might an article be written on Mathematical Theology. The virtue of charity is a good example of intensive quantity, for it can not only be extended to a greater or smaller number of objects, but it can also embrace each of the objects of its love with a greater or less intensity. So writes St. Thomas in S. *Th.* II, II, Q. 24, a. 4, ad 1.

13. "There is for all the qualities and forms the common term of magnitude which is spoken of as their perfection in the subject. However, there are some qualities, which have besides that magnitude or quantity which *per se* belongs to them, another magnitude or quantity which *per accidens* belongs to them. And this admits of a two-fold division. One way on account of the subject. As for example when whiteness is called a quantity *per accidens* because its subject is quantified. Thus any increase in the subject brings an increase in the whiteness *per accidens*. However, on account of this increase a thing is not said to be more white, but there is a greater (amount of) whiteness (major albedo), just as a larger thing is said to be white . . . In another way quantity and augmentation are attributed to a quality *per accidens*, and that is the object towards which its activity is directed. This is called virtual quantity, and this especially, too, because of the quantity of the object or its ambit (continentiam). So one is considered strong who can carry a heavy weight, or who can do any other striking thing which is great on account of its dimensional quantity or the greatness of its intrinsic perfection, or on account of its discret quantity . . . But we must bear in mind that those quantities are the same, if one of them can exert great effects and the other is great in itself. This is clear from what has been said. Thus also the quantity of one's perfecion mav be said to be the quantity of one's power." (*Q. D. de Virt. in comm.*, a. 10, ad 10).

The Mathematical Philosopher Explains The Mathematization of Qualities
The Application of Analysis Situs

That quantity *per accidens*, i.e., the type whose increase is proportioned to the increase or augmentation of its subject, is subject to measuration and thus to mathematization, is so clear in itself and from the example of the white plank, that we scarcely need pay any further attention to it. The law of the refraction of light—a phenomenon which occurs when light passes from one medium to another of a different density, is another good example of this in view of the fact that both light and the media are qualities.

It is also well to realize that from this type of quantity arises the possibility of what is called "an accidental heterogeneity." This means that in an extended object one quality can be localized in one portion of it, and another elsewhere, or again the same quality can be of different intensities at different sections of the same object. This heterogeneity allows the widest sort of localized distribution of the various qualities in all parts of any body. It is readily possible here to apply the mathematical method of *analysis situs* (topology) to this localized distribution of qualities.[14]

Modern science, if it has stressed anything, has stressed the importance of measuration or pointer readings—sometimes to the extent of making experimental and mathematical physics nothing else. This is unfortunate for at times it makes scientists forget the rich ontological realities behind these pointer readings. Extensive quantity is, of course, the foundation of all measurements and pointer readings. Since, however, certain qualities can be reduced to or classified as intensive quantity, they too can be measured for "measure is nothing else than that by which the quantity of a thing is known."[15]

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14. Hoenen, op. cit., pp. 192-193. Dr. Hoenen cites two examples in which this method has been used with great success: (1) the theory of chemical and steriochemical structure and (2) the theory of crystal structure. He makes one other very pertinent remark here: "Again it is clearer than daylight that this method preserves the nature of qualities intact, and it does not break the bond between theory and reality. Rather it comparably better achieves this end than the non-mathematical methods of approach. These things are so evident that it is foolish to insist further. From this short treatment of quality *quantified per accidens* on account of the extension of the subject, we can conclude with complete certitude that those err who in a general way assert that mathematical physics does not reach (considerare) reality because it is mathematical." p. 193. Dr. Hoenen here is most likely referring to M. Maritain.

15. *In Met.* X, lect. 2, Cath. 1938. As just stated, we are not trying to maintain here that in the recorded measurements of intensive quantities (qualities) all the reality is represented by this process for such a "reduction" is always a "reducing." Still in the light of our own proper experience of the various qualities and by interpretation, the recorded measurements mean much more to us than mere figures.

St. Thomas shows by various examples, as velocity, gravity, pitch of sound, etc., how his philosophy of measure can be applied to qualities.[16]

Let us further analyse this possibility and prove that qualities can be mathematized by another method of approach.[17]

Pure Ordinal Numeration of Qualities

With the same mathematical evidence and certitude which we have in the case of ordinary extensive quantity, we can say with reference to *intensive* quantity that if intensity A is greater than intensity B, and this latter is greater than intensity C, then A is greater than C. Mathematically this is expressed: if $A>B$ and $B>C$, then $A>C$. It likewise here too follows that if $A=B$ and $B=C$, then $A=C$.

This analogy proves the possibility of expressing qualities mathematically (or by mathematical symbols). Whence it follows that just as we can properly arrange a hundred soldiers of different heights according to their respective heights in a line, so too we can arrange a hundred similar bodies with varied temperatures (or varied qualities) in a similar series, based on their respective quantity of heat(or other quality). We denominate each of these bodies, as we do with the soldiers, by *some number*, and this number tells us whether *this* object or body has a greater or less temperature or grade of quality than *any other* object or body in the same series. This rigorously follows from the postulates of the preceding paragraph.[18]

It is plain that we must use *here* ordinal numbers (1st, 2nd, 3rd, . . .), and not the cardinal type (1, 2, 3, . . .).For in both examples we are not concerned whether the difference of height or temperature or quality (something which may greatly vary) are the same or not, but we are rather concerned with the *order* in which the soldiers and bodies are. Thus the first body may be 20° F., the 99th 60° F. and the 100th 100°F. And yet these numbers or symbols give us a true picture of the intensity of the heat (or other quality) in these bodies. These numbers, it is true, do not tell us whether the difference of temperature between the 10th and the 12th bodies be equal to or greater than that between the 20th and the 30th bodies. But they do state that the variation of temperature between the 10th and 30th bodies is greater than it is between the 20th and 30th.[19]

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16. *In Met.* X, lect. 2, Cath., n. 1948. "Pitch in sound (gravitas et acuitas in sonis) is determined by the relative frequency of vibrations,—a matter explained in the science of sound (in musica)."

17. In the matter which follows I am very particularly indebted to Dr. Hoenen. I largely follow him in theory, approach, and order of treatment. See Hoenen, op. cit., pp. 194-204.

18. Cfr. Hoenen, op. cit., p. 194.

19. Hoenen, op. cit., p. 195.

Now if the intensity of a quality increases or decreases with a continuity—something which St. Thomas defends[20]—then between any two intensities an indefinite number of intermediate intensities can be thought of or found, just as there are an indefinite number of points between any two non-coincident points in a straight line. It is the function of the science of physics, and not of metaphysics, to determine further how a graduated series or scale of these intensities in individual cases can be handled.

Intensity of Qualities Known from Their Effects

Along with St. Thomas scientists arrive at the intensity of an agent (cause) by observing its quantitative effects. Thus temperature is proportioned to and determined by the expansion it causes in bodies. Now just as this expansion can be divided indefinitely, so too the cause of this expansion admits a similar divisibility. Then by mathematically designating to each of these possible points of this latter divisibility an *ordinal* number, we can know (just as we knew in the case of the soldiers) the relative intensity of each advance or degree in temperature. This approach can similarly be used for other qualities according to the particular setup in each case.[21]

It is clear that this approach of judging from the agent's effects provides us with a method of determining very accurately each intensity with reference to all the others, according to the norm of *greater* or *less*. The *ordinal* numbers here not only provide us with sufficiently detailed results, but they also leave qualities as such intact. Then, too, consistent with St. Thomas' definition of measure in qualities(that medium by which we know the dimensions of some object), they give a true measure of the virtual quantity which is present in this or that quality. And, therefore, if a scientist speaks of e. g., *a certain quantity* (say 25 pounds) of stress or strain (and it is a question here of an experimental setup which can be reduplicated), he will be speaking in terms which have a very definite significance to other scientists.

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20. *In Met.* XI, lect. 10, Cath. n. 2354: "In the case of change, [the presence of the infinite] is not so clear. And yet in some true sense it is here, because qualities in which change takes place, are *per accidens* divided after the fashion of quantity. Besides, the intensity and abatement (remissio) of a quality is reckoned insofar as a quantified subject in some sort of a more or less perfect fashion participates of the quality." Cfr. also S. *c. G.*, II, 19.

21. Dr. Hoenen, op. cit., p. 196, makes a remark which is pertinent here: "We realize too, provided we have at hand that type of procedure which, to speak in the language of the physicists, can be "reduplicated" ("reproduci"), that we have here a method which *per se* has a universal application." By the word "reproduci," Dr. Hoenen no doubt refers to "a reduplication of the original setup and controlled conditions necessary for repeating any experiment."

Intensity of Qualities Known from Their Causes

We can also obtain the same results by the opposite approach of noting the quantity or magnitude of the agent (cause).[22] Thus we determine the relative degrees of illumination which will result at a definite distance from the source of light when one, two, three, etc., electric lights of the same amperage and voltage are lit at the source. If we use one electric light and vary both the voltage and amperage within the limits which the electric bulb will stand, we will note a *continual* variation in illumination at a definite distance from this source of light. On the other hand, if the amount of light is not changed, but the distance from the light-source is changed progressively (*modo continuo*), then also progressively the illumination will be increased or decreased.[23]

In the first case (where the distance is constant) the numerical amount of illumination (light flux) is directly proportional to the number of electric lights or to the *varied* intensity of one of them. In the second, however, where the distance is allowed to vary, if the amount of light flux is so adjusted that it is inversely proportional to the square of the distance, the results will be the same as in the first case. We should not forget likewise that we can approach this problem satisfactorily by determining or calculating the intensity of the source through observing the variation of the illumination (*in effectu*). Further ramifications here and elsewhere are matters which belong to the science of physics.

Hence the ordinal numbers provide a basis whereby a table of intensities can be tabulated for the various qualities.[24] From these tabulations mathematical graphs, functions (vector or scalar), and equations can be further accurately determined, although, of course, in a restricted sense. And here we have *experimental* physics as understood today,—a science both experimental and mathematical.

"If then," to use Prof. Hoenen's words, "this same method which is used in explanatory theoretical physics, is applied to hypotheses which are intended to determine the causes of phenomena and so to explain these phenomena as they are now mathematically (quantitatively) known, not even here is there any need for any other numbers, except the pure ordinals, to express the various intensities. Here quality as such remains also intact (as the qualitative state of the ether

22. S. *Th.*, II, II, Q. 24, a. 5, ad 2: "The addition of light to light can be understood in this sense that there is an increase of illumination on account of the change effected in the light-source."

23. Dr. Hoenen uses a candle as an example. See op. cit., p. 197. I believe an electric light is better calculated to bring out all the details which we are looking for.

24. See Dr. Hoenen, op. cit., p. 198, for a pertinent example in physics.

which is measured by the electric and magnetic forces in the Maxwell theory), nor is it confused with dimensional quantity, and still it is subjected to mathematical analysis. With this we have the foundations of *theoretical* and mathematical physics."[25]

At this point in the development of this article, we might with Dr. Hoenen lay down the following certain conclusion:

The application of measure and mathematical analysis (methodi) to qualities in either experimental or theoretical physics does not destroy qualities as such and it permits an investigation of them and interrelationships in a way incomparably more strict and accurate than could be obtained by any other approach.[26]

The Application of Cardinal Numbers

We might well further examine whether or not the *distances* or differences (similarities or dissimilarities) between the various intensities (for so far we have only examined the *order* of the intensities) can have any meaning. Of course, we realize that this further analogy is not necessary to satisfy the demands of mathematical physics. That we have already done.

Now if given a series of intensities in which A>B>C, it follows at once that the distance AC>distance AB from the very nature of ordinal numeration. Further it is certain that AB can be compared analogically to AC as a part to its whole as in extensive quantity. AB does not, however, admit of divisibility[27] nor can it exist alone in some subject.

Here, however, we are rather interested in the relationship of AB to BC. Can we speak of equality, or inequality with reference to them? Can we say that the difference between the 30th and 20th degree can in any intelligible way be equated or not to the difference between the 20th and 10th degree on any standard type of thermometer? In a word, does equality of distances have any meaning?[28]

We believe that it is no idle reasoning to say that it can have a true meaning. Suppose, given A>B>C, that B starting from A, is

25. Hoenen, op. cit., p. 198.

26. Hoenen, op. cit., p. 199.

27. Cfr. I *Dist.* 19, Q. 4, a. 1, ad 1: "Virtual quantity is not reckoned according to the intrinsic division of a force, but it's division is rather computed with reference to exterior objects, be it by the number of objects or the intensity of an act, or by the mode of action. Thus it is evident that in virtual quantity there is no reference (ratio) to the whole and an integral part, because integral parts are contained within their whole."

28. See Hoenen, op. cit., pp. 200-201.

allowed to vary between A and C, and finally to reach C. In the course of this variation the difference (diversity, disparity, dissimilarity) between B and C, is at first greater, then it gradually gets smaller, and finally it vanishes. So too, the difference between A and B is at first smaller, and later on greater than the difference between B and C[29]

Likewise if B would start from C and vary toward A, then at least at an infinitesimal distance from C say at B″ in the diagram, it seems evident that, since $B''>C$ and $A>B''$, A/B'' and B''/C are ratios of an altogether different value because A is much greater than B″, and B″ is only infinitesimally greater than C. Thus the further evident conclusion: $A:B''>B'':C$, or expressed in another way, $A/B''>B''/C$.

To revert again to the case where B varies from A to C. At any infinitesimal distance from A, say B′ (we will use B′ for B here for the sake of following the diagram), we can say that $A>B'$, and $B'>C$. Now since B′ nearly approaches A, the ratio A/B' is nearly equivalent to 1, whereas, since B′ is clearly much greater than C, the ratio B'/C is certainly much greater than 1. Whence we conclude

$$B'/C>A/B'.$$

Now dropping the primes from B, we see clearly that as B varies from A to C, we have the following relationships,

$$B/C\gtrless A/B.$$

Since this is true there will be one critical point for B as it varies continually between A and C, at which

$$B/C=A/B.$$

Whence the general conclusion is, given B to vary from A to C:

$$B/C\gtreqless A/B.$$

Of course, here we suppose that we know the intensities from the quantitative effects which for all purposes form a *continuum*. Where this is not verified, no deduction can be made. Nor does it necessarily follow that B will be midway in distance between A and C. It may be. But the point is that I can locate B so that $A/B\gtreqless B/C$ and that is mighty important in this matter of intensive quantity.

In view of what we have just laid down, the relationship between two different intensities, even in the case where one is not a part of the other, but is wholly out of its sphere of intensity, should have a significance.[31]

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29. A here signifies what is represented in the figure by OA; B by OB, and C by OC. It is all important to keep this in mind, for when we write $A>B>C$, geometrically this would be expressed: $OA>OB>OC$. Failure to keep this in mind, is certain to beget misconceptions in what follows. Otherwise we will seem to affirm what we before denied.

30. In mathematical parlance $B/C\gtrless A/B$ means that the ratio B/C is either greater or less than the ratio A/B; and $B/C\gtreqless A/B$ means that the ratio B/C is either greater than, or equal to, or less than the ratio A/B.

31. Hoenen, op. cit., pp. 201-202.

It now remains our task to find a means of properly orientating such a comparison. In the case of extensive quantities there are two methods of comparison. For given A>B>C, then the comparisons of their respective difference (A—B) and (B—C) can be made. Or we can form a comparison by the ratios, A/B and B/C. The first of these two methods does not seem to have applicability in the solution of our problem. For otherwise it would seem necessary that we should perceive smaller intensities in greater intensities which should virtually contain the former. Thus we ought to perceive the 20th degree in the 30th degree of heat. However, the very nature of quality, as we have seen, does not permit any such direct divisibility or discernment of its parts.[32]

This comparison brings out one striking difference between dimensional and intensive quantities in bodies: the difference of two extended things or objects is extension, while the diversity of two intensities is not intensity.[33]

The other method of comparison by ratios (*proportiones*) can be used in both extensive and intensive qualities. Here it is a matter of comparing an individual whole (totum indivisum) A with another individual whole B, and entire B with entire C, and finally comparing the first ratio with the second one.[34] There is no need here that a smaller intensity be perceived as a part of a greater one. However, it certainly can be said of these ratios that they are either equal or unequal; otherwise our axiom on intensities[35] cannot be verified. It might

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32. Hoenen, op. cit., p. 202, gives two other good reasons. He concludes: "Extension can be divided in parts, but not intensity." It is well to remember that *a part* in intensity can only be had by a continual change (intensio vel remissio) of the quality. Again since *equality* is rather predicated of quantities, while *similarity* more applies to qualities, subtraction, and addition too, seems to be here without *strict* significance.

33. Hoenen, op. cit., p. 203, makes this timely observation: If one, erroneously of course, should not admit this and wish to apply the method of arithmetical subtraction to determine the different intensities of a quality, he would not still identify quality with dimensional quantity, as long as he admits that these subtracted "parts" are neither homogeneous with the whole nor among themselves (v.g. heat between the 20th and 30th degree); [and], moreover, that they cannot exist by themselves in a subject, nor that the total intensity is composed from the parts taken in any order. All these, indeed, are found verified in extended objects. Wherefore, even if he should admit these erroneous ideas, still he would not yet identify extension and quality."

34. In the case of the differences arising from subtraction, B was compared with a part of A, to which it was equal, and the difference (A — B) remained. These differences in the matter of intensities neither by themselves nor in any subject can be conceived as existing. Thus a comparison or relationship is meaningless. See Hoenen, op. cit., p. 203.

It will be noted here that we use the English word *ratio* as a translation of the Latin word *proportio*, because the English word *proportion* signifies the equality of two ratios. On the other hand the Latin word *proportionalitas* is concerned with the relationship between two *proportiones*—a thing which figures so prominently in the Thomistic analogy of being.

again be stated in another way: If B is infinitesimally close to A, then the ratio A/B will surely be less than B/C, given initially A>B>C; likewise if B is infinitesimally close to C, then the ratio A/B will surely be greater. Since therefore, $A/B \gtreqless B/C$ according as B moves between A and C, it follows that at one critical point A/B=B/C.[36]

Tracing as we have the applicability of ordinal and cardinal numbers as well as the various kinds of comparisons to intensive quantities, we are brought to this conclusion of Prof. Hoenen:

The application of pure ordinal numbers to the intensities of a definite quality are sufficient to satisfy the demands of mathematical physics. Nevertheless, these qualities have a closer analogy with dimensional quantity, for just as in extended bodies, so too in qualities there are ratios (*proportiones*) of intensity which can be compared with one another to determine whether they are equal or not. Therefore, the numbers which express these intensities, can (at least theoretically) be so determined so that they are no longer pure ordinals, but they more properly resemble (*accedant*) the cardinal numbers, even though they do not form a part (*attingant*) of them. For they can (at least theoretically) be so determined that their ratios (*proportiones*) are at least analogous to the ratios of intensities. The analogy with dimensional quantity, however, does not go to the extent that there can be subtraction in intensities just as in the case of extended bodies.[37]

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35. See pp. 137-138.

36. We are not concerned here whether the ratios (*proportiones*) are either univocally or analogously predicated of extensive and intensive quantities. For further study on this subject see *Q. D. de Ver.* Q. 26, a. 1, ad 7; ibid. Q. 2, a. 3, ad 4; Q. 8, a. 1, ad 6; Q. 23, a. 7, ad 9; IV *Sent. Dist.* 49, Q. 2, a. 1, ad 6; *In Boet. de Trin.*, Q. 1, a. 2, ad 3; *In Eth.* V, 1. 5; *Anal. Post.* I, 1. 12.

37. Hoenen, op. cit., p. 204. No doubt at this point some difficulties will have come to the reader's mind, especially on the point of the subtraction of qualities. I know that I have not clarified all my difficulties. Here an attempt has been made to give, follow, interpret, and build on St. Thomas' principles, but not to go contrary to them. If one admits what he says in I *Dist.* 19, Q. 4, a. 1, ad 1, and elsewhere (see ref. 27 of this article), then it would seem that one cannot go beyond what has been laid down here. However, in any case, it is well to remember that St. Thomas' principles and the further development that they admit, satisfy the demands of mathematical physics—a thing surely insisted on in this article.

Moreover, no attempt has here been made to establish a *rigorous* definition of Mathematics since that would require another article—and besides, who has succeeded so far if we would look at the matter *rigorously*? But Mathematics is obviously something more than the science of the functional relationships between *pure* abstract quantities. Imaginery numbers are a good example of *analogous* quantities. Vectors, involving as they do direction besides extension, size, shape, succession—things in the subject-matter of mathematics—are surely in the realm of quality. And what shall we say of tensors, or mathematical operators as the *gradient, curl, divergence?* These latter involve operations upon quantities in a way that they seem to modify the quantities in question *qualitatively*. Thus mathematics extends beyond mere *pure* extension or quantity, as implied in the beginning of this article.

GRAVITATIONAL, ELECTRIC AND MAGNETIC FIELDS OF FORCE

A Consideration of their Extent and its bearing on Molecular and Atomic Continuity.

REV. JOHN S. O'CONOR, S.J.

Two recently published discussions, the first Father James A. McWilliams' article in the November 1940 "Modern Schoolman" entitled "The Supposit in the Inorganic World" and the second an abstract of Mr. George M. Tipton's paper, "Molecules, X-Rays and The Continuum", found on page 41 of the Jesuit Science Bulletin for October 1940, raise a very interesting point which is summarized in the title of this note.

Unless I have misunderstood the argument, in Father McWilliams' treatment it is implied that the de facto continuity of the molecule follows from the fact that the parts of the molecule (atoms, electrons, protons, etc.) act upon one another with forces of attraction (and repulsion?) and that since action at a distance is most probably physically repugnant there must be a medium between the interacting parts. This medium is then to be considered as a continuous "something" which permits the predication of continuity with respect to the molecule as a whole and gives it therefore the unity necessary to constitute it a supposit, (admitting other prerequisites).

A similar line of reasoning is used by Mr. Tipton but applied to crystalline structure, specifically employing the field of force of a bar magnet as example or rather as an analogy. The notion of the dimensions of a magnet is extended, and a distinction made between "actual dimensions" and the extension of the field of force. It is said that the "quid substantiale" of the magnet must be coextensive with the "larger continuum, the *field of force*."

Since the entire discussion is qualitative we can for the sake of simplicity disregard the quantitative differences between the point charge of electrostatics and the dipole nature of the magnet. It would also seem that *as far as the arguments for continuity are concerned* we can also priscind from the fact that the gravitational and electrostatic fields are lamellar and the magnetic field is solenoidal. Finally it does not seem pertinent to the question at issue whether the forces are Newtonian (Coulombian), of the Van der Waals or of the exchange type. The sole point of the argument appears to be the fact that the particles in question *act upon one another*, and therefore are connected by a continuous medium.

What *is common* to all the above enumerated type of fields of force is the fact that their intensity is some function of distance and (outside the bodies themselves) as far as this writer knows, the intensity of the field of either a sphere of gravitational matter, of electrostatic charge or of a magnetic dipole reduces to zero *only at an infinite distance from the source of the field*. That these fields may become negligible at great distances, as far as our measuring instruments are concerned, is of Physical but not of Philosophical significance.

Two inadmissible conclusions would seem to follow from the admission of the previously outlined arguments for continuity in the molecule and the form (for example) of the inverse square law of electric and gravitational attraction.

For if the interaction through an inferred medium according to fixed laws constitutes a sufficient condition for continuity then not only our solar system but the entire universe is a single continuum. There must be therefore some other requirement which is absent from the solar system and present in the molecule which excludes the former and establishes the latter in the category of the "unum per se". That molecules are any more "natural units" than is the solar system,—that they are governed by regular law in any stricter sense than is the motion of the planets would not have been admitted by the "classical" physicists of the last century. Today the motion of electrons about nuclei is expressed as a probability function, because the indeterminacy postulate of quantum mechanics has opened up approaches to knowledge of atomic and sub-atomic phenomena which remained closed as long as the Physicist clung to determinism. Thus according to present day notions (right or wrong) it is the law of chance which enables us to predict (with probability) the position and velocity of an electron in the field of force of proton or nucleus, to within specified lower limits. Whereas on deterministic principles complicated problems of this type were either insoluble or gave incorrect predictions.

The second inadmissible conclusion which would seem to follow from the position adopted by Mr. Tipton is the fact that if the "quid substantiale" of every attracting body is coextensive with its field of force and its field drops to zero only at infinity then the material world is in a state of universal compenetration.

For it is a demonstrable fact of physics that fields of force overlap and may be superimposed.

GEOPHYSICS

THE SEISMOLOGICAL OBSERVATORY AT SPRING HILL COLLEGE

W. J. Rhein, S.J.

When Fr. Cyril Ruhlmann, S.J. had to give up teaching at Spring Hill College, Mobile, in 1926 because of ill health, he brought to a close many years of accurate and efficient observations of earthquakes. He had kept the Wiechert horizontal seismograph in excellent running order and had obtained a large number of excellent seismograms. From 1926 to 1939 the Wiechert was only occasionally in operation because of lack of trained personnel. In 1939 Fr. Anthony J. Westland, S.J. after four years of graduate work in the department of geophysics at St. Louis University took over the observatory. Fr. Westland put the Wiechert in running condition again, and introduced a group of Philosophers to the field of seismology by giving them an elementary course and teaching them the routine of making minor adjustments and changing the records. Much effort was expended to improve the station, and finally in the summer of 1940 definite activity along that line was started.

Through contact with the Coast and Geodetic Survey, Fr. Westland obtained as a loan the Neumann-Labarre vibration meter to determine the suitability of Spring Hill as a site for a high-magnification teleseismic instrument. Through the kindness of Fr. Macelwane a recorder was obtained to supplement the vibration meter. The meter, which is similar to a vertical seismograph and records by means of an optical lever, has a natural period of about 0.7 second, and a possible magnification of 10,000. The magnification used at Spring Hill was of the order of 3,500. The meter was set on a pier in the vault under the Administration Building. This vault proved to be admirably suited for the tests, because the air was dry and the temperature almost constant. During a week in October when there was an outside temperature change of forty degrees, the temperature in the vault, as determined by a thermograph, varied only three degrees. The vibration meter had not been used as a recording instrument before it was installed at Spring Hill. In its use to test locations in New England, only direct observations were made by watching the motion of the light spot on a screen three meters distant. At Spring Hill runs of as long as twelve hours were made without serious difficulties due to wandering of the light spot. The character of the records showed that artificial vibrations with a period of 0.2 second caused by freight trains passing at a distance of one mile were very predominant. Microseisms with a period of about two seconds were also evident, but their amplitude was much smaller than that of the vibrations due to the train.

On Nov. 9, 1940, the meter made an excellent 'gram of the Rumanian 'quake. This record, as well as the other test records, were sent to the Coast and Geodetic Survey for examination. The men in Washington who examined the 'grams were well satisfied with the results obtained and made favorable comments on the tests in the December issue of "Earthquake Notes."

The conclusions reached after numerous tests indicate that since there is so much disturbance of very short period near the Spring Hill station it would not be wise to operate a short period instrument of high magnification in that vicinity. Further tests with both short and long period instruments are to be made, and at present a short period Wood-Anderson, lent through the courtesy of Fr. Stechschulte of Xavier University, is being operated.

The Neumann-Labarre meter was returned to the survey office in the last week of November. It will be used, we understand, for tests in Philadelphia and then will be sent out to the West Coast for tests at the various government dams. The interest and effort of the members of the Coast and Geodetic Survey in the observations made at Spring Hill is thoroughly appreciated by those in charge of the station.

The station has benefited greatly from the presence of Mr. Louis Eisele, S.J. on the physics staff at Spring Hill. Mr. Eisele has constructed a radio with crystal controlled oscillator to receive the time signals from Arlington. It includes an automatic relay which puts individual second marks on the seismograms. Mr. Eisele has completely overhauled the Wiechert by renewing the pivots, increasing paper speed, and making necessary adjustments. He is now working on recording drums driven by spring wound phonograph motors. He is experimenting with mechanico-optical seismographs of high magnification.

Having progressed to such an extent in the last four months of 1940, the staff of the Spring Hill station is confident of success for the future. It feels that Mobile is an excellent site for a teleseismic observatory and for the study of microseisms, and it is anxious to establish associate stations in Florida, Louisiana and Texas to take care of the seismically active regions of Mexico, The Caribbean and South America.

RECENT EARTHQUAKES IN NEW ENGLAND

JAMES J. DEVLIN, S.J.

Earthquakes, like fires, are always news, as the seismologists in New England learned last December when this corner of the United States quivered in response to a twitch of the rock beds in New Hampshire. Since the morning of the twentieth of December when the first shock brought the seismologist over to the vault at an all too early hour we have been answering endless questions, foolish and profound, relative to the cause, epicenter, and likelihood of a repetition of these quakes. Some people want to know whether or not they should buy earthquake insurance, and one lady wanted to know the exact time when the next earthquake was coming because she was planning a party and thought that the earthquake would be an added feature if she timed it properly.

As yet it is a little premature to write any final scientific conclusions on these two shocks, since we are still engaged in a study of the records and field data. But at least an indication on the trend of the study is of sufficient interest at present. The member stations of NESA have all forwarded their grams to Weston and the U. S. Coast and Geodetic Survey has swelled the field data which were gathered by Weston through postcards soliciting information, and from data gained from direct observation. The study is being made in conjunction with Dr. Leet of Harvard. In the near future it is expected that a full report will be published.

That New England should experience an earthquake of the proportions of the two in December is not surprising. The northeastern section of North America, including New York State, New England and Canada from the Saint Lawrence Valley to the Atlantic Coast, has a seismic record. For our present purposes we can designate a major quake in these regions as one which was felt over an area of 100,000 square miles or more and which has an intensity of at least seven on the modified Mercali scale in the vicinity of the epicenter. A quake of intensity seven would cause people to run outdoors, damage poorly constructed buildings and break chimneys. Since 1663 sixteen quakes of the types just referred to have occurred in this region; nine of them in the Saint Lawrence Valley. Four of these quakes in the Saint Lawrence Valley have occurred since 1900; one on Feb. 10, 1914 with an intensity of seven and felt over an area of 200,000 square miles, one on Feb. 28, 1925 with an intensity of nine and felt over 2,000,000 square miles, a third on Nov. 1, 1935 of intensity nine and felt over 1,000,000 square miles and the fourth on Oct. 19, 1939 of intensity 8 and felt over 600,000 square miles. It is worthy of note that the quake of 1925 seems to have been of normal depth focus; it was accompanied by a representable number of aftershocks. It certainly would have been more devastating in its effects had the epicenter been near a large city such as Montral. The quake of 1935 was determined as a quake of deep focus, approximately 200 kilometers. Again we have a fair number of aftershocks but none of serious proportions. The final word on the quake of 1939 is not available as yet but it was felt over an area of considerable extent though causing little damage. There were relatively few aftershocks and the question of its depth leaves room for debate at the present stage of our information.

In 1904 there was a strong quake of intensity 8, felt over some 300,000 square miles with an epicenter in the vicinity of Calais, Maine. This shock had relatively few after shocks. The quake of August 12, 1929 was felt over an area of approximately 100,000 square miles, had its epicenter in the vicinity of Attica, New York, and was not followed by many after shocks from the reports made on it. The Grand Banks quake of Nov. 18, 1929 was felt throughout New England and Eastern Canada. It was apparently of shallow focus since it caused landslides in the ocean bed which resulted in damage to the trans-Atlantic cables. It gave rise to a seawave that swept about 100 miles of the coast with loss of life and great property damage.

The latest quakes of December 20th and 24th of 1940 were felt over an approximate area of 500,000 square mlies. That these quakes were of nearly the same intensity is deduced both from the field data and the records of the seismographs. There was relatively little damage. A visit to the area most seriously affected showed consistent damage to chimneys which had not been recently pointed; monuments in a graveyard at Whittier, N. H. were rotated in a Northeasterly direction about two inches from their normal position. One house of modern construction but resting on an old foundation was moved slightly on its base. All of this data was collected in the towns in the immediate vicinity of West Ossippee, N. H. The same results were observed by others in Conway and towns to the north of the ones we visited. About seventeen aftershocks have been reported to date from the residents of this region, but not all of these have shown on the instrumental records. All except one which immediately preceded the second large shock were felt after the shock of the 24th.

While the data cited thus far are sketchy one thing is clear, namely that the recent shocks show the same characteristics as those that have been felt in this region, with the exception of one or two as was noted above. These quakes are felt over an amazingly large area with little damage even in the vicinity of the epicenter. This cannot be totally attributed to the sparse settlement of the regions affected. There have been relatively few aftershocks despite the opinion to the contrary of the residnts of New Hampshire. And there is a marked absence of surface effects, such as faulting or large cracks in the soil. This becomes all the more apparent if we study the characteristics of the California quakes. Even though these quakes are felt over a comparatively small area, such for instance as the Long Beach quake of 1933, they produce much more damage to property and life; show marked surface effects; and are accompanied by hundreds of aftershocks. In some instances the course of the faults of these California disturbances can be plotted from the field data. Now, these quakes are definitely known to be shallow or of normal depth. The obvious explanation for the difference in characteristics of the New England quakes seems to be depth of focus.

Influenced by such considerations we have examined the records available at Weston from the member stations of NESA. Two things are immediately evident on these records, first the absence of surface waves and secondly the excessive amplitude of the "S" or transverse waves, as compared with the "P" waves. An attempt to get a reasonable time of origin and an accurate epicenter from the record data in the routine way leads into some blind alleys. So at the present stage of the study of these recent New England quakes we are assuming that they are of deep focus origin and are well on the way to a rational solution of the data. These will be published at a later date in a joint report with the Harvard Seismic Station.

But the technical solution of the quake is not the only problem facing us. For the public is very much excited and in some cases alarmed over this recent trembling of what is commonly believed to be staid New England. This is abundantly clear from the number of requests for advice on the question of earthquake insurance. There is a report that one insurance agent sold \$100,000 worth of insurance in New Hampshire. Then there is the pressure from the newspapers to predict the time and place of the next shock. Since we are not gifted with grace of prophesy we have naturally shunned all ventures at a prediction. While it seems very likely from the statistical evidence that New England will experience some further quakes even of the magnitude of these December shocks, we have no reasonable basis of even a long range prediction. So that it seems inadvisable to purchase the rather expensive insurance that is being sold. The insurance companies themselves have no more data than we have and consequently cannot afford from their viewpoint to take too big a risk. Thus the rates are rather high. This excitement has had its counterpart in the past with the advent of a totally unexpected shock, and if no further shocks of this magnitude appear on the records within the next couple of years the fears and the interest in them will die out.

On the other hand it would be rash so to allay all concern about quakes that nothing at all would be done to take reasonable precautions. For if a quake of the proportions of these recent quakes should occur close to one of the larger cities, there is every reason to believe that poorly constructed and badly designed buildings resting on filled-in land would suffer serious damage. Consequently with the completion of the study and the ordering of all data an attempt will be made to have municipal and state authorities take steps to provide against that eventuality. The program adopted by several citizen committees in California will serve as the working basis for such a plan. Even if the quakes should not come, and we certainly hope that they will not, the efforts would not be in vain for the plans could be put into operation in almost any emergency and would result in buildings of better design and construction in the future.

MATHEMATICS

GENERAL SOLUTION OF A COMMON DIOPHANTINE PROBLEM*

REV. EDWARD C. PHILLIPS, S.J.

THE PROBLEM

Three boys went out to gather nuts. They put all the nuts in one basket with the agreement that at the end of the day they would divide them equally. Growing tired they hid the basket of nuts in the bushes and lay down in the shade for a rest—and promptly fell asleep. One of the boys on waking decided to take his share and go home. He ate one nut and then divided the rest into three equal parts, took one part for himself and put the rest back in the basket and went away without disturbing the other two. The second boy on awakening did the same thing, not knowing that the first had taken his share. Finally the third boy, not knowing that the other two had helped themselves, ate one nut, took one-third of the remainder and left the rest, as he thought, to be divided among the other two. Determine how many nuts the boys had gathered, supposing that it was the smallest number satisfying the indicated numerical conditions.

SOLUTION

Calling the total number of nuts N; the number left by the first boy N_1, and that left by the second boy N_2 we can express by the following equations the procedure indicated in the problem:

1) $N-1=3x$ (because the number in the basket after the removal of one nut was exactly divisible by 3)
2) $N_1=2x$ (because he took one-third of the nuts and left two-thirds)
3) $2x-1=3y$ (same reason as in 1)
4) $N_2=2y$
5) $2y-1=3z$

We now have three Diophantine equations 1), 2) and 3) by means of which we can eliminate x, y and z. We solve 5) for y in terms of z with the condition that y and z must be positive integers; we place this value of y in 3) and solve for x as a positive integer, and place this derived value of x) in 1) thus obtaining the integer form which N must have to satisfy the conditions of the problem.

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*Cf. a more extended article on Diophantine Equations in THE JESUIT SCIENCE BULLETIN for March, 1938, (Vol. XV, pp. 108-120).

From 5) we have $y=z+(z+1)/2$
and since y and z must be integers $(z+1)/2$ must also be an integer, hence z must be an odd number of the form $2m+1$, m being an integer; hence

6) y must be of the form $y=(2m+1)+(2m+2)/2$ or $3m+2$
Putting this in 3) we have $2x=3y+1=9m+6+1$, and therefore
7) $2x=9m+7$

since 2x is even, $9m+7$ must be even, which requires that 9m be odd; and this can be verified only if m is an odd number; let it be $2k+1$, where k may be either zero or any positive integer. Putting this value of m in 7) and dividing by 2, we get $x=9k+8$; so that finally we get
8) $N-1=27k+24$ or $N=27k+25$.
Putting k equal to zero we obtain the least value of N satisfying the conditions of the problem and the solution is therefore
9) $N=25$.

To test the correctness of our answer we start with this number and go through the procedure of the boys: The first finds 25 nuts in the basket, eats one leaving 24; he takes one-third of these, namely 8, and puts back the remaining 16 into the basket. The second boy eats one and takes one-third of the remaining 15, leaving 10 in the basket. The third boy eats one, takes 3 and leaves 6, which he erroneously thinks are to be divided between the other two boys.

Whilst this is a perfectly good *arithmetical* solution, it is not a *practically admissable* one, since the boys would have known that they had certainly gathered more than ten nuts apiece during the afternoon. Hence practically speaking we must make N much larger, which we do by giving to k a reasonably large value so as to make N agree with what the boys would expect the number of nuts to be. Suppose we make k equal to 20; then N would be equal to 565, and the first boy would take 188 (besides the one he ate), leaving 376; the second takes 125 and leaves 250; the third boy takes 83, leaving the remaining 166 nuts in the basket as *bona derelicta.*

It should be noted that the equation $N=27k+25$ *is a general solution of the problem for three boys.* Can we express this as some simple function of three? Inspection shows at once that 27 is the cube of 3, and that 25 is 27 less 3 diminished by 1, and hence we may express the value of N by the function $3^3k+3^3-(3-1)$. A more condensed and convenient form can be obtained by dropping the second term, 3^3, and making the requirement that k cannot be zero but must be 1 or some greater positive integer. With this restriction on k, our general formula becomes
10) $N=3^3k-(3-1)$.

Seeking for greater generalization of the problem we may ask whether this same functional solution is correct not only for the case of three boys, but of four or five, or of any number n. We answer affirmatively; and state that the general solution is
11) $N=n^n k-(n-1)$, n being the number of individuals in the party.

Furthermore, if each boy had eaten p nuts, instead of only one, before taking his share, the general solution would be
12) $N=n^n k-p(n-1)$.

Finally, if each boy had *added* instead of taking away p nuts before making the division, the solution would retain the same form, subject to the provision that p must, in this case, be given a negative value, or what is equivalent that we use a plus instead of a minus sign, giving us
12′) $N=n^n k+p(n-1)$.

For example, if each of the boys had added 2 nuts before taking his (putative) share, the formula 12′) would give us 27+4 or 31. Then the first boy would add 2 getting 33, and take 11 as his share and leave 22. The second boy on adding 2 would have 24 and take his 8, leaving 16; finally the third boy by adding 2 nuts brings the number up to 18 and takes 6 as his supposed share.

The proof that the above formulae, 11), 12) and 12′) are universally valid is left to our readers as an exercise in integral (diophantine) arithmetic. And if any readers of the BULLETIN send in proofs or comments I will be glad to prepare for the next issue a synopsis of their contributions.

PHYSICS

CORNU'S SPIRAL AND DIFFRACTION ANALYSIS (PART I)

Stanley J. Bezuszka, S. J.

The diffraction patterns of the Fresnel class (patterns produced by screens bounded by straight lines of infinite length, such as straight edges, wires and slits) were analyzed graphically by Cornu and present an elegant application of the results obtained from the mathematical discussion of the problem. Historically, the analysis of diffraction phenomena produced a new theory on the propagation of light. The foundation of the wave theory of light was laid through diffraction observations and even the acquired present information upon light, spectra and wave length values, both in the visible and invisible ranges, has been made possible by diffraction studies.

Since every theory of light and every theory of the propagation of wave-like disturbances is based on the differential equation of motion, a review of the fundamentals will serve as a readily accessible reference to the main discussion. The equation of wave motion which serves as the basis for the development of theories in sound and light is:

$$\frac{\partial^2 s}{\partial t^2} = v^2\left(\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2}\right) = v^2\,\Delta s \qquad . \ . \ . \ (1)$$

From this equation we can obtain the analytical forms of s for plane and spherical waves.

(A) Plane waves:

Let the x-axis be in the direction of the normal to the wave front, that is, in the direction of propagation. Then s is independent of y and z and can depend only upon x and t since in every plane x is a constant which is a wave front, and the conditions of vibration for a given value of t is everywhere the same.

Equation (1) then reduces to:

$$\frac{\partial^2 s}{\partial t^2} = v^2\,\frac{\partial^2 s}{\partial x^2} \qquad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ (2)$$

The general solution of this equation can be derived in a number of ways, and the integral must contain two arbitrary functions. Since the functions $f_1[t - (x/v)]$ and $f_2[t + (x/v)]$ satisfy the equation, the general integral is:

$$s = f_1\left(t - \frac{x}{v}\right) + f_2\left(t + \frac{x}{v}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\frac{\partial s}{\partial t} = f_1' + f_2' \qquad \frac{\partial s}{\partial x} = -\frac{1}{v}f_1' + \frac{1}{v}f_2 \qquad v^2\frac{\partial^2 s}{\partial x^2} = f_1'' + f_2''$$

$$\frac{\partial^2 s}{\partial t^2} = f_1'' + f_2'' \qquad \frac{\partial^2 s}{\partial x^2} = \frac{1}{v^2}f_1'' + \frac{1}{v^2}f_2'' \qquad \frac{\partial^2 s}{\partial t^2} = f_1'' + f_2''$$

Thus, equation (2) is satisfied and $f_1[t - (x/v)]$ corresponds to a wave moving in the direction of the positive x-axis and $f_2[t + (x/v)]$ in the direction of the negative x-axis.

If the relation between s and t is of the simple harmonic form, that is, proportional to $\cos 2\pi(t/T)$ equation (3) becomes:

$$s = A_1 \cos 2\pi\left(\frac{t}{T} - \frac{x}{vT} + \delta_1\right) + A_2 \cos 2\pi\left(\frac{t}{T} + \frac{x}{vT} + \delta_2\right) \quad (4)$$

This corresponds to the normal equation for a plane wave of wave length $\lambda = vT$.

(B) Spherical waves:

For spherical waves whose center is at the origin, s depends only upon t and the distance r from the origin. The analysis then reduces to:

$$\frac{\partial s}{\partial x} = \frac{\partial s}{\partial r}\cdot\frac{\partial r}{\partial x} = \frac{\partial s}{\partial r}\cdot\frac{x}{r}$$

$$\frac{\partial s}{\partial y} = \frac{\partial s}{\partial r}\cdot\frac{\partial r}{\partial y} = \frac{\partial s}{\partial r}\cdot\frac{y}{r}$$

$$\frac{\partial s}{\partial z} = \frac{\partial s}{\partial r}\cdot\frac{\partial r}{\partial z} = \frac{\partial s}{\partial r}\cdot\frac{z}{r}$$

where

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\frac{\partial(r)}{\partial x} = \frac{x}{\sqrt{x^2 + y^2 + z^2}} = \frac{x}{r}$$

$$\frac{\partial(r)}{\partial y} = \frac{y}{\sqrt{x^2 + y^2 + z^2}} = \frac{y}{r}$$

$$\frac{\partial(r)}{\partial z} = \frac{z}{\sqrt{x^2 + y^2 + z^2}} = \frac{z}{r}$$

$$\frac{\partial^2 s}{\partial x^2} = \frac{x^2}{r^2}\cdot\frac{\partial^2 s}{\partial r^2} + \frac{\partial s}{\partial r}\left(\frac{1}{r} - \frac{x^2}{r^3}\right)$$

$$\frac{\partial^2 s}{\partial y^2} = \frac{y^2}{r^2} \cdot \frac{\partial^2 s}{\partial r^2} + \frac{\partial s}{\partial r}\left(\frac{1}{r} - \frac{y^2}{r^3}\right)$$

$$\frac{\partial^2 s}{\partial z^2} = \frac{z^2}{r^2} \cdot \frac{\partial^2 s}{\partial r^2} + \frac{\partial s}{\partial r}\left(\frac{1}{r} - \frac{z^2}{r^3}\right)$$

Equation (1) becomes:

$$\frac{\partial^2 s}{\partial t^2} = v^2 \left(\frac{\partial^2 s}{\partial r^2} + \frac{\partial s}{\partial r}\left[\frac{3}{r} - \frac{x^2 + y^2 + z^2}{r^3}\right]\right)$$

$$\frac{\partial^2 s}{\partial t^2} = v^2 \left(\frac{\partial^2 s}{\partial r^2} + \frac{2}{r} \cdot \frac{\partial s}{\partial r}\right) \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ (5)$$

Multiplying by r:

$$r\left(\frac{\partial^2 s}{\partial t^2}\right) = v^2\left(r\frac{\partial^2 s}{\partial r^2} + 2\frac{\partial s}{\partial r}\right)$$

$$\frac{\partial^2 (rs)}{\partial t^2} = v^2 \frac{\partial^2 (rs)}{\partial r^2} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ (6)$$

Equation (6) is of the same form as (2) only that rs replaces s, and r replaces x. The general integral will therefore be similar:

$$rs = f_1\left(t - \frac{r}{v}\right) + f_2\left(t + \frac{r}{v}\right) \quad . \ . \ . \ . \ . \ . \ (7)$$

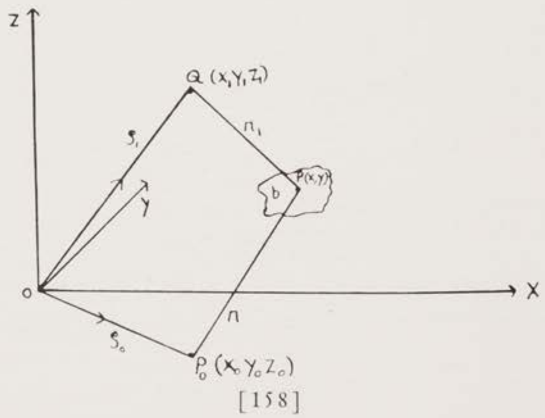
If homogeneous light of period T is used, it follows:

$$s = \frac{A_1}{r}\cos 2\pi\left(\frac{t}{T} - \frac{r}{vT} + \delta_1\right) + \frac{A_2}{r}\cos 2\pi\left(\frac{t}{T} + \frac{r}{vT} + \delta_2\right) \quad . \ (8)$$

This is the equation for spherical waves. One train of waves moves from the origin and the other moves toward it and the amplitudes A_1, A_2 are inversely proportional to r.

After this preliminary discussion of the plane and spherical wave forms, the following development of diffraction will lead to the type we are to study in detail.

(C) General diffraction treatment:



Assume that between the source Q and the point P_0 there is introduced a plane screen S which is of infinite extent and contains an opening b of any form. Let this opening be small in comparison with its distance r_1 from the source Q and also in comparison with its distance r from the point P_0 at which the disturbance s_0 is to be calculated by the equation (analytically deduced from Gauss's and Green's theorem):

$$s_0 = \frac{A}{2\lambda}\int \frac{1}{rr_1} \sin 2\pi\left(\frac{t}{T} - \frac{r + r_1}{\lambda}\right)[\cos (nr) - \cos (nr_1)]\, ds$$

In performing the integration over b (and because it is very small) the angles (nr) and (nr_1) are considered constant; likewise the quantities r and r_1 when they are not divided by λ.

Therefore:

$$s_0 = \frac{A}{2\lambda} \cdot \frac{\cos (nr) - \cos (nr_1)}{rr_1} \int \sin 2\pi\left(\frac{t}{T} - \frac{r + r_1}{\lambda}\right) db \quad . \quad (9)$$

Using the rectangular coordinate system x, y, z:

Let the xy plane coincide with the screen S,
some point P in the opening b have the coordinates (x, y),
$(x_1 y_1 z_1)$ be the coordinates of the source, z_1 being positive
and $(x_0 y_0 z_0)$ those of P_0, z_0 is then negative.

Then:

$$r_1^2 = (x_1 - x)^2 + (y_1 - y)^2 + z_1^2 \qquad \rho_1^2 = x_1^2 + y_1^2 + z_1^2$$
$$r^2 = (x_0 - x)^2 + (y_0 - y)^2 + z_0^2 \qquad \rho_0^2 = x_0^2 + y_0^2 + z_0^2$$

Writing r_1 and r in terms of ρ_1 and ρ_0:

$$r_1 = \rho_1 \sqrt{1 + \frac{x^2 + y^2 - 2(xx_1 + yy_1)}{\rho_1^2}}$$

$$r = \rho_0 \sqrt{1 + \frac{x^2 + y^2 - 2(xx_0 + yy_0)}{\rho_0^2}}$$

Since the dimensions of b and its distance from the origin are assumed small with respect to ρ_0 and ρ_1, in the integration over b, x and y are small with respect to ρ_1 and ρ_0. If we develop the above two forms in a series with increasing powers of x/ρ_1, y/ρ_1, x/ρ_0, y/ρ_0

$$(1 + \epsilon)^{1/2} = 1 + \tfrac{1}{2}\epsilon - \tfrac{1}{8}\epsilon^2 + \tfrac{1}{16}\epsilon^3 + \cdots +$$

and neglect powers higher than the second order:

$$\left.\begin{aligned} r_1 &= \rho_1\left(1 + \frac{x^2 + y^2}{2\rho_1^2} - \frac{xx_1 + yy_1}{\rho_1^2} - \frac{(xx_1 + yy_1)^2}{2\rho_1^4}\right) \\ r &= \rho_0\left(1 + \frac{x^2 + y^2}{2\rho_0^2} - \frac{xx_0 + yy_0}{\rho_0^2} - \frac{(xx_0 + yy_0)^2}{2\rho_0^4}\right) \end{aligned}\right\} \quad (10)$$

Denote the direction cosines of ρ_1 by $\alpha_1\beta_1\gamma_1$ and those of ρ_0 by $\alpha_0\beta_0\gamma_0$ and then adding the equations in (10) we arrive at:

$$\left.\begin{aligned} r_1 + r = \rho_1 + \rho_0 &+ \frac{x^2 + y^2}{2}\left(\frac{1}{\rho_1} + \frac{1}{\rho_0}\right) \\ &- x(\alpha_1 + \alpha_0) - y(\beta_1 + \beta_0) \\ &- \frac{(x\alpha_1 + y\beta_1)^2}{2\rho_1} - \frac{(x\alpha_0 + y\beta_0)^2}{2\rho_0} \end{aligned}\right\} \quad . \; . \; . \; . \; (10a)$$

$$\left.\begin{aligned} & r_1 + r = \rho_1 + \rho_0 + f(xy) \cdot \frac{\lambda}{2\pi} \\ & \frac{t}{T} - \frac{\rho_1 + \rho_0}{\lambda} = \frac{t'}{T} \\ & \frac{A}{2\lambda}\,\frac{\cos(nr) - \cos(nr_1)}{rr_1} = A' \end{aligned}\right\} \quad . \; . \; . \; . \; . \; (10b)$$

Equation (9) becomes:

$$s_0 = A'\left\{\sin 2\pi\frac{t'}{T}\int \cos[f(x,y)]\,db - \cos 2\pi\frac{t'}{T} \int \sin[f(x,y)]\,db\right\} \quad . \; . \; . \; . \; . \; . \; (11)$$

may thus be conceived as due to the superposition of two waves whose amplitudes are proportional to:

$$C = \int \cos[f(x,y)]\,db \qquad S = \int \sin[f(x,y)]\,db \quad (11a)$$

and whose phase difference is:

$$\frac{\pi}{2}$$

The intensity is given by:

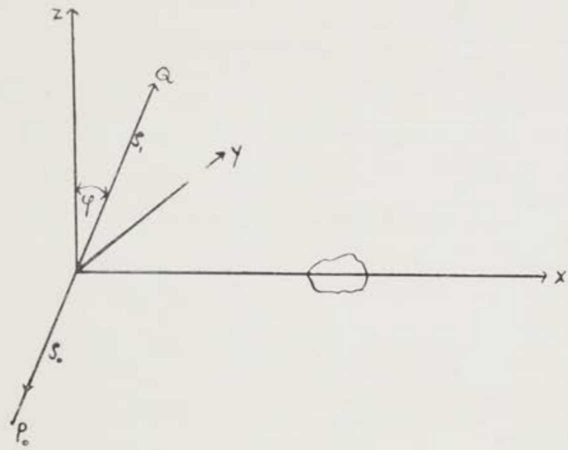
$$I = A'^2(C^2 + S^2)$$

Two cases can be distinguished:

1. Both the source and the point P_0 lie at finite distance (Fresnel diffraction type).

2. The source and P_0 are infinitely apart (Fraunhofer diffraction type).

D) Fresnel's diffraction phenomena:



Let the origin lie upon the line QP_0 and in the plane of the screen; then ρ_1 and ρ_0 lie in the same straight line but have opposite signs; hence

$$\alpha_0 = -\alpha_1$$
$$\beta_0 = -\beta_1$$

A comparison of equations (10*a*, 10*b*) after simplification by choosing as the x-axis the projection of QP_0 upon the screen (then $\beta_1 = 0$) and calling the angle which ρ_1 makes with the z-axis φ (then $\alpha_1 = \cos(\rho_1 x) = \sin\varphi$) we have:

$$f(xy) = \frac{\pi}{\lambda}\left(\frac{1}{\rho_1} + \frac{1}{\rho_0}\right)[x^2 + y^2 - x^2 \sin^2\varphi]$$

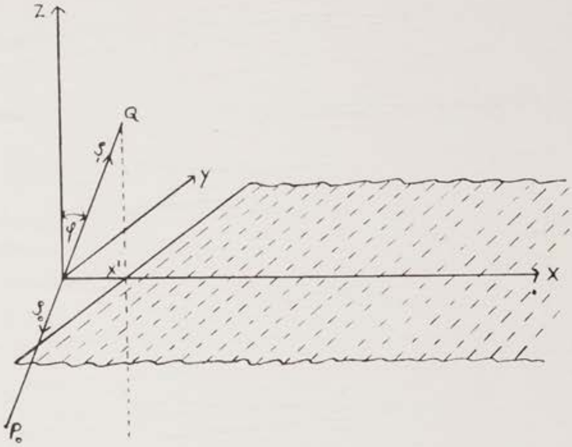
$$f(xy) = \frac{\pi}{\lambda}\left(\frac{1}{\rho_1} + \frac{1}{\rho_0}\right)[x^2 \cos^2\varphi + y^2] \quad . \; . \; . \; . \; . \; . \quad (12)$$

For the special cases:

1. *Diffraction by a straight edge:*

We assume a screen whose edge is parallel to the y-axis, and which extends from $x = x'$ to $x = +\infty$, $y = -\infty$ to $y = +\infty$.

$x' > 0$, that is P_0 lies outside of the geometrical shadow of the screen. We consider the intensity of the light in a plane which passes through the source Q and is perpendicular to the edge of the screen. Equation (12) is applicable here and gives in combination with (11*a*) the expression:



$$\left.\begin{aligned} C &= \int_{-\infty}^{x'} \int_{-\infty}^{+\infty} dx\,dy \cos\left[\frac{\pi}{\lambda}\left(\frac{1}{\rho_1}+\frac{1}{\rho_0}\right)(x^2\cos^2\varphi + y^2)\right] \\ S &= \int_{-\infty}^{x'} \int_{-\infty}^{+\infty} dx\,dy \sin\left[\frac{\pi}{\lambda}\left(\frac{1}{\rho_1}+\frac{1}{\rho_0}\right)(x^2\cos^2\varphi + y^2)\right] \end{aligned}\right\} \quad . \quad . \quad (13)$$

$$\frac{\pi}{\lambda}\left(\frac{1}{\rho_1}+\frac{1}{\rho_0}\right) x^2 \cos^2\varphi = \frac{\pi v^2}{2} \qquad \frac{\pi}{\lambda}\left(\frac{1}{\rho_1}+\frac{1}{\rho_0}\right) y^2 = \frac{\pi u^2}{2}$$

$$x = v\,\frac{1}{\cos\varphi\sqrt{\frac{2}{\lambda}\left(\frac{1}{\rho_1}+\frac{1}{\rho_0}\right)}} \qquad dx = dv\,\frac{1}{\cos\varphi\sqrt{\frac{2}{\lambda}\left(\frac{1}{\rho_1}+\frac{1}{\rho_0}\right)}}$$

$$y = u\,\frac{1}{\sqrt{\frac{2}{\lambda}\left(\frac{1}{\rho_1}+\frac{1}{\rho_0}\right)}} \qquad dy = du\,\frac{1}{\sqrt{\frac{2}{\lambda}\left(\frac{1}{\rho_1}+\frac{1}{\rho_0}\right)}}$$

$$\left.\begin{aligned} C &= \frac{1}{\cos\varphi\cdot\frac{2}{\lambda}\left(\frac{1}{\rho_1}+\frac{1}{\rho_0}\right)} \int_{-\infty}^{v'} \int_{-\infty}^{+\infty} dv\,du \cos\left[\frac{\pi}{2}(v^2+u^2)\right] \\ S &= \frac{1}{\cos\varphi\cdot\frac{2}{\lambda}\left(\frac{1}{\rho_1}+\frac{1}{\rho_0}\right)} \int_{-\infty}^{v'} \int_{-\infty}^{+\infty} dv\,du \sin\left[\frac{\pi}{2}(v^2+u^2)\right] \end{aligned}\right\} \quad . \quad (14)$$

$$v' = x' \cdot \cos\varphi\sqrt{\frac{2}{\lambda}\left(\frac{1}{\rho_1}+\frac{1}{\rho_0}\right)}$$

Expand the sin and cos functions and integrate with respect to *u*, arriving at:

$$C = f \cdot \left\{ \int_{-\infty}^{v'} \cos \frac{\pi v^2}{2}\, dv - \int_{-\infty}^{v'} \sin \frac{\pi v^2}{2}\, dv \right\} \quad \ldots\ldots \quad (15a)$$

$$S = f \cdot \left\{ \int_{-\infty}^{v'} \sin \frac{\pi v^2}{2}\, dv + \int_{-\infty}^{v'} \cos \frac{\pi v^2}{2}\, dv \right\} \quad \ldots\ldots \quad (15b)$$

where

$$f = \frac{1}{\frac{2}{\lambda} \cos \varphi \cdot \left(\frac{1}{\rho_1} + \frac{1}{\rho_0} \right)}$$

Hence:

$$I = 2A'^2 f^2 \left\{ \left(\int_{-\infty}^{v'} \cos \frac{\pi v^2}{2}\, dv \right)^2 + \left(\int_{-\infty}^{v'} \sin \frac{\pi v^2}{2}\, dv \right)^2 \right\} \quad \ldots \quad (16)$$

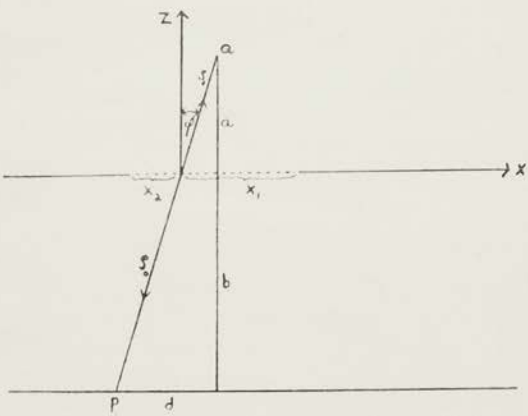
Since we consider only those portions of the xy-plane which lie near the origin, it is possible to set in the expression for A':

$$r = \rho_0 \qquad r_1 = \rho_1$$

$$\cos(nr) = -\cos(nr_1) = \cos \varphi$$

$$A'f = \frac{A}{2\lambda} \cdot \frac{2 \cos \varphi}{\rho_0 \rho_1} \cdot \frac{\lambda}{2 \cos \varphi \cdot \left(\frac{1}{\rho_1} + \frac{1}{\rho_0} \right)} \qquad A'f = \frac{A}{2(\rho_0 + \rho_1)}$$

2. *Diffraction through a narrow slit:*



To investigate the intensity of light in a plane which passes through the source Q and is perpendicular to the edges of the slit. This plane is the xz-plane.

If the point P_0 at which the intensity is to be calculated, lies in the geometrical shadow of one of the screens which bound the slit on either side, then x_1 and x_2 are either both positive or both negative. But if the line joining Q and P_0 passes through the open slit, then the signs of x_1 and x_2 are opposite. We assume that the source Q lies directly above the middle of the slit. Let δ be the width of the slit.

$$x_1 - x_2 = \delta \qquad \frac{x_1 - \frac{1}{2}\delta}{d} = \frac{a}{a+b} \qquad \begin{aligned} a &= \rho_1 \\ b &= \rho_0 \end{aligned}$$

$$v' = x' \cos\varphi \sqrt{\frac{2}{\lambda}\left(\frac{1}{\rho_1} + \frac{1}{\rho_0}\right)}$$

Introducing the quantity v and calling v_1 and v_2 the values of v which correspond to the limits of integration (cf. eq. 16) x_1 and x_2, the intensity of light at P_0 is:

$$I = \frac{A^2}{2(\rho_0 + \rho_1)}\left\{\left(\int_{v_1}^{v_2} \cos\frac{\pi v^2}{2}\,dv\right)^2 + \left(\int_{v_1}^{v_2} \sin\frac{\pi v^2}{2}\,dv\right)^2\right\} \tag{16a}$$

$$v_1 = x_1 \cos\varphi \sqrt{\frac{2}{\lambda}\left(\frac{1}{a} + \frac{1}{b}\right)} \qquad v_2 = x_2 \cos\varphi \sqrt{\frac{2}{\lambda}\left(\frac{1}{a} + \frac{1}{b}\right)}$$

$$\cos\varphi = 1 \qquad v_1 - v_2 = \delta\sqrt{\frac{2}{\lambda}\left(\frac{1}{a} + \frac{1}{b}\right)} \quad . \; . \; . \tag{17}$$

$$\frac{v_1 + v_2}{2} = \frac{x_1 + x_2}{2}\sqrt{\frac{2}{\lambda}\left(\frac{1}{a} + \frac{1}{b}\right)}$$

$$\frac{x_1 + x_2}{2} = \frac{x_1 + x_1 - \delta}{2} = x_1 - \tfrac{1}{2}\delta = \frac{ad}{a+b}$$

$$\frac{v_1 + v_2}{2} = \frac{ad}{a+b}\sqrt{\frac{2}{\lambda}\left(\frac{1}{a} + \frac{1}{b}\right)} = \frac{ad}{a+b}\sqrt{\frac{2}{\lambda}\frac{(a+b)}{ab}} = d\sqrt{\frac{2}{\lambda}\frac{a}{b(a+b)}}$$

$$P = \sqrt{\frac{\lambda b(a+b)}{2a}} \quad . \; . \; . \; . \; . \; . \; . \tag{18}$$

$$\frac{v_1 + v_2}{2} = \frac{d}{P} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \tag{19}$$

It will be necessary to treat briefly the terms:

$$\xi = \int_0^v \cos\frac{\pi v^2}{2}\,dv \qquad \eta = \int_0^v \sin\frac{\pi v^2}{2}\,dv \quad . \; . \tag{20}$$

which are commonly known as the "Fresnel Integrals." ξ and η are functions of the parameter v and may be considered as the rectangular coordinates of the point E. Then as v changes continuously, E describes a curve whose form we will determine. When $v = 0, \xi = \eta = 0$ the curve passes through the origin. When v changes to $-v$ the expression under the integral is not altered; but the upper limit of the integral and hence the sign of ξ and η change. Therefore the origin is a center of symmetry, for to every point $+\xi, +\eta$ corresponds a point $-\xi, -\eta$. For an element of arc ds we have:

$$ds = \sqrt{d\xi^2 + d\eta^2}$$

$$d\xi = dv \cdot \cos\frac{\pi v^2}{2} \qquad d\eta = dv \cdot \sin\frac{\pi v^2}{2}$$

$$\therefore ds = dv$$

The angle γ which is included between the tangent to the curve at any point E and the ξ-axis is:

$$\tan\gamma = \frac{d\eta}{d\xi} = \tan\frac{\pi v^2}{2} \qquad \gamma = \frac{\pi v^2}{2} \quad . \; . \; . \; . \; (21)$$

For $v = 0, \gamma = 0$: at the origin the curve is parallel to the ξ-axis. When $v = 1$ it is parallel to the η-axis.

The radius of curvature ρ of the curve at any point E is:

$$\rho = \frac{ds}{d\gamma} = \frac{1}{\pi v} = \frac{1}{\pi s}$$

At the origin where $v = 0$ there is a point of inflection. As v increases, that is, as the arc increases, ρ continually diminishes. Hence the curve is a double spiral with double points, which winds itself about the two asymptotic points F and F' whose position is determined by: $v = +\infty, v = -\infty$.

Moreover:

$$\xi_F = \int_0^\infty \cos\frac{\pi v^2}{2}\, dv \qquad \eta_F = \int_0^\infty \sin\frac{\pi v^2}{2}\, dv$$

Let

$$\int_0^\infty e^{-x^2}\, dx = M \qquad \int_0^\infty e^{-y^2}\, dy = M$$

$$\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dx\, dy = M^2 \qquad \begin{aligned} x^2 + y^2 &= r^2 \\ dx\, dy &= r\, dr\, d\varphi \end{aligned}$$

$$M^2 = \int_0^{\pi/2} d\varphi \int_0^{\infty} r \cdot e^{-r^2}\, dr \cdots \text{where} \begin{cases} \int_0^{\infty} r \cdot e^{-r^2} = \frac{1}{2} \\ \int_0^{\pi/2} d\varphi = \frac{\pi}{2} \end{cases}$$

$$M^2 = \frac{\pi}{4} \qquad M = \tfrac{1}{2}\sqrt{\pi}$$

Let

$$x^2 = -i\,\frac{\pi v^2}{2} \qquad x = v\sqrt{\frac{-i\pi}{2}} \qquad dx = dv\sqrt{\frac{-i\pi}{2}}$$

Then:

$$\int_0^{\infty} e^{-x^2}\, dx = \sqrt{\frac{-i\pi}{2}} \int_0^{\infty} e^{(i\pi v^2)/2}\, dv = \tfrac{1}{2}\sqrt{\pi}$$

Employing suitable transformations, we arrive at:

$$\int_0^{\infty} e^{(i\pi v^2)/2}\, dv = \frac{1+i}{2}$$

Since

$$e^{(i\pi v^2)/2} = \cos\frac{\pi v^2}{2} + i\sin\frac{\pi v^2}{2}$$

$$\int_0^{\infty} e^{(i\pi v^2)/2}\, dv = \int_0^{\infty} \cos\frac{\pi v^2}{2}\, dv + i\int_0^{\infty} \sin\frac{\pi v^2}{2}\, dv = \tfrac{1}{2} + \tfrac{1}{2}i$$

Equating the real and imaginary parts it follows:

$$\int_0^{\infty} \cos\frac{\pi v^2}{2}\, dv = \tfrac{1}{2} \qquad \int_0^{\infty} \sin\frac{\pi v^2}{2}\, dv = \tfrac{1}{2} \quad \ldots \quad (22)$$

Thus the asymptotic point F has the coordinates

$$\xi_F = \tfrac{1}{2}, \qquad \eta_F = \tfrac{1}{2}.$$

Part II will contain the graphical analysis of this discussion and a comparative study of a few experimental results obtained in the laboratory.

NEWS ITEMS

HOLY CROSS COLLEGE

Department of Chemistry

The W G Y Science Forum again invited Fr. Sullivan to give one of his talks on popular scientific subjects. Fr. Sullivan spoke at 7.30 P.M., February 18th over W G Y on *"Some Curious Costs of Water."*

Two more publications have been added to the long list of articles by men of Holy Cross. Mr. Edwin T. Mitchell in the recent issue of the Journal of Chemical Education describes several interesting lecture experiments for students in inorganic chemistry. In last December's issue of the same periodical Mr. Olier L. Baril wrote on *"An Improved Method For The Preparation Of Chloroform."*

Members of the faculty have been giving seminars at Clark University. Last November, Mr. Mitchell spoke on *"Elimination Reactions of Olefins."* On January 15th, Mr. Tansey spoke on *"The Peroxide Effect."* On February 19th, Mr. Baril spoke on *"The Chemistry of Hormones."* The talk on March 19th will be given by Mr. Mitchell on *"Reactions in Liquid Ammonia."*

In 1941, Holy Cross College will offer six graduate Fellowships in Chemistry, each Fellowship carrying a stipend of $450.00. These Fellowships are offered to graduates of colleges and technical schools properly qualified to undertake graduate work in chemistry. Application must be made to the Chairman, Faculty of Chemistry, on or before April 15th.

On February 27th the Worcester Chemists' Club held its meeting at Holy Cross. The Rev. M. J. Ahern, S.J., popular Jesuit scientist and lecturer, gave a very interesting discourse on color photography. The lecture was illustrated by numerous color transparencies of which he can justly be proud.

BOSTON COLLEGE

Physics Department

Ten more students received their Private Pilot Certificates after completing the Fall Session of Civil Pilot Training at Boston College. This brings up the number who have obtained their Private Pilot licenses to 45. The Spring Session started here February 6th.

Graduate Students in Physics gave seven seminars on their experimental work. The Senior B. S. in Physics class is now giving the seminars on their experimental problems for their Senior Theses.

On November 3rd in the Science Building the Science Committee of the Alumni Council of Catholic Action gave demonstrations and motion pictures of their work. The following program was well received by over two hundred who attended the meeting.

"Contributions of the Church to Medicine" .... Paul Coughlin
"Recent Developments in Modern Physics" .... Leo Landrey

TECHNICIANS ROUND TABLE DISCUSSION
Elizabeth Eichorn, *Chairman*

Eleanor Boylan, Mary Raftus, Barbara Benson, Rita McNeil
"Biology in Recent Times" .... John Riordan
"The Soilless Growth of Plants" .... Paul J. Boylan

The Physics Research Academy heard a lecture on "The Effects and Remedies of Ice Formation on Planes" by Professor Henry P. Houghton of M. I. T. at a dinner meeting on Jan. 16th. The research work in methods of removing ice from the wings, propeller and ailerons of modern aircraft was described by Professor Houghton, and then there was a discussion on Meteorology.

Dr. Reinheimmer of the Staff has obtained some very interesting measurements of the densities of various types of photographic films. He built a Sensitometer that gives 13 different steps of exposure on the film. Then these 13 different exposures are measured on a Densitometer that consists of a light source, microscope, and photo electric cell and galvanometer.

BOSTON COLLEGE
MATHEMATICS DEPARTMENT

On January 15, 1941, the Ricci Mathematics Club of Boston College was host to the Association of College Mathematics Clubs of Greater Boston for their semi-annual meeting. Over seventy members were present including representatives from M.I.T., Northeastern and Boston University, Tufts, Regis and Emmanuel College. The program, originally scheduled to include as its principal speaker, Mr. Harold A. Zager of our Mathematics Department, had to be changed at the last minute due to the latter's illness. Mr. Cedrone of our Mathematics Department substituted in his place, contributing a paper on "Determinants of the Third Order". The remaining contributions included papers from the student members, Mr. Corcoran of Northeastern who spoke on Indeterminate Equations, and Mr. Wingeiski of

Tufts who spoke on "Constructions in Advanced Geometry". A novel "Information Please" program followed with first prize going to Mr. Oliver Smith of M.I.T. A copy of Bell's Men of Mathematics was given as the prize. Refreshments were served and an informal meeting followed. The Boston College Mathematics Library was opened for inspection and drew highest praise from visiting professors.

The regular meetings of the Ricci Mathematics Club are held twice a month. At one of these meetings papers are presented and read by the members. These papers are afterwards published in the Ricci Journal. For this year the program for the Sophomores includes an investigation into the foundations and historical development of Analytic Geometry and Calculus; for the Freshmen, an investigation in the history and development of the Trigonometric Function Tables and the Logarithmic Tables. At the other regular meeting an informal discussion takes place on the solution of problems and questions related to class matter.

GEORGETOWN UNIVERSITY

The Graduate School

Department of Chemistry

This Department specializes in Bio-Chemistry, and in one of its weekly Seminar meetings, Dr. Othmar Solnitzky, of the Georgetown Medical School Faculty, presented a very remarkable moving picture history of the short life of an infant born with the heart completely outside the chest. This is one of those rare cases (about two dozen are on record in the world literature) in which the heart during the development of the foetus descends from its original precranial position to its normal location in the chest at a subnormal rate, so that on its arrival there it finds the foetal opening between the two parts of the sternum (the breast bone) already closed or so narrow that entrance into the chest is barred. This child was born in Gallinger Municipal Hospital, Washington, D. C., last November, and lived for twenty-two days. During its life Koda-chrome moving pictures were taken by Dr. Solnitzky at weekly intervals, so that there is a lasting record of the action of the heart and of its development. The pictures being in color, show clearly the blue color of the venous blood as it is being pumped into the lungs by the right lobe of the heart, and the rich red color of the arterial blood as it is pumped through the left lobe into the arterial circulatory system. A complete description of the case is to be published in the *American Heart Journal.*

Department of Mathematics

A new work by Fr. Frederick W. Sohon, S. J., Head of the Department of Mathematics, entitled *The Stereographic Projection*, is now in press and is scheduled to appear soon. The publishers are:

The Chemical Publishing Company,
148 Lafayette Street,
New York, New York.

The problem of grouping (in threes) of deaths among Jesuits, was outlined in the May, 1940 issue of the BULLETIN (Vol. XVII, pp. 196-199) and further developed at the national meeting of the Association in Chicago (September 5-6, 1940): an abstract of the Chicago paper will be found in the BULLETIN for October, 1940 (Vol. XVIII, p. 52).

Father Joseph T. O'Callahan, whose statistical work and some preliminary conclusions were presented at the Chicago meeting, has had to give up further research on the problem due to his appointment as Chaplain in the Navy. A further study of the problem has been taken over by Dr. Josef Solterer, Head of the Graduate Department of Economics and Professor of Statistics. The very interesting results of this study will probably be published soon.

On Feb. 3, Fr. Edward C. Phillips, read, by invitation, before the Columbia Mathematics Club, a paper on "Christopher Clavius and his Manuscripts." The paper was illustrated with lantern slides of the autographs of seven of his works still preserved in the Archives of the Gregorian University, and also of some letters of historical interest in the same Archives. Among these are the two earliest known letters of Galileo, and a contemporary (early 17th century) copy of the letter in which Galileo announced his first observation of the Satellites of Jupiter.

Department of Physics

Dr. Henry M. O'Bryan, Executive Officer of the Physics Department, was granted a leave of absence for the present scholastic year, in order to respond to an urgent request of the Government to conduct some of the physical investigations being carried on as a part of the National Defence Program. It is expected that he will have fulfilled his engagements with the Government before the autumn and will resume his full-time work at the University this coming September.

The following is a synopsis of a report prepared by Dr. Archie Mahan, for the February Meeting of Physical Society, on his investigations in infra-red spectrometry.

Until recently, little work had been done on the dispersion of liquids in the infra-red region because of their relatively high opacity, so some additional investigations were undertaken. Measurements were extended out to the region of 8 microns on CH_2Cl_2, CH_2Br_2, and CH_2I_2. The methods used consisted of the utilization of two spectrometers in series, the first serving as a monochromator for the second. The second spectrometer contained a hollow rock salt prism into which the liquids for study were inserted. Due to the low transmission of liquids in this region, a Moll Relay was used to amplify the galvanometer deflections received. Since measurements on $CHCl_3$ and $CHBr_3$ have already been completed by Pittman, it was possible to compare the dispersion curves as a function of the number of hydrogen atoms present in the molecule. It was found that the magnitude of the anomaly in the refractive index near three microns increased as the number of hydrogen atoms increased thus supporting the classical theory of absorption and dispersion.

Fellowships and Scholarships

The Graduate School carries seven Fellowships (four in Biochemistry and one each in Economics, History and Mathematics) and about the same number of Service Scholarships. The Fellowships carry stipends ranging from \$500 to \$800 a year in addition to freedom from Tuition but not from other Fees. The Scholarships carry freedom from Tuition but not from other Fees, and the Scholars have to give service of a non-teaching character, mainly in supervising the Departmental Libraries. The Fellows have to act as Laboratory Instructors, Quiz Masters and in similar teaching capacities for a limited number of hours a week. There have been a number of applicants from our Jesuit College Students graduating from Colleges in all three of the Eastern Provinces.

GEORGETOWN UNIVERSITY

Progress Report of the Chemo-Medical Research Institute

The Chemo-Medical Research Institute functioning also as the Department of Chemistry of the Graduate School of Georgetown University has continued work on improvement of tests for important biological constituents and on the relation of sulfur compounds to health and disease.

In the field of cystine stones and cystinuria in man new knowledge has been added. Contrary to all previous work in this field as given in the literature, feeding various amino acids to humans was found to increase the cystine output through a stimulation of the general metabolism in a system unable to metabolize cystine properly.

In addition to work with man two cystinuric dogs were obtained. Upon these detailed experiments are underway to find out the exact meaning of cystinuria and to devise if possible means of offsetting this abnormality.

A relatively simple series of sulfur compounds has been synthesized and these compounds have been found to have a marked sterilizing action on streptococcus veridans in vitro. This streptococcus work will be carried further in bacteriological and pharmocological studies.

Considerable progress has been made in tests for differentiating beta and alpha naphthoquinones, the latter of which plays a great role in vitamin K activity regulating blood coagulation. For vitamin K itself, a test has been devised but the degree of specificity of this test is still to be determined.

A paper "The bacteriostatic and the bactericidal action of certain organic sulfur compound by Edward L. Everitt and M. X. Sullivan" was published in the Journal of the Washington Academy of Sciences. Vol. 30, page 457, Nov. 1940.

A paper covering the effect of variation in the diet on the wool and skins of lambs in cooperative work with the Bureau of Animal Industry, U. S. Department of Agriculture, will appear shortly in the Journal of Agricultural Research. The authors are M. X. Sullivan, W. C. Hess and Paul E. Howe.

A short paper by W. C. Hess and M. X. Sullivan covering a new compound, glycylmethionine, a new derivative of an important food constitutent, methionine, will appear shortly in the Journal of the American Chemical Society.

A paper dealing with the cystine and sulfur content of tobacco virus was given by W. C. Hess and M. X. Sullivan before the Washington branch of the Society for Experimental Biology and Medicine, Dec. 5, 1940. Another paper "The estimation of cystine and cysteine in the presence of large amounts of carbohydrates" was given before the same society, Feb. 6, 1941 by Hartley W. Howard and M. X. Sullivan.

A talk on "The Culture and Philosophy of Chemistry" before the Secchi Academy of Georgetown University, Nov. 1940, was given by M. X. Sullivan who also gave a lecture on "The Cultural Value of Chemistry" at Trinity College, Washington, D. C., Jan. 15, 1941.

The two following papers will be given at the annual meeting of the American Society of Biological Chemists in Chicago, Illinois, April 15-19, 1941.

(a) Further studies in cystinuria in man, by W. C. Hess and M. X. Sullivan.

(b) The cystine content of crystalline pepsin, by M. X. Sullivan and Melvin Goldberg.

As regards the candidates for higher degrees, M. S. and Ph. D. progress is being made by a number of advanced students in various types of problems. This progress seems very satisfactory when it is considered that course laboratory work and special problems work have to be done in one laboratory under crowded conditions. Plans and specifications for an extension of the laboratory have been prepared and it is hoped these can be executed during the coming summer vacation period.

WOODSTOCK COLLEGE, MD.

The American Association of Jesuit Scientists was represented at the Conference on Science, Philosophy and Religion in their relation to the Democratic Way of Life by Fr. John S. O'Connor. The meeting was held the second week in September, 1940 at the Jewish Theological Seminary in New York City.

An article entitled "A Scientific Approach to Religion" by Fr. O'Conor, which appeared in the October 1940 issue of the Scientific Monthly has provoked a reply from Chauncey D. Leake, M.D., Professor of Pharmacology at the University of California Medical School and retiring chairman of the section on Historical and Philological Sciences of the A.A.A.S. This reply together with a rejoinder by the author of the original article will appear in one of the forthcoming issues of the same magazine.

LOYOLA COLLEGE, BALTIMORE, MD.

Chemistry Department

On Friday, February 7th, Dr. Alexander O. Gettler, toxicologist of the city of New York, addressed a large audience in Library-Hall on the subject: Chemistry in Crime Detection. This lecture was presented under the auspices of the Loyola Chemists' Club. The audience included among others the Police Commissioner, members of the Supreme Bench, prominent physicians, surgeons and attorneys.

The speaker for the March meeting of the Loyola Chemists' Club will be Dr. G. E. Lundell, Chief Chemist of the National Bureau of Standards. The topic for this lecture will be: Chemical Analysis and Its Problems.

The Physical Chemistry Laboratory has recently acquired the latest models of Kemicott-Campbell-Hurley Colorimeter, and the Leitz G and D Electro Titrator.

ST. JOSEPH'S COLLEGE
Chemistry Department

A few changes in the Chemistry Curriculum have been introduced. A two semester course in Biochemistry is now required of all chemistry majors. Pre-medical students may elect this subject. The chemistry majors now receive a full year Physical Chemistry course and the pre-medical students are offered a special two semester course in the same subject. The first quarter of the Sophomore year is given to semi-micro qualitative analysis. The success we have had this year with the latter subject encourages us to continue the same plan next year.

The following additions have been made to our chemistry faculty. Mr. George Beichl, B.S. '39 has been appointed Professor of Inorganic Chemistry and Mr. Donald Cooke B.S. '40 is now Instructor in Chemistry and is in charge of the stock room and supplies. Mr. Beichl continues his studies at the University of Pennsylvania.

On October 21st, the official inspector of the Committee on the Professional Training of Chemists of the A.C.S. visited our department. No report on the result of his visit has as yet been received.

On November 9th, the Chemist Club held its first public meeting. Our guest speaker was Hugh S. Taylor D. Sc., Chairman of the Department of Chemistry of Princeton University and Professor of Physical Chemistry at that institution. Dr. Taylor lectured on "Catalysis and its Modern Developments." He reviewed summarily, but fully, the history of this interesting topic and outlined its effect upon the economical development of the world. His lecture was replete with illustrations of important chemical reactions, each involving a different type of catalyst. He closed his talk by informing the audience that today, because of successful catalytic studies, the United States government is erecting a plant to manufacture toluene from aliphatic substances. This plant will produce more toluene per day than the entire country formerly manufactured in a year. Modestly, he did not reveal that this tremendous asset to our national defense is the direct result of his own research.

On December 4th, Dr. Arthur L. Fox, associate director at the Jackson Laboratories of the DuPont Co. addressed the club on taste differences in individuals. He titles his talk "Why Johnny Likes Spinach and Mary Doesn't." Among other points of interest he told our members that many eminent students of heredity claim that the taste reactions of individuals to phenylthiocarbamide is a very specific hereditary trait.

Plans are under consideration to conduct the sectional meetings of the club in the form of seminars in the various topics of undergraduate chemistry. It is proposed to limit the attendance at these seminars to students who attain and maintain an average of at least 75 per cent in their science subjects.