A. M. D. G.

BULLETIN

of the

American Association of Jesuit Scientists

(Eastern Section)

For Private Circulation

HOLY CROSS COLLEGE
WORCESTER, MASS.

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BULLETIN OF AMERICAN ASSOCIATION OF JESUIT SCIENTISTS

EASTERN STATES DIVISION

VOLUME VII NO. 4

Maturation and Fertilization in Ascaris Megalocephala

CHARLES A. BERGER, S.J.

The maturation and fertilization of Ascaris megalocephala is a study that has an important place in at least four different biology courses, namely, General Biology, Embryology, Heredity and Cytology. In spite of its importance and widespread use, and in spite of the fact that it has been treated of in many books, papers and pamphlets, I have yet to find after four years' search an account of the process that is not incomplete or inaccurate on some details. The same may be said of many diagrams and charts on the subject. The Laboratory Instructions given below were made from various sources and checked up by examining thousands of eggs.* They can be placed on three mimeographed sheets and provide more than sufficient material for three full laboratory periods.

* Wilson—The Cell in Development and Heredity.
Lillie—Problems of Fertilization.
Sharp—Introduction to Cytology.
Hegner—The Germ-cell Cycle in Animals.
Muckermann—Grundriss der Biologie.
Sigmund—Microcosmos Soc., Stuttgart, Pamphlet on Worms.
Turtox—Handbook on Ascaris.

Laboratory Instructions—

For Maturation and Fertilization in Ascaris

Note: Each slide contains all the stages of maturation and fertilization. There are five levels or lines of sections on each slide and each level contains several closely related stages. Examine the slide under low power and become familiar with the different sections. When looking for any particular stage do not be satisfied with the first example you find; the eggs are so numerous that there are many of all stages present, look for the most perfect specimen you can find.

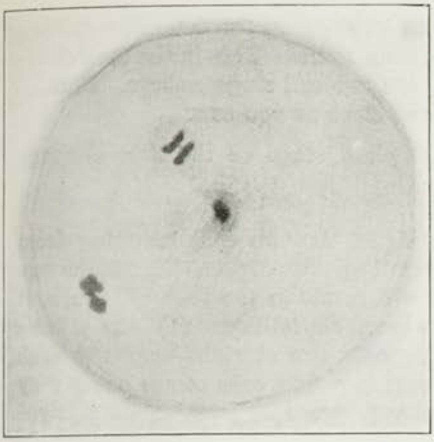
I. *Ripe Egg Unfertilized.* Examine the section containing the earliest stages. Pick out a good specimen of a ripe unfertilized egg. Note the vitelline membrane, plasma membrane, cytoplasm (much vacuolated), and the dense nucleus. In among the eggs will be found large numbers of short, bluntly conical sperm. Find a good specimen and examine it. The head contains a nucleus with a centrosome in close connection with it and a small amount of cytoplasm in which are scattered a number of small granules (mitochondria). The mitochondria and centrosome are extremely difficult to see. The short conical tail contains the crystalline body which is discarded after fertilization and absorbed.

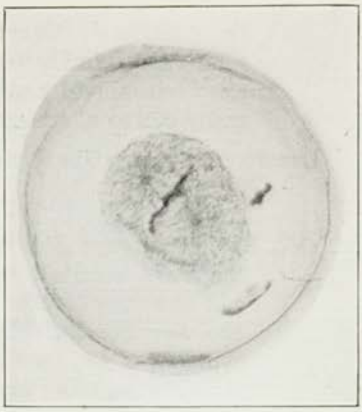
Draw: (½ page) A ripe unfertilized egg and sperm, both in detail, label thoroughly.

II. *First Polar Body.* As soon as the sperm penetrates the egg the vitelline membrane thickens considerably and becomes known as the fertilization membrane, and a space appears between it and the cytoplasm known as the perivitelline space. At the same time the egg nucleus begins its maturation divisions. In Ascaris the first maturation division is the reduction division, the second the equation division. The spindles are formed near the periphery of the egg at first tangential and later radial to it. In giving off the first polar body the nucleus moves towards the periphery of the cell and breaks up into four chromosomes which unite in pairs. Each chromosome also becomes constricted in the center giving it a dumb-bell appearance (the dyad), a pair of chromosomes in this condition looks like four small spherical chromosomes and the group is called a tetrad. There are two tetrads in the spindle that gives off the first polar body. The spindle is of the anastral type, i. e. it has no centrioles or asters at its ends but is barrel shaped. It is at first tangential to the periphery of the cell but before the anaphase is completed is radial to it.

Draw: (½ page) An egg with the first polar spindle in the anaphase condition. Make several smaller drawings showing stages immediately preceding and following the above stage.

III. *Second Polar Body.* After giving off the first polar body the egg immediately enters on its second maturation division, the equation division. The spindle is again of the anastral type but the division is mitotic not a reduction division, i. e. the reduced number of chromosomes left after the reduction division (two in this case) split as in regular cell division and give off the second polar body which will have two half chromosomes. Dyads are formed during this division and the spindle is again at first tangential then radial. Find a specimen in which the second polar spindle is in the anaphase and in which the first polar body can be seen as a short heavy streak disintegrating along the inner surface of the vitelline membrane. The sperm head and nucleus should also be seen undergoing the transformations that will result in the male pronucleus. You may find specimens in which the first polar body has divided into two





small first polar bodies which are disintegrating. Find as many as possible of the stages described above.

Draw: (½ page) A specimen in the anaphase of the second polar division also showing the first polar body and the sperm. Add smaller drawings of other stages as mentioned in II.

IV. *Egg with Male and Female Pronucleus.* After giving off the second polar body the two remaining chromosomes of the egg break up into threads and granules and become enclosed in a nuclear membrane in which condition the egg nucleus is called the female pronucleus. The sperm has also undergone a similar transformation into a small resting nucelus and is now called the male pronucleus. The egg now appears to have two small round nuclei each having a prominent nuclear membrane and chromatin in granules or a coarse network. Sometimes the male pronucleus has a small cloud of bacteria-like bodies near it, these are the mitochondria brought in by the sperm. The remains of the partly absorbed crystalline body of the sperm may be seen near the male pronucleus. The disintegrating polar bodies can still be seen at this stage. Find specimens showing the steps mentioned above.

Draw: (½ page) An egg with male and female pronuclei formed and approaching each other, and with the remains of the polar bodies and any other structures visible.

V. *Fertilization, First Cleavage Spindle.* The male and female pronucleus do not fuse to form a first cleavage nucleus as is common in other animals. An amphiastral spindle is formed, two chromosomes are reformed from each pronucleus while the nuclear membranes are dissolving and the four chromosomes take up their position on the equatorial plate. The amphiastral spindle is formed by the division of the centrosome brought in by the sperm, there were no asters present in the maturation spindles of the egg. The chromosomes now split longitudinally and move towards their respective centrosomes. The fertilized egg now begins to constrict off into two daughter cells. While the division of the cytoplasm is going on the daughter chromosomes are reformed into resting nuclei. This is the first cleavage division and results in the two cell stage embryo. Find as many of the above mentioned steps as you can.

Draw: (½ page) The equatorial plate stage of the first cleavage division.

VI. *Early Cleavage Stages.* Each of the two cells resulting from the first cleavage division now begins to divide. This is the second cleavage division and will result in the four cell stage, it has two peculiarities, however, as follows: (1) the mitotic spindles of the two dividing cells are at right angles to each other, hence when the division into four cells comes about they will roughly assume a T shape, two cells forming the upright and the other two the crossbar. (2) Second peculiarity. In the second cleavage division (2-4 cell stage) one of the cells divides by the ordinary mitotic process, the second cell, however, behaves quite differently. When its chromosomes take their position on the equatorial plate the outer half of each chromosomes remains intact and the inner half of each breaks up into granules. The outer intact portion of the chromosomes is thrown off and absorbed while the granular portion is retained and passes on in all the following divisions of these cells. Such a division is called a diminution division. In the third cleavage division (4-8 cells) one of the two cells that were formed by normal division in the second cleavage again shows the diminution phenomenon, the other divides regularly. This process continues for the first five cleavage divisions i. e. up to the 32 cell stage after which all the divisions are regular, not diminution divisions. Hence there are two kinds of cells in the Ascaris larva, one kind with diminished chromatin content gives rise to the soma, the other kind with the full amount of chromatin gives rise to germ cells. Find as many as possible of the early cleavage stages described above.

Draw: (½ page) The two cell stage with spindles for the second cleavage division forming at right angles to each other.

When and Where Mosses Grow at Woodstock

H. L. Freatman, S.J.

Jan.—Anomodon minor (r)........................base of trees.
Jan.—Hylocomium proliferum...................on boulders, ledges.
March—Fissidens adiantioides (r) (c)........moist earth.
March—Fissidens taxifolius (smaller).........moist earth.
March—Bartramia pomiformis......................hothouse, base of cliffs.
March—Physocomitrium turbinatum............hothouse, base of cliffs.
March—Ceratodon purpureum (c)................everywhere.
May—Mnium affine (largest) (r)............lawns, rocks in deep woods.

May—Mnium cuspidatum (rosettes) (c) .... lawns, rocks in deep woods.
May—Mnium punctatum (larger rosettes) (r) (c) .... lawns, rocks in deep woods.
May—Webera nutans (c) .... in sand, loam.
May—Webera albicans .... in sand, loam.
May—Rhacomitrium asciculare (r) (c) along brooks

June—Funaria hygrometrica .... in burnt-over soil.
June—Funaria clavicans (smaller) .... in burnt-over soil.
June—Bryum argenteum .... everywhere.
June—Bryum caespiticium .... everywhere.
June—Bryum roseum (large rosettes) (r) .... rotten stumps.
June—Ulota americana .... on rocks

July—Encalyptra ciliata .... in hard soil.
July—Polytrichum (c) commune .... open dry woods
July—Polytrichum (c) juniperinum (lighter) .... open dry woods
July—Pogonatum urnigerum .... clay bank or roadside.

August—Ditrichum pallidum .... sandy or clay soil.
August—Ditrichum vaginans (smaller) .... sandy or clay soil.
August—Hypnum imponens (r) .... base of trees.
August—Hypnum reptile (smaller) .... base of trees.
August—Hypnum haldianum .... base of trees.
August—Hypnum curvifolium .... rocky soil.
August—Hypnum schreberi (c) (r) .... on rocks
August—Thuidium delicatulum (like above) .... on rocks
August—Hypnum splendens .... on rocks, fallen trees.
August—Hypnum triquetrum .... on decayed wood.
August—Hypnum boscii .... on sides of great boulders.
August—Hypnum cordifolium .... swampy places.
August—Sphagnum .... wet places.

Sept.—Homalia jamesii .... on rocks
Sept.—Plagiothecium denticulatum .... on decayed wood.
Sept.—Isopterygium micans .... soft loam.
Sept.—Rhynchostegium serrulatum .... on decayed wood.
Sept.—Brachothecium starkei (like above) .... on decayed wood.
Sept.—Leucobryum glaucum (r) .... on rocks.
Sept.—Climacium americanum (r) .... wet soil.
Sept.—Entodon cladorrhizans .... on decayed wood.
Sept.—Entodon seductrix .... on decayed wood.
Sept.—Catharinea undulata (e) .... dry or moist soil.
Sept.—Catharinea angustata .... smaller moist soil.
Sept.—Catharinea crispa (largest but rarer) .... moist soil.
Sept.—Dicranum scoparium (c) .... decayed wood, stones, soil.
Sept.—Dicranum flagellare (c) .... decayed wood, peaty banks.

c=plant common in locality— r=fruit is rare.

Objective Methods of Teaching High School Biology

A. V. DOWD, S.J.

"The Objective Method" of teaching is, I believe, a technical expression in modern educational books. Having but a slight acquaintance with these, it might be well to describe what I mean by the objective methods used in teaching High School Biology at the Ateneo from 1926 to 1929. The reason for this description is twofold; first that they might prove helpful to others; and second to distinguish, if there is a distinction, between the methods that I used and what is usually meant by the term "Objective Methods."

Firstly, regarding examinations, the methods are practically the same as used in the so-called Intelligence Tests. Definitions or other matters of information are to be given in sentences of different types as follows:

Type A; Part of the definition is given, part is omitted, as;

A cell is a minute particle of ———— ———————— in which is found a ————————

The student must fill in the omitted parts.

Type B; Biology is a Science, which treats of Numbers, Politics, Morals, Life.

The student is to underline the correct word.

Type C; The Science which treats of Matter in the Living State, is called ——————

The student fills in the omitted word or words. (This is nearly the same class of sentence as Type A except here the definition is given, the thing defined must be stated. This sentence here is given as an example; it would be too easy to give this one in a test.)

Type D; A Sporophyte is a plant that reproduces by means of sex cells.

Obviously this sentence is wrong. Such a sentence would be read off, or written on the board; the student is requested to place on his paper W, if he thinks this is wrong; or he is told to write a C on his paper, if he thinks it is correct.

Type E: This is a follow-up sentence, and always comes after a Type D sentence.

All that is done in this type, is to request the student to state the reason for his answer in the previous question. (This does away with guess work.)

Other *methods* to be used with the sentence method.

1) Drawings of the specimens, upon which the examination is based, are put on the board; the student is requested to label the different parts.

2) Draw a specimen on the board; label half of the parts, and leave blanks after or alongside of other parts. The student must fill in the blanks.

3) Another follow-up question; state the functions of the parts named in the preceding question:

4) Draw a specimen, and label incorrect; ask the student what he thinks of the picture. (This is being done now by asking "What is wrong with this picture?")

5) This requires a sort of chart as:

	Reproductive	Circulatory	Pulmonary	Sensatory
Heart		X		
Lungs			X	
Ovaries	X			
Arteries		X		

The student is required to place an *X* for each organ named under the column of systems to which he thinks the organ belongs. Cf. above.

It is well in this type of question to use a large number of names of systems. This does away with guess work, as the student who does not know will almost invariably put an *X* where it does not belong, especially since there are now many spaces where no *X* is to be placed.

If your school has a museum, or if you can find enough specimens of plants and animals, place them where all can be seen; each one is numbered, mimeographed papers are given out to the class, and each question is numbered corresponding to the number of the animal or plant. After the number on the paper the following words will appear; Phlyum, and then a blank; Class, and a blank; Order, and then a blank. Then the student fills in the blanks, as he thinks correct, for the Phylum, Class, and Order of the specimens he is being examined on. (This classification is sufficient for high school students.)

In oral quizzes, both in the subject of plants and animals, place a number of specimens on your own table or desk, and while the boys are working in the laboratory, where you can supervise them, call one at a time to the table. Ask each one to identify one of the specimens, tell why he identifies it as he did; ask him to tell the different functions of the animal, how they are performed, and without any dissection, pick out different parts of the plants or animals, and ask him what they are for, how does he know, and so on. E. g. If the specimen is an insect; where are the antennae; what are they for; how can you tell; how would you distinguish this animal from a spider, etc.

The different methods given here for written examinations have the following advantages. To the teacher, this kind of examinations can be corrected quickly. Besides in a short space of time, a fairly large number of questions can be asked and answered, so that you can be sure to cover in a test or examination, all that has been seen during class and laboratory exercises.

Advantages to the pupil. As far as I know, this method of examinations is unusual in our schools, and in this lies the advantage to the pupil. He is forced to handle his knowledge in a new way, is forced to use his reasoning faculties, his powers of observation, as well as his memory. When first tried I found that the results were not so successful; the boys seemed "flabbergasted" for a time, but when this fear wore off, the results were about the same as obtained in an ordinary examination of the question and answer variety. However, this ordinary method must be used alternately with this objective method, as there is one advantage to the teacher; he will, if not careful, follow almost unconsciously a certain scheme in making out his questions, and when the boys become aware of this, as they will, the high marks are alarmingly on the increase. But with sufficient thought in preparing these examinations and with a few questions, of the ordinary method interspersed, the results will be such as to warrant the teacher using the objective methods, viz. that the student is acquiring the knowledge of the subject he is being taught.

I am indebted to Mr. Mariano Dumontay and Mr. Vincente Ruiz, assistant professors at the Ateneo, for their valuable suggestions in these objective methods of examinations. By experimenting I found them useful and helpful, and in my case where I was pressed for time in order to cover all matter required, I found that these methods could be used for short tests, in which very little time was consumed, and a great deal of matter covered.

In the case of the oral tests in the laboratory, most teachers use them, but perhaps less formally. They are a great help. You can be absolutely sure then the student is learning his subject, it is not a question of guess work, or mere memory of names; the animal is there before him, and he must know about it before he can give any kind of an answer. Of course this is the object of laboratory work, but the formal oral quiz allows the teacher to find out if the boy is actually working in the laboratory correctly or only wasting his time.

These methods are not new, but they may give a suggestion to some of our high school biology professors, so that they can make their subject more interesting.

Shall We Have More Chemical Elements?

GEORGE F. STROHAVER, S.J.

At the outset it may be advisable to elucidate the present significance of the term "chemical element." When the chemical atomic theory was first advanced, the indestructibility of the chemical elements was regarded as a proof that they are built up of only one kind of atom. Chemical investigation of the radioactive substances showed for the first time that this conclusion was misleading, since elements with properties so diverse as those of Radium D, on the one hand, which emits B-rays and undergoes transformation into Radium E, and those of ordinary, inactive lead on the other, once mixed with each other, cannot be separated by any known method, nor can their relative concentrations be changed to a perceptible degree. A mixture of these two kinds of atoms, which are certainly different from each other, is therefore just as incapable of decomposition as a chemical element. Such substances, which, from their chemical behavior, belong in the "same place" in the natural system, are called "isotopes." Work upon the isotopes of the radio elements establishes the fact that for the ten elements, 81, 82, 83, 84, 86, 88, 89, 90, 91, 92, there are forty-two types of atoms. In certain cases, such as that of 82 (lead) or 84 (polonium), no fewer than seven kinds of atom, essentially different in their radioactive properties, and in their atomic weights, are practically identical in the chemical sense. The differences in atomic weight amount in some instances to as much as eight units.

Results of similar work upon all the elements studied up to the present time (sixty-six) lead to the conclusion that the old assumption of Dalton, that a chemical element consists of only one entirely definite kind of atom, actually holds true for part of the elements, for example, nitrogen, and oxygen. Others, however, including indeed the greater number studied up to this time, and therefore probably the greater number of all the elements, are composed of more than one kind of atom. The conjecture that chemical inseparability of a form of matter may be no proof of its atomic homogeneity, which was first proposed on the basis of chemical inseparability of different radioactive substances, is therefore entirely confirmed by the investigations of Aston. The theoretical view of the nature of a chemical element has accordingly changed greatly after a century-long acceptance of the fundamental Daltonian atomism, on the basis of which the number of atomic types is considered equal to the number of chemical elements. That the complex character which so many elements possess, could have been overlooked entirely by chemists for so long a time, and was finally discovered by investigations so remote from the field of chemistry as those upon canal rays, is a circumstance that shows at once how very slight is the practical significance of this knowledge for chemistry.

The systematization of chemistry, the natural system, must therefore be built up as before upon the concept of the chemical element. Since chemical elements, according to our present day notions, are no longer absolutely undecomposable, since the separation of them into their kinds of atom is in principle possible, the definition of the concept element may no longer be based upon its complete stability. Today it is suitably defined as follows:

A chemical element is a substance which by no chemical process can be decomposed into simpler substances.

For precisely this chemical stability of the elements valid for all practical purposes, is the reason why these substances may be considered to persist in all chemical changes, and may therefore be correctly called "chemical elements." In investigations of other than a chemical nature, these same substances are not of necessity undecomposable. With the aid of certain physical devices the separation of a mixture of isotopic atoms may be effected, and with others an atomic disintegration may be accomplished. Since neither method, however, plays a part in chemistry, this complexity of the elements in its twofold sense may be neglected in the entire enormous domain of practical chemistry.

The definition of the term "chemical element" that we have given is for practical use probably the most suitable, as it rests on that characteristic which forms the true basis for retaining even at the present time the concept of the chemical element, namely, indivisibility by chemical means. There are, however, two patent objections to this definition. In the first place no very sharp line may be drawn by this definition inasmuch as the distinction between chemical and physical behavior is to a certain degree arbitrary. In the second place there is the disadvantage that it cannot be used as a means of determining the elementary character of that small group of substances, of little practical importance, which show absolutely no chemical properties—the rare gases.

The Rutherford-Bohr atomic theory, of which the basic features are generally known, permits a more concise definition of element, although this definition involves a departure from the strictly chemical meaning of the concept. It may be stated with entire precision that: *A chemical element is a substance all of whose atoms have the same nuclear charge.* This is rather more theoretical than the earlier definition, which is based directly upon experimental observation. Concerning the means by which the uniformity of the nuclear charge is determined in any individual case, nothing is stated. This can be accomplished by a determination of the nuclear charge with the aid of the characteristic Roentgen rays; more indirectly, but usually more simply and in many cases no less certainly, by a demonstration of chemical undecomposability; and with the rare gases, by the constancy of the spectrum in all fractionation experiments.

As we have said before, it makes practically no difference, in the realm of chemistry, whether an element is composed of one or more kinds of atom, and the definition of the concept "chemical element" therefore includes both groups of elements. From a theoretical viewpoint, however, the elements consisting of only one kind of atom and accordingly inseparable by either chemical or physical means, occupy a separate position with reference to those elements that contain more than one kind of atom, and thus belong to the class of elements only because of their chemical indivisibility. This is taken into account by introducing the distinction between simple elements and complex elements, based on these definitions:

A simple element is an element composed of only one kind of atom.

A complex element is an element composed of more than one kind of atom.

A simple element is accordingly, in the thermodynamic sense, a homogeneous substance, while a complex element is not. Since both complex and simple elements are regarded as "chemical elements," it is obvious that the necessity of retaining the chemical system, in spite of the discovery of isotopy, has compelled a separation of the concept of the chemical element from the thermodynamic concept of homogeneous matter. The members of the natural system of the elements are the chemically undecomposable but for the most part thermodynamically inhomogeneous substances. The present question is whether we shall have more of these "chemical elements," thermodynamically homogeneous or inhomogeneous.

The number of spaces established for the elements from hydrogen to uranium, as a result of Moseley's fundamental investigation, has been found to total ninety-two. Of these ninety-two places, ninety are occupied by elements already known, and the two still missing, eka-iodine (85) and eka-caesium (87) would in the event of their discovery offer very little new information of a chemical nature, since their properties can be predicted quite accurately from general relationships within the natural system. Consequently our question, shall we have more chemical elements, does not mean shall these two be discovered. More interesting is our question, namely, is the discovery of elements outside of the series 1 to 92 possible or probable.

On the basis of the Rutherford-Bohr theory of the atom, elements with integral nuclear charge only are conceivable. More precisely stated, the question is whether nuclear charges of elements may correspond with the integers of the numerical series below + 1 and above +92.

An element with a nuclear charge of zero does not seem to be excluded theoretically; it may be conceived as the result of a plunging of the electron of a hydrogen atom from orbit one into orbit zero, the hydrogen nucleus; or as the result of a corresponding neutralization of several nuclear protons, by an equal number of electrons. Such a neutral particle, or "neutron" as it has been occasionally termed, would have the remarkable property of possessing mass like other atoms but only vanishingly small dimensions. It would therefore be able easily to penetrate any kind of matter. Attempts to establish the existence of such neutrons have as yet been fruitless, and it is therefore unnecessary to enter upon further theoretical speculations connected with them.

Further progress in the same direction in the numerical series leads to the negative integers, and hence to nuclear charges characterized by an excess of electrons over protons. It is very doubtful whether such structures would be stable and whether protons could revolve around them by way of electrical compensation. In spite of emphatically expressed opinions to the contrary, such theoretical-physical ideas have been already discussed, but without leading as yet to definite results.

Far more promising, therefore, would appear a pursuit of the numerical series in the other direction, and an inquiry as to whether elements with a positive nuclear charge greater than 92, may not be capable of existence. The Rutherford-Bohr theory of the atom may certainly be developed even beyond the nuclear charge 92; no convincing theoretical explanation for a "boundary to the natural system" just at uranium, is as yet available, and it is therefore possible at this time to discuss the probable existence of elements higher than uranium solely on the experimental basis.

Let us first consider the probability of the discovery of such elements on the earth. It can be said at once that such elements will probably be radioactive, since no stable elements are known above 84, polonium. It can be asserted further, that such a hypothetical radio-element cannot be a mother substance of uranium or thorium, since if short lived, it must already have undergone disintegration, while if long lived, from the laws of radioactive equilibrium it must occur in amounts comparable with those of uranium and thorium, and so considerable an amount of an admixed element would not have escaped the attention of chemists. Nor is there evidence of the formation of a later member of the uranium or thorium series with nuclear charge higher than 92, by branching from the chief series, as a result of β-radiation. It is not impossible, however, that an element of higher nuclear charge than 92 may be the mother substance of a disintegration series as yet unknown; this element might be represented in much smaller amounts than uranium, even if very long lived, since the constancy of the product of the disintegration constant and the mass is valid only within one and the same disintegration series.

The assumption of such an element, present in traces, belonging to no known series, very weakly active (or even inactive), of nuclear charge higher than uranium, is, however, supported as yet by no observations whatsoever.

We may conclude, therefore, that such elements beyond uranium do not seem to exist on the earth, or exist in extremely small traces. One possibility nevertheless remains. Such elements may occur as essential constituents of certain celestial bodies. What can be said concerning the present state of our knowledge of this matter?

Here it is well to recall that messages relating to the chemical composition of celestial bodies are transmitted to the earth in two well known, independent ways; by meteorites, and by the light radiated from the stars.

It is primarily of interest to learn, as a result of the analysis of meteorites, that no element unknown on earth has ever been obtained from this source. Moreover, the elements of the meteorites show a distribution frequency strikingly similar to the frequency of their occurrence on earth (the entire earth, not merely the crust). The same rules hold, that the elements with low atomic numbers greatly predominate in quantity, and that the elements with even atomic numbers are more abundant than those with uneven,—a regularity which was first pointed out by Harkins.

This speaks strongly for the idea that the material of the meteorites, and of the celestial bodies of which they are fragments, has its origin under the same conditions as the material of which the earth is composed. The efficacy of the test is appreciably enhanced, however, by the knowledge, resulting from the discovery of the phenomena of isotopy, that the formation of new elements would manifest itself by an abnormal value of the combining weights. This has been observed to be the case with the radio-elements. Deviations in the case of lead and thorium of radioactive origin, together with those artificially produced in the case of the isotopes of mercury, chlorine, and zinc, constitute the only variations of the combining weights, which have been established up to the present time. In all other cases the combining weights of the elements are natural constants, and not only indeed in samples of material from different geological strata and from different geographical areas but also in samples obtained from meteorites. It must be concluded, therefore, that the different varieties of atom were formed prior to solidification in the solar system and thus had opportunity for thorough mixing; the assumption of a molten igneous, or a gaseous, initial condition of the solar system, plausible on other grounds, is known to afford room for such a possibility.

The first kind of message from other heavenly bodies, the meteorites, therefore strongly contradicts the probability that unknown elements may exist outside of the earth. What is true of the second kind of message, the light emitted?

Not so long ago it was assumed from the extremely varied appearance of the star spectra, that these heavenly bodies are of different composition, and might even contain new types of element. It has been found possible, however, during the last few years, on the basis of the Bohr theory of the atom in conjunction with thermodynamic considerations, to explain the diverse character of the star spectra in an entirely different way. The appearance of unknown lines in the spectrum of a star raises the question, not of new chemical elements, but of known elements under abnormal conditions of excitation. The omission of the lines of a known element does not prove its absence from the star under consideration, but indicates simply that the conditions for the appearance of these lines are unfavorable. The notable discovery of helium, and the discovery of scandium by not unrelated methods were cases of especial good fortune; spectroscopists no longer believe in the existence of a nebulium, coronium, or geo-coronium, and particularly, since among the lighter elements in the natural system there is no longer a single vacant place.

If in spite of this, the assumption of hypothetical, unknown elements has been made within the last few years from the view-point of astro-physics, the basis is to be found not in the experimental results obtained by the analysis of meteorites, or in the observation of the spectra, but in purely theoretical considerations. Such properties are ascribed to the unknown elements as will make possible an explanation of the long life period of the stars.

One of the most difficult problems of astro-physics is the question as to the source of the energy continuously emitted by a star. The greatest minds have devoted themselves to this problem, and it is interesting to observe that each newly discovered, especially powerful source of energy has been promptly utilized in the attempted explanations. Recourse to an unknown element has been sought for this reason. Nernst attributes to hypothetical elements heavier than uranium an abnormally high radioactivity. Jeans goes a step farther still in considering that such elements are not radioactive in the ordinary sense, but have the ability to convert their mass partially into energy. This is not the place to enter into detail as to how such hypotheses may serve also to explain other cosmic problems; we need only mention here that the question of the elements heavier than uranium, in spite of the present lack of experimental evidence, is likely to play an important rôle in the future.

The Rutherford-Bohr theory of the atom seems of little use in connection with questions concerning the stability of the atomic nucleus; and yet its aid in clarifying the problem should not be overlooked. It is known today that everything related to the frequency of occurrence and the stability of the elements is a question of the atomic nucleus, while all that concerns the chemical properties is a question of the outer electrons. In discussions like the present, therefore, it is necessary to consider only the possibility of the existence of the corresponding nuclei; it is not to be doubted that as soon as an atomic nucleus with a positive charge of 93, for example, is formed, 93 outer electrons, according to the quantum laws, will group themselves appropriately around it. The chemical properties which element 93 would show, are therefore rather a secondary matter.

It is not necessary to go back far in the history of chemistry to recognize how complex the question of the genesis of the elements appeared, before this theoretical separation of nuclear and electronic properties was proposed. Contrast the lemniscatic spiral of Crookes with any modern theory of element formation. The latter is obviously concerned solely with the origin of the nuclei of atoms with positive charges from 1 to 92 (and eventually still further). The theory of Bohr can explain fundamentally, and with entire satisfaction, the transition of the linear arrangement of the numbers of nuclear charges, to the periodicity of the properties of the elements.

Today, when it may be demonstrated on principle, that to each of the numbers of nuclear charges, 1 to 92 (and even higher), there correspond perfectly definite chemical and spectroscopic properties, the celebrated old dictum uttered by de Chancourtois in a half scientific and half mystical sense, appears with a new significance: Les propriétés des corps sont les propriétés des nombres. However, there remains this fundamental difficulty which we leave unsettled: the numerical series is infinite (syncatagorically), while the number of "chemical elements" is certainly finite. At what point in the numerical series the correspondence ceases, remains yet to be told.

What Price Pure Ether?

VINCENT A. GOOKIN, S.J.

Some recent chemical investigations of samples of "pure ether" have brought to light the causes underlying one of the dangers following its use as a general anaesthetic. Post-operative irritation and sometimes pneumonia are among the anxieties of the hospital in surgical cases, and despite all care death has often resulted. We now know that ether is not the cause of this condition, and it will be possible to avoid the dangerous agents as a result of accurate chemical knowledge. These investigations have, moreover, caused us to rearrange and rewrite one of our old and standard experiments in physiology on the action of the cilia that line the bronchial tubes.

The experiment had for its purpose the demonstration of the activity of cilia, those minute hair-like processes that line the bronchial tubes and mucous membrane, and also to show the effect of various conditions and agents on this activity. The post-operative danger of ether was among these last.

The frog was used and it was found easier to experiment on the cilia of the mucous membrane of the mouth than on those of the tube. After pithing the brain and cord, cut through the lower jaw with heavy scissors and continue the cut to the stomach. It is better to remove the ventral body wall and all the viscera except the stomach and oesophagus. Draw back the flaps of the lower jaw and pin out the oesophagus to form a flat surface with the level of the roof of the mouth when the whole body is fastened back downward on a board. Place a small piece of cork on the mucous membrane covering the roof of the mouth. It will be moved along towards the stomach and even if the board be tilted the piece is carried up the incline by the beating of the cilia. It is an interesting demonstration of the fact that the cilia are capable of doing a surprising amount of work.

The effect of varying conditions is shown by the application of isotonic salt solutions of different degrees of temperature. The time consumed in moving the cork 2 centimeters was taken. The time was again taken after the application of the normal salt solution warmed to 30. I found the following:

2 centimeters required 2 minutes and forty seconds.
But after applying the 30° solution
2 centimeters required 1 minute and thirty seconds.

The effect of ether was now tried. Hold above the mucous surface a piece of filter paper which has been dipped in ether and blow the vapor down on the surface. After a few seconds it will be found that the ciliary action has ceased. A paralysis has resulted.

The connection with post-operative dangers is apparent. These cilia by their constant vibrations carry foreign bodies up to the throat and prevent the entrance of such bodies into the lungs. If after an operation these millions of guards stand paralyzed the way is free for all particles or bacteria to enter and bronchial irritations and even pneumonia are proximate. The lowered tone of the whole system and the shock to the nervous system make resistance to disease almost nothing.

It has been found, however, that if the pure ether is really so there is no paralyzing effect on the cilia. Acetaldehyde, to the amount of 0.001%, will cause it. This along with peroxide is one of the oxidation products of ether. Richter (Vol. 1, p. 123) gives the following concerning vinyl alcohol, a form of acetaldehyde: "It is produced simultaneously with hydrogen peroxide when ether is oxidized with atmospheric oxygen." According to Reid vinyl alcohol "is believed to exist as the tautomeric enol form of acetaldehyde:

$$H_2C{:}\,CHOH \rightleftharpoons H_3C\cdot CHO."$$

The problem before the manufacturers is the prevention of this oxidation and the duty of the hospital is the accurate analysis of their ether. The prevention of the oxidation has been attempted by using carbon dioxide to expel the air and by using various types of containers that will prevent its entrance and that will not constitute a catalytic system. Amber glass is being tried, and the introduction of antioxidants is another suggestion under experiment.

Whatever the final solution may be the work that has been done presents another striking example of the results of chemical investigation. It will, in this case, mean the saving of sickness and life. 0.001 per cent is a very small amount. But it may mean a life. Says Robert Browning:

> Oh, the little more and how much it is!
> And the little less, and what worlds away!

New Vertical Galitzin at Canisius College

Copy of a letter from Mr. R. L. Faris, Acting Director of U. S. Coast and Geodetic Survey, to Mr. Howard C. Menagh, President of Greater Buffalo Advertising Club.

My Dear Sir:

It has recently been called to my attention by Rev. John P. Delaney, S.J., director of the seismological station at Canisius College, Buffalo, New York, that it is through the good offices of your organization that there has been made possible a vertical component seismograph of the Wilip-Galitzin type at that institution.

My purpose in writing you is to call attention to the important service that you have rendered to seismology in the United States by this gift. During the past five years there has been a wonderful development of interest in seismology. There is every reason to think that future progress will be even more rapid. One of the essential elements is the existence of an adequate number of seismograph stations with installations of modern instruments of such type as to be able to record accurately distant earthquakes, and the station in Buffalo is important, not only from this viewpoint, but in its relation to the earthquakes which have occurred not infrequently in the St. Lawrence Valley, the Great Lakes and the Mississippi Valley regions, and in recording such important occasional earthquakes as that at Attica, New York, last August.

A particular service is rendered in installing a vertical component instrument, since there are comparatively few of these at American stations and this has been a cause of criticism by European seismologists. Since the earthquake wave occurs in a medium of three dimensions, the record is incomplete unless the measurement can be made in three dimensions.

The competent direction of the work at Canisius College and its association with the Jesuit Seismological Association, which as a whole is making a most important contribution to seismology in the United States, make it certain that adequate use will be made of the records obtained.

As you probably know, this bureau has been charged with seismological investigation on behalf of the Government. In this capacity it considers its function not only to do the work which logically belongs to it, but to encourage in every practicable way the activities of existing organizations and the addition of new activities. It is for this reason that your effort adds to the effectiveness of its work.

New Equipment at Weston College

FRANCIS W. POWER, S.J.

Through Father Ahern's generosity the Chemical laboratory at Weston College has recently acquired several new pieces of apparatus.

Most important are: a 50 mm. Klett Calorimeter which we use for ammonia in water; and sugar, uric acid, and creatinine in blood; an L210 Gaertner Spectroscope, with scale illuminator, for qualitative analysis of the last two groups of metals; a Leeds and Northrup Co. hydrogen-ion potentiometer with Eppley standard cell, lamp and scale, galvanometer for work on electrometric titration; a set of tube rheostats to accompany it, and also two small Weston measuring instruments for D. C.—an ammeter and a voltmeter. A barometer and a small low-sensitivity balance have also been purchased, and smallest of all in size although not in value a 25 cc. platinum crucible.

The following "jobs" are in progress at the laboratory: synthesis of the three isomeric fluorhippuric acids (which have never been synthesized) starting with the corresponding aminobenzoic acids; an analysis of the water used throughout the house; an analysis of feldspar for potash and soda, using the J. Lawrence Smith method, determining the K_2O as $KClO_4$.

Alloys as Unknowns in Basic Analysis

F. D. DOINO, S.J.

The following account is not given with a view to completeness or details. It is merely intended to open up certain general lines of thought and suggest a few of the possibilities of giving our students what we call "the training."

The alloys may be easily made up in the laboratory by an intelligent assistant or else purchased from a dealer with the request for a certificate of analysis to accompany each article.

Some of the more common ones that suggest themselves are the following:

1. Ordinary lead and zinc solders.
2. Brazing solder.
3. Wood's metal.
4. Rose metal.
5. Type metal.
6. Yellow and red brass.

7. German silver.
8. Nichrome.

And any number of others which can be found in the ordinary handbooks of physics and chemistry.

Things may be livened up a bit by throwing in also:

1. Some bits of cheap jewelry for curiosity's sake.
2. Counterfeit coins and the like.
3. Worn-out instruments, like cast-off crystal sets (Wood's metal), plates and grids of old radio tubes, and so on.

And many other articles that ingenuity or interest may suggest.

The scheme may be introduced at the end of the Third or Zinc group, since Magnesium is the only other metal of the remaining groups that ever finds its way into the ordinary alloys. The test for magnesium may then be postponed with a mere reference to its place in the scheme.

Coming as it does at such a time in the course, experience shows that you have with these alloys a new stimulus for each of the two kinds of students. To the more brilliant may be given the more difficult products and any number of them if they are of the kind that need slowing down. To the backward the scheme serves as a positive relief after the awful grind of the Copper Group, A, and B, and also the Zinc Group. As they may not yet know what the whole course is about they have a chance to go back over it and "get" it.

The scheme is given below rather sketchily, because it makes use of the already familiar procedure thus far studied. One or two notes here and there will be all that is needed.

The Scheme

The Ions Included Are:

Ag; Pb; Hg; Bi; Cu; Cd; As; Sb; Sn; Co; Ni; Mn; Fe; Cr; Al; Zn; Mg.

(Mercury and other bases which have more than one valence are tested for in the higher state of oxidation.)

1. THE ORIGINAL SUBSTANCE: About 0.5 gm. of the metal or alloy is prepared by shaving off with an old knife, filing it down, or else, beating a small piece to a very thin sheet and then dropping it into a casserole under the hood.

2. PREPARING THE SOLUTION: Enough conc. HNO_3 is added slowly and in small portions at a time to just cover the substance in the casserole. Heat gently and keep stirred. More acid is added until the action subsides or ceases.

3. THE RESIDUE FROM NO. 2: If a residue is given at this point it may be the oxides of Sn and Sb which are white to gray in color and notable in bulk. Small amts and black in color may be particles of C and may be filtered immediately in absence of any other residue.

4. THE SOLUTION FROM NO. 2: May contain the nitrates of all other Bases except of Sn and Sb.

5. REMOVING EXCESS HNO_3: The mixture resulting in No. 2 must be freed from all excess oxidizing acid as this would prevent subsequent pption with H_2S. It is done by adding to this mixture slowly and in small portions conc. HCl until no more Cl is given off and then evaporating to gentle dryness, not igniting.

6. RESIDUE FROM NO. 5: A white curdy or pasty ppt on addition of HCl is due to chlorides of lead and silver. When pption of these is complete the mixture is cooled and filtered. The white residue on the filter is washed as usual and tested for Group I.

7. SOLUTION FROM NO 5: Contains the chlorides of all bases present after Group I. Add to this the first washings from the Group I ppt in No. 6 and prepare the whole solution for the usual separation of Group II by making acid as usual and passing in H_2S.

8. SOLUTION FROM NO. 7: The filtrate separated from the Group II precipitate is tested for Group III as usual by making alkaline with NH_4OH and passing in H_2S.

9. SOLUTION FROM NO. 8: The filtrate separated from the Group III precipitate is tested for Mg as usual.

The scheme is brief as it appears on paper. No doubt it is a simple one and certainly within the grasp of any student who has finished a year of general chemistry. But as in all work of analysis care and patience add to the amount of time required to make the method both practical and successful.

A few of the more obvious advantages of the plan may now be enumerated:

1. A satisfactory means of review, practical as well as theoretical.
2. Of completing within the ordinary time allotted for the analytical laboratory the entire anlysis—and not at the expense of care.
3. Of introducing a student for the first time into the mysteries of a continued analysis.
4. Of educating him to catch on to the hundred and one tricks of omitting unnecessary routine steps when efficiency and intelligence demand it.
5. Of "mixing brains" with chemicals, as a distinguished author advises elsewhere, e. g. testing only small portions of a solution for the presence of any group before proceeding to dump in wholesale quantities of group-reagents when such a group might not happen to be present.
6. Of gaining a useful amount of information and learning a certain amount of technique.

A Recent Publication: Laboratory Construction and Equipment

LAWRENCE C. GORMAN, S.J.

A book which may prove to be of more than passing interest to certain readers of the BULLETIN has just been published at cost by the Chemical Foundation of New York. "Laboratory Construction and Equipment" is its title and it consists of 340 pages of invaluable information for the laboratory planner or builder. The book presents the final report of a committee appointed in 1924 by the National Research Council of Washington, D. C., to investigate the subject of laboratory building. The committee was headed by Fr. George L. Coyle, S.J., of Georgetown University and much of the credit for the success of the work is due to his energetic labors.

The aim of the work, as set forth in the preface, is to aid chemists in preparing preliminary plans for new laboratories or the improvement of those already built, to furnish architects and engineers with dependable information of the special needs of chemical laboratories, and to discuss critically the various materials available for their construction and equipment: in a word, to supply the data of general interest regarding the present status of chemical laboratory construction and equipment.

How thoroughly the book covers its subject may be judged from a glance at some of its chapter headings which include: The Planning of a Chemical Laboratory. Arrangement of Interiors. Types of Buildings. Ventilation. Electrical Equipment. Service and Storage Rooms. Lecture Rooms. Class Rooms. Libraries. Laboratories for General, for Analytical, for Organic, for Physical, for Electro-chemical, for Biochemical, for Microscopical Chemistry, etc.

The book is well illustrated with diagrams, floor plans and halftones of modern Science Buildings,—among the illustrations are exterior and interior views of the Science Building of Boston College, and of the Jenkins Science Building of Loyola College, Baltimore.

A Small Capacity Water Still

A. B. LANGGUTH, S.J.

The still is of the continuous variety and will furnish about one liter of distilled water in an hour. It is easy to construct and not too costly and hence might find use in laboratories where the demand for distilled water is small and intermittent. A still of this type, one consisting of an electric heater coil in a two liter flask, siphon-fed, and a third manually fed, supplied the water for our laboratories over a considerable period, from the breaking of the regular still to the installation of a new one.

In Figure 1. A is a five liter Pyrex Balloon Flask. A spray-trap is shown at B. C is a ¾ in. Pyrex tube, but could equally well be made of copper or brass. Ring supports are shown by letter D. The four inch ring burner E has 25 holes of a diameter of 2 mm. which gives a total area of burner holes equal to the area of a circle of 10 mm. diameter, about equal to the area of the small laboratory burner. The condenser jacket F is made of 1½ in. brass tubing, is twenty inches long and has ¼ in. outlets (gas inlet tubes from old bunsen burners) soldered ¾ in. from the ends. The water from the condenser flows through G to the reservoir I which feeds the still

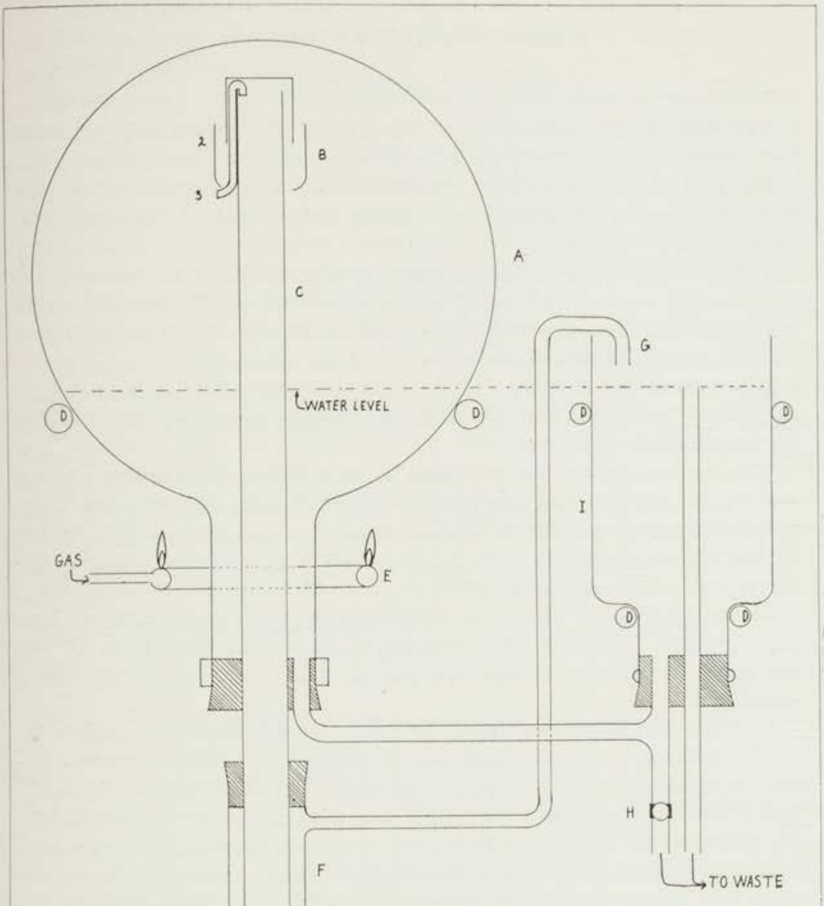


FIG. 1

and spills the excess into the waste pipe. Pinchcock **H** permits of draining the whole apparatus.

The spray-trap **B** is composed of three units—No. 1 is a section cut from the bottom of a homeopathic vial, the diameter of which is 1¼ in. No. 2 is the upper section of a four ounce saltmouth bottle, the neck of which has been removed at the shoulder and has three grooves filed in it so that it can slip over the three (only one shown in the figure) glass hooks No. 3 which are made from glass rod. In constructing the spray-trap this must be borne in mind that the annular space between **B** 1 and **C** minus the area of the three glass hooks equals the area of the cross section of **C** and the area of the annular opening between **B** 2 and **B** 1, otherwise a back-pressure may be produced which would drive the water into the reservoir. Three small pads of folded asbestos cushion flask **A**. No screen or gauze was used between the flame and the flask.

Father Frederic Faura, S.J.

Bernard F. Doucette, S.J.

The following brief sketch of the life of Fr. Faura was compiled for publication in Universal Knowledge and is submitted to the readers of the Bulletin who are interested in the founder of an important scientific institution now under the control of the Maryland-New York Province.

Frederic Faura, S.J., the founder of the Manila Observatory, was born in the town of Artes, province of Barcelona, Spain, on December 30, 1840. He obtained his education at his native town and in the Seminary-College of Vicli.

He entered the Society of Jesus in October, 1859. His course in philosophy was at Tortosa and he studied his theology at Saint Carion, Toulouse, France.

While a scholastic, he was sent to the Philippine Islands (1866), and when teaching there in the Ateneo de Manila he was (1867) appointed director of the new observatory established at the Ateneo by Fr. Vidal, superior of the Mission and Rector of the Ateneo. Observations of atmospheric pressure and of temperature were taken with the purpose of studying typhoons and finding a method of predicting them. His work was recognized in a short time when (1868) the governor of the archipelago placed him in charge of an eclipse expedition.

In 1871 Fr. Faura returned to Spain to complete his studies for the priesthood. He was ordained in 1875 at Toulouse, France. Before returning to the Philippine Islands, he studied under Fr. Secchi and Fr. Perry, besides visiting all the best observatories in Europe. He returned to Manila in 1878 and at once began studying the data collected during his absence in Europe. The results of his labors gave the public a typhoon warning (1879) which was received for the most part in a sceptical manner. The port officials, however, trusted him and kept the shipping in the harbor. The arrival of the typhoon, as Fr. Faura predicted, made the people understand that a new order of things was at hand.

In 1880, Fr. Faura's study of some very severe earthquakes obtained for him even more respect for his ability. The government (1884), also recognized his talents when the observatory was established as an official branch of the government service and had charge of all weather reports. The promptness and accuracy of the warnings given when an approaching typhoon threatened destruction, increased still more the reputation of the observatory. Neighboring countries asked for these weather reports and typhoon warnings when cable service was introduced and the Manila Observatory soon had an enviable reputation and the public's confidence. Fr. Faura did his hardest work, namely the study of the nature of typhoons, when there were no telegraph stations to help him by rapid communication of weather observations, and yet he was able to build up a trustworthy and dependable system for warning the public of the approach of the terrible storms of the region. The result of all his studies was crystallized into the Faura Barometer, a good aneroid barometer with special dial markings to indicate the approach of a typhoon as the barometer went down.

Astronomical and magnetic departments were added to the observatory and many scientific articles were written by Fr. Faura during this time. In connection with the astronomical department, which Fr. Faura wished to be of the highest quality, a new telescope was planned. Seven thousand pesos ($3500.00) were collected and promptly lost by the failure of the bank where the sum was deposited. More money was collected but the erection of the telescope was left to Fr. Algué, the successor of Fr. Faura.

In 1892, Fr. Faura attended the International Meteorological Congress held in Chicago as part of the Columbian Exposition. He returned to Manila with Fr. José Algué, S.J., who had then finished his studies at Georgetown University under Fr. Hagen, S.J. In Manila, Fr. Algué became sub-director and took many burdens from Fr. Faura's shoulders. In 1896 Fr. Faura's health began to fail. Later when the Revolutionists were planning an attack on Manila, he was removed from the observatory, which was in the line of battle, to the Ateneo, within the walls of the city, and there he died on January 23, 1897.

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An All Electric Long Wave Converter

REV. JOHN A. BLATCHFORD, S.J.

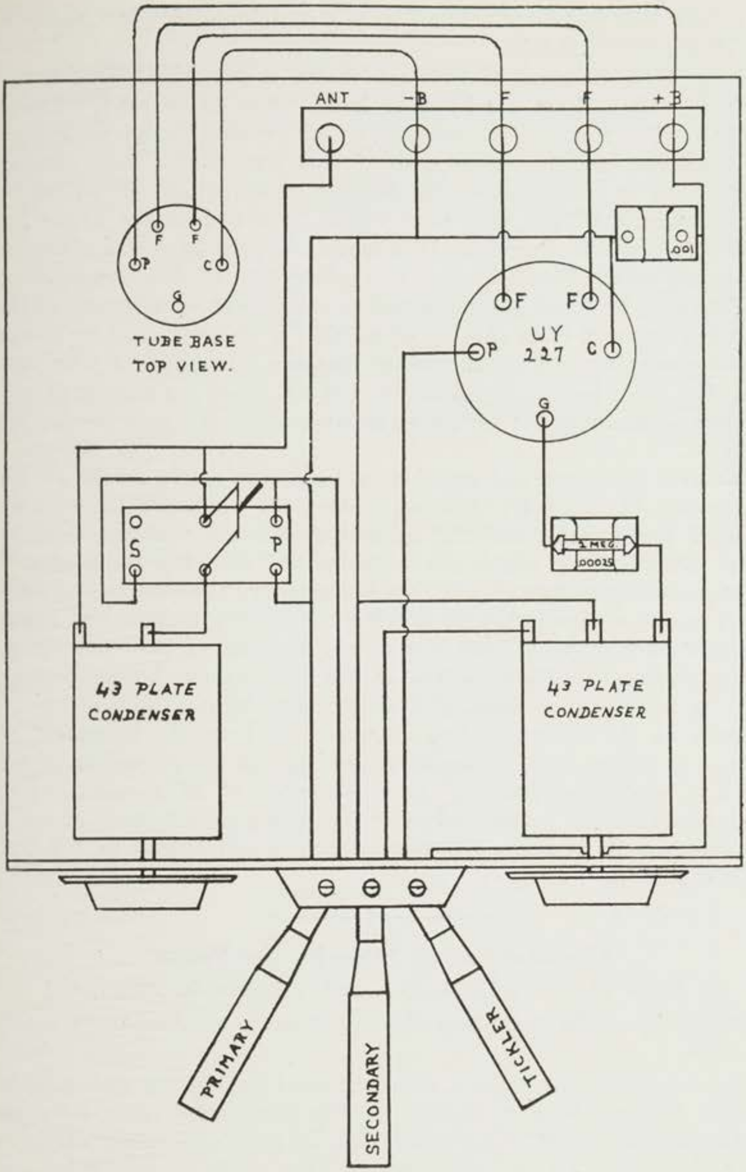
Being desirous of starting the regular observation of occultations of stars by the moon at Weston College, we soon discovered that one very essential requirement was missing—an accurate knowledge of the correct time. The rapidly diminishing rebroadcasting of the U. S. Naval Observatory time signals and the general untrustworthy and contradictory substitutes of gongs and cuckoo-clocks caused us to lose every occultation which the weather permitted us to observe up till about Christmas, 1928. In order to get over this difficulty we decided to build a set which would not be dependent on the broadcast stations but would be capable of receiving the Government time signals on their own wave-lengths.

These signals are sent out on three short waves (a fourth was added March 17, 1930), one broadcast wave of low power and two long waves of high power. In addition there were several very powerful foreign stations which we hoped to be able to pick up. A set was soon knocked together from the remains of various other decrepit receivers and abandoned batteries, and completed on Jan. 3, 1929, in good time for the 10.00 P. M. signals. Our very first effort was a complete success. The time signals came in with good volume from NSS on 16,855 meters. Great credit is due to Rev. Edward S. Hauber, S.J., for picking up, on this precarious contraption, the morning and afternoon time signals of Jan. 10 from Rugby, England. Two days later he also picked up the French ones from Bordeaux and finally on Jan. 16 the time signals from Nauen, Germany. After that we received these signals regularly. The success of our observations of occultations was now assured as we could obtain the clock correction eight times a day if necessary with all the accuracy necessary for our work.

About this time a new child of radio appeared on the scene. It was called the short wave converter and was exhibited at the eighth annual radio show held in Boston. It consisted of a single regenerative detector with an adapter plug which would fit into the socket of the ordinary direct current operated, broadcast receiver and thus procure the reception of waves below the two hundred meter mark. Three or four interchangeable coils were provided for the converter so as to cover the short wave range with as little crowding of stations as feasible. As our "A" batteries were constantly needing charging and the strength of the "B" batteries was rapidly waning, the success of the new converter suggested the possibility of making a long wave converter and if possible an all electric one. The writer's information on AC circuits was admittedly obscure at this period, but by patient and careful study of the little data obtainable the idea finally materialized into the desired receiver and the first time signals received on it were those of Arlington on 2655 meters, June 7, 1929.

The construction of this converter is quite simple as can be seen from the accompanying illustration. Except for the honey-comb coils, their mount and the 43 plate condensers, all the parts can be obtained from chain stores as Grant's or Kresge's or from the regular dealers in radio supplies. Charles Branston, Inc., of Buffalo, manufactures a complete set of honey-comb coils and several varieties of mounts. There are probably no firms that carry 43 plate condensers in stock—at least we were unable to locate any. The condensers used in this set were made by the Charles Freshman Co. of New York and were the opportune gift of Rev. William R. Crawford, S.J. The largest condensers carried in stock today have about 23 plates. Most of the better makes have one-quarter inch removable shafts so it ought to be a simple enough matter to procure a longer shaft and extend it through two of the modern 23 plate condensers thus obtaining the larger capacity of a 43 plate condenser.

The double pole double throw switch, shown just behind the primary condenser, would probably be much more conveniently located on the front panel but is shown on the baseboard in order to bring out the connections more clearly. When the switch is thrown to the left the primary coil and the condenser are in series. This diminishes the volume but makes for better separation of the stations. When the switch is thrown to the right the full volume of the set is obtainable and the tuning is rather broad, making it easy to pick up stations. We find the most useful combination of coils is 1250, 1500 and 750 for the primary, secondary and tickler respectively. This combination allows us to tune in the time-signals from Annapolis, Rugby, Bordeaux and Nauen without changing the coils. To get Arlington use about 300, 200 and 100 turn coils. Separating the primary and secondary coils helps for sharper tuning. Separating the tickler and



LIST OF PARTS

1 set of honey-comb coils from 100 turns up.
1 triple mount for honey-comb coils
2 43 plate condensers
1 grid leak, 2 megohms
1 grid leak condenser with clips, .00025 microfarads
1 fixed condenser, .001
1 five prong tube socket
1 old tube base for adapter
1 series parallel or d.p.d.t. switch
5 binding posts
1 panel, about 7x14 inches.

secondary coils increases volume up to a certain point when regeneration drops off completely.

Notice that the rotor of both condensers is grounded. This avoids detuning effects from the hand or body, when adjusting the tuning controls. The minus B is connected to the ground, though very likely the broadcast receiver takes care of this connection. The adapter, which is to be plugged into the detector socket of any broadcast receiver using a UY227 tube as detector, is shown in the upper left hand corner of the diagram. It is made from the base of a discarded five prong tube. Flexible wires are soldered to all the terminals except the grid terminal and connected to the respective terminals on the set. Sealing wax from the top of an old "B" battery may be poured into the adapter to keep the wires separate. It makes a very neat job. The wires leading to the filament of AC tubes are usually twisted in order to minimize the danger of picking up the AC hum on adjacent wires.

The grid condenser and grid leak are common values .00025 μf. and 2 megohms respectively. Instead of the .001 condenser between the plate of the detector and the ground a variable condenser may be used. Twenty-three plates are sufficient and it makes the control of regeneration very smooth. It is not necessary, however, as the same result can be accomplished by varying the spacing between the tickler and secondary coils. If used it should be mounted on the front panel.

The antenna used with a set of this type should be rather long. Ours is about three hundred feet and five stories high. Put the antenna on the converter and disconnect it from the broadcast receiver. It makes little difference on which unit the ground is placed. Work is now being done on the construction of an all-electric long wave receiver with a stage of radio frequency amplification ahead of the detector. If it turns out successfully, it will appear in due time in this bulletin.

Occultations of Stars by the Moon

By JOHN A. BLATCHFORD, S.J. and THOMAS D. BARRY, S.J.

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The following occultations were observed during the past year at Weston College, Weston, Mass. All the stars were taken from the *American Ephemeris*. Only immersions at the dark limb of the moon were observed. All the observations were made with a three-inch portable telescope.

The formulae used in the reductions were those of INNES, as modified by PROFESSOR ERNEST W. BROWN. The computations were duplicated by THOMAS H. QUIGLEY, S.J.

All the occultations, with the single exception of No. 4, were observed with a stopwatch. The stopwatch was compared as soon as possible, usually within two minutes, with the electric clock system of the college. The clock rate was determined by comparisons with the time signals sent out from station NSS before and after the occultations. On some occasions, in order to eliminate the possibility of mistaking the minute of the occultation in the reading of the stopwatch and also to test another method, the following procedure was also adopted. One observer was stationed at the telescope with the stopwatch, while the other stood nearby with an ordinary watch illuminated by a flashlight. When the observer at the telescope pressed the stopwatch, he also gave a signal to the other man, who read off the time from his watch and then compared it with the electric clock. The times determined by this method are given below in the notes. The reader may judge for himself whether this method is a safe one to adopt in general for the observation of occultations. The times agree within one second, which is the error allowed by PROFESSOR BROWN. This method was used exclusively in the observation of No. 4.

The star positions were taken from the *American Ephemeris.* The longitude and latitude of the place of observation were taken from the Framingham sheet of the topographic maps of the United States Geological Survey. They are:

Longitude = $4^h\ 45^m\ 18^s$, Latitude = $42°\ 22'\ 52''$.

Although the telescope was set up in slightly different places in order to avoid obstructions, the places were not so far apart as to cause any change in the values of $\log \rho \sin \varphi'$ and $\log \rho \cos \varphi'$, which were taken as equal to 9.82644 and 9.86912 respectively.

The correction to the moon's longitude was changed during the year

No.	Star	Mag	α	δ	Obs
			h m s	° ′ ″	
1	τ Tau	4.3	4 37 59.11	+ 22 49 24.3	Bl.
2	145 B Ari	6.5	3 0 42.97	+ 15 34 52.7	Bl.
3	103 Tau	5.5	5 3 47.22	+ 24 10 26.9	Br.
4	σ Ari	5.4	2 47 33.35	+ 14 47 25.1	Br.
5	13 Tau	5.6	3 38 12.52	+ 19 28 29.3	Br.
6	14 Tau	6.2	3 39 40.16	+ 19 26 33.5	Br.
7	300 B Tau	6.2	4 41 24.96	+ 23 30 3.1	Br.
8	η Leo	3.6	10 3 28.24	+ 17 6 37.2	Br.
9	X Sgr	4.4	17 43 6.89	− 27 48 28.4	Br.

No.	Date	Imm. G. C. T.	H. A.	X − ρ	σ′ − σ	△v
	1928	h m s	° ′	° ′	″	″
1	Dec. 25	7 11 43.3	+ 60 40	− 7 50	− 0.7	− 0.7
	1929					
2	Jan. 20	0 3 24.6	+ 3 14	+12 50	− 2.6	− 2.7
3	Jan. 22	1 38 55.7	− 1 37	− 5 5	− 0.8	− 0.8
4	Feb. 16	1 4 6.1	+ 48 21	+ 2 25	− 0.6	− 0.6
5	Feb. 16	22 42 52.3	+ 1 16	+73 1	− 1.3	− 4.4
6	Feb. 16	23 3 56.1	+ 6 11	−21 51	− 0.9	− 1.0
7	Feb. 18	0 28 22.6	+ 12 54	−26 16	− 0.3	− 0.3
8	May 16	0 57 7.1	+ 25 21	−24 40	− 2.6	− 2.9
9	Oct. 9	0 48 48.9	+ 52 15	+ 0 55	− 0.5	− 0.5

from $+ 7''.00$ to $+ 6''.00$, according to a communication by PROFESSOR BROWN to A. J. No. 922. Therefore in the computations for the first seven occultations, the coefficients of $\triangle\alpha$ and $\triangle\delta$ in steps 22 and 23 were $- .212$. For the last two occultations this value was reduced to $-.182$.

ABBREVIATIONS:

Column 6: Bl stands for Blatchford; Br for Barry.

NOTES:

No. 1. Seeing good.

No. 2. Br (on signal from Bl at telescope) $24^s.3$.

No. 3. Bl $56^s.4$.

No. 4. G. C. T. given is that of Bl from signal of observer at telescope.

No. 5. Disappearance sudden. Limb visible. Time probably 1 second late. Bl $53^s.0$.

No. 6. Disappearance sudden. Limb visible.

No. 7. Disappearance sudden. Limb visible. Bl $22^s.6$.

No. 8. Disappearance sudden.

No. 9. Seeing fair, although only half an hour before moonset. Disappearance sudden. Bl $48^s.8$.

Weston, Mass.,
January 20, 1930.

Successive Averaging

CHARLES A. ROTH, S.J.

$$\text{Average of n numbers} = \frac{\text{Sum of the nos.}}{n} \text{ or m} \tag{1}$$

$$\text{Average of n + a (another no.)} = \frac{\Sigma\,(N + a)}{n + 1} \tag{2}$$

$$\text{e.g. Av. } 3, 6, 9 = \frac{3 + 6 + 9}{3} = m$$

$$\text{Av, } 3, 6, 9 \text{ and } 12 = \frac{3 + 6 + 9 + 12}{3 + 1}$$

$$\text{Now } \frac{\Sigma N}{n} \text{ or } m > \frac{\Sigma N}{n+1} \text{ by } \frac{m}{n+1}$$

$$\text{or } \frac{\Sigma N}{n+1} = \frac{\Sigma N}{n} - \frac{m}{n+1} \tag{3}$$

$$\text{e. g } \frac{20}{5} = \frac{20}{4} - \frac{\frac{20}{4}}{5}$$

To put the problem in graph form, how can I get the next successive average of m with a?

Let m be zero (for convenience in graphing).

From (2) and (3) $$\frac{\Sigma\,(N + a)}{n+1} = \frac{a}{n+1} + m - \frac{m}{n+1} \qquad (4)$$

Then by subtracting m from both sides of (4)

$$\frac{\Sigma\,(N + a)}{n+1} - m = \frac{a}{n+1} + m - \frac{m}{n+1} - m$$

$$= \frac{a-m}{n+1} = \frac{1}{n+1} \text{ of } (a - m)$$

Therefore on the graph I simply have to take $\frac{1}{n+1}$ of $a - m$

(e. g. on a ten foot rule if 60 is the 6th av. of 6 nos. then the 7th av. of 60 with 74 will be $\frac{1}{7}$ (74 − 60) or 2 which will be 62 on the rule).

With the foregoing data a chart can be easily constructed for reading six or seven successive averages rather accurately. For convenience we will start with the half average of two numbers.

Construct A C and B D parallel and perpendicular to A B (respectively). The larger A B the more accurate the 4th, 5th, etc. readings.

A C and B D should be spaced to include all possible values in our discussion, e.g. between zero and 100 for the marks in five or six subjects in class.

Call B D the line of reference
and A C the line of first reading
Let the two nos. be m and a

Then on B D, $\frac{m + a}{2}$ will be read at

$$\text{pt. } \frac{a - m}{2} \text{ or } x \qquad (5)$$

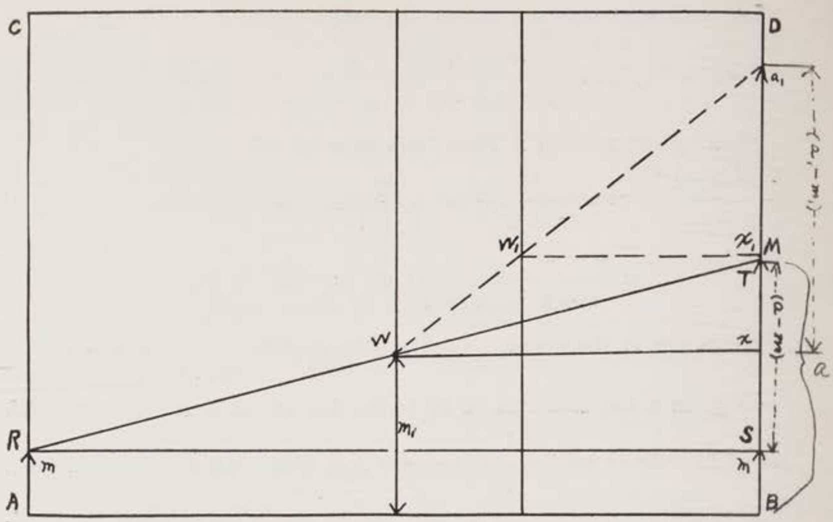
Where does x lie on the st. edge between m on A C and a on B D?
Construct Rt. $\triangle$ R S T and X W T (cf. fig. 1).

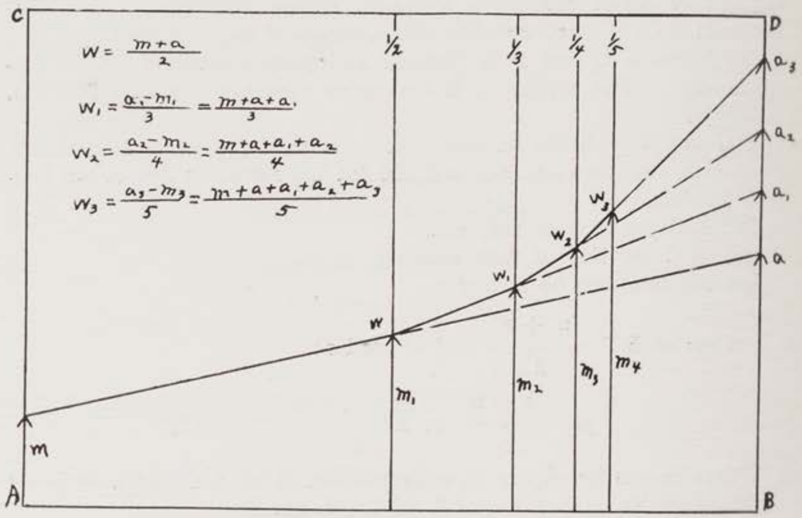
They are similar. (parallel sides.)
Furthermore xT is half of ST. (cf. (5))
∴ W x is half of R S on the base line.
∴ x may be read at W.
But W is the locus of mid-points between A C and B D
∴ W may be read at the intersection of the st. edge, through the two nos. and the parallel to A C and B D at the half way distance between them.

For the third successive average the process is the same, starting of course with W as the new zero, it becoming now m, and the 3rd no. a.

On B D the 3rd av. of m, and a, will be

$$\frac{a_1 - m_1}{3} \text{ or the pt. } x_1$$

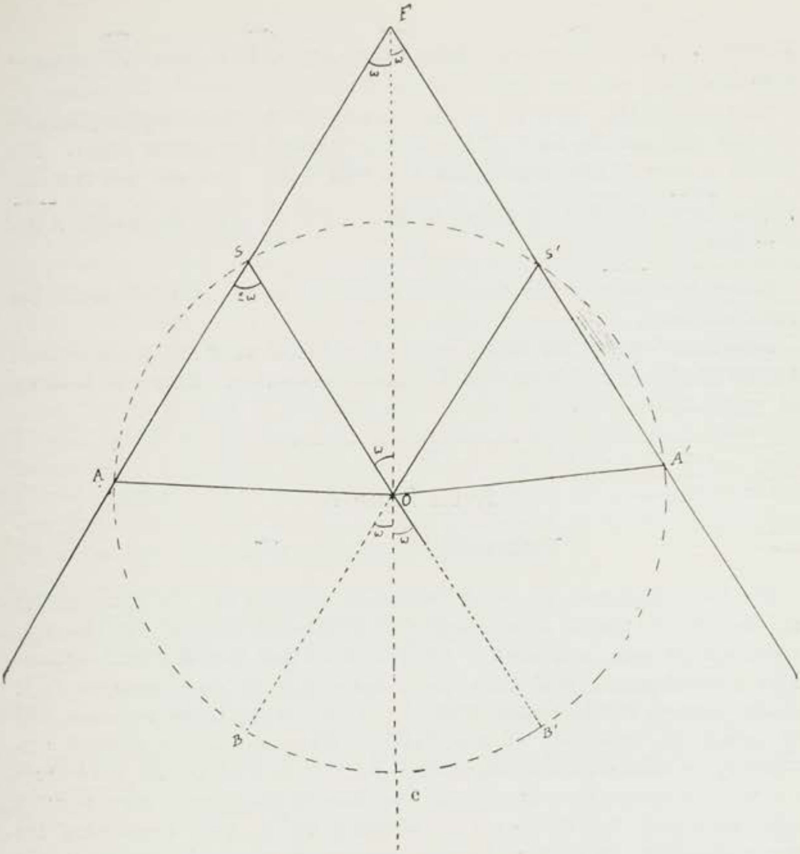




Then by similar st. △ as before the 3rd av. may be read at the intersection of a st. edge from m, or ½ way line to a, on B D and the parallel, between ½ way line and B D at ⅓ the distance from the ½ way line to B D, which we will call the third way line.

In like manner the ¼ av. and ⅕ av. etc. will be read on lines parallel to B D which are ¼ way and ⅕ way from the last reading.

In fig. 2 the successive averaging of five numbers is indicated. The numbers are arranged in ascending order for convenience only. The operation is true regardless of the order of value. Thus with st. edge on m and a put the stylus on its intersection with the half-way line, i. e. w then run st. edge from w to a, and mark the intersection with the third-way line, w_2. Continuing thus, w_3 will be the 5th av. required.



Trisection of an Angle

Henry Pollet, S.J.

I have found in my notes something which might be interesting, if it is not known yet in this country. When I was in China, an Austrian Jesuit showed to me an apparatus which he had invented to divide an angle into three equal parts. As a matter of fact, it is impossible to solve this problem with ruler and compass only.

As the History of Science shows that many inventions have been made several times, it is quite possible that this apparatus is already known. However, I include a figure, a short explanation, and a model of that apparatus.

Division of an angle into three parts by a mechanical process.

In the figure, $FS = FS' = R$ (radius of the circle). Therefore FSOS′ is a diamond.

The angle $\widehat{OFS}$ (which we shall call ω) is equal to the angle $\widehat{COB}$ (parallel sides), and also to the angle $\widehat{SOF}$.

We must prove that $\widehat{COA} = 3\omega$.

The exterior angle $(\widehat{ASO})$ of the triangle $\widehat{SOF}$ is equal to the sum of the two other angles, that is to say, to 2ω.

Therefore Arc $\overset{\frown}{AB'} = 4\omega$. Arc $\overset{\frown}{AC} =$ Arc $\overset{\frown}{AB'} -$ Arc $\overset{\frown}{CB'} = 4\omega - \omega = 3\omega$.

The apparatus, used to divide an angle into three equal parts, is formed with all the lines of the figure (except the dotted ones). The points A and A′ can slide along F A and F A′. One can see that the angle $\widehat{SFS'}$ $(= 2\omega)$ is equal to the third part of the angle $\widehat{AOA'}$ $(= 6\omega)$.

Limit for $\widehat{AOA'} = 270$ degrees (when the points A et A′ reach the points S et S′.

Editor's Note: The model which Mr. Pollet sent me could be best appreciated by exhibiting it at the next convention. This will be done. It works quite well.

Even Squares

Frederick W. Sohon, S.J.

We have just seen that the Tubo-tape pattern for the construction of the 25 cell magic square leads to a general method for the construction of any odd square from a so-called fundamental square. The present paper will show the construction of even squares from easily formed fundamental squares. The method to be followed will be called the principle of association. This method is applied differently in oddly even squares such as 6×6, 10×10, 14×14 from the way in which it is applied in the case of the evenly even squares such as 4×4, 8×8, 12×12. Leading up to, and accounting for this difference, we first prove that there exist no oddly even squares that are perfect magic squares.

Non-Existence of Perfect Oddly Even Squares

Let n be the number of rows, and the case to be considered is where $n/2$ is an odd whole number. Then $2N = 1 + n^2$ is always an odd number. The sum of all the numbers in any row or column will be nN, and if the square is to be perfect, the sum of two numbers diametrically opposite and equally distant from the center will be 2N.

Now, since the square has an even number of columns and rows, let us divide it into four quarters, each containing $n^2/4$ numbers. Let the sum of the numbers in the upper left hand quarter be P, the sum of the numbers in the upper right hand corner be Q, and the sum of those in the lower right hand corner be R. Now P and Q together contain $n/2$ rows, while Q and R contain the same number of columns.

Hence
$$P + Q = Q + R$$
or
$$P = R.$$

If the square is to be perfect, each pair of opposite numbers must sum to 2N. We ought to have then

$$P + R = 2N \times n^2/4$$
and
$$P = R = 2N \times n^2/8 = 2N \times (n/2)^2 \times (1/2).$$

If n is oddly even, $n/2$ is odd, $2N$ is odd, and therefore $P = R$ must be fractional. Hence we have no perfect magic squares with an oddly even number of columns or rows.

Principle of Association

If R is the number in any cell, and N the average of all the numbers, then the complement of R is $2N - R$ and is found diametrically opposite to R and equally removed from the center. If there are n rows and if R is in the p row, its complement is in the row $1 + n - p$. These two rows are said to be complementary as each contains the complements of the numbers in the other. In like manner the columns q and $1 + n - q$ are complementary columns. The number R' in the same row as R but in the column complementary to the column of R, will be called the horizontal alternate of R. In like manner the number $2N - R'$ which is situated in the same column as R but in a complementary row, is called a vertical alternate. Or if we let p be the row and q the column of R, we may designate R by its coordinates (p, q) and the related numbers will then be:

$(p, 1 + n - q)$ = horizontal alternate of (p, q)
$(1 + n - p, q)$ = vertical alternate of (p, q).
$(1 + n - p, 1 + n - q)$ = complement of (p, q).

The horizontal and vertical alternates of the same number are complements of each other.

Consider the combined weight of a number together with its alternate.

Let $$R + R' = m$$

then in the complementary row, if

$$m' = 2N - R' + 2N - R = 4N - m$$
$$m' - 2N = 2N - m$$

In other words the sum of a number and its alternate falls short of the average weight of a pair by the same amount that their complements exceed this value. Now let

$$R + R' = m = S + S'$$
then $$R + 2N - S' + 2N - S + R' = 4N$$

so that if each pair of alternates could be associated with the complements of another pair of the same weight, the inequalities would be removed. This could be accomplished as follows:

A fundamental square is first to be constructed in which complementary numbers will be properly situated with respect to each other, and in which the sum of any pair of alternates will be the same for a given row or column.

Upon this fundamental square is laid a pattern by which each pair of alternates will be associated with another pair in the same column or row. Keep within the upper left hand quarter and start with R. Its associate we shall call S. The associate of S will be considered another R, its associate another S, etc. A closed chain of associates must be formed. If the numbers are not exhausted, another chain must be formed giving a series of T numbers with its associates the U numbers.

When the associates have been all selected, we replace each S number by its complements and similarly for U numbers.

The square is now reassembled, but not necessarily by the same patterns according to which the associates were selected. The result is a perfect magic square.

For example:

take the fundamental square

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Start with 1. Its horizontal associate is 2. The vertical associate of the latter is 6, and the horizontal associate of 6 is 5 whose vertical associate is 1, the number we started with. The R numbers are 1, 6, and of course 4 and 7, 13 and 10, the alternates with 11 and 16 the complements. Their associates are the S numbers 2, 3, 8, 12, 15, 14, 9 and 5. There is only one pattern possible. The S numbers are replaced by their complements and the square is completed.

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	15

The Fundamental Square

Some knowledge of the properties of a fundamental square will assist in the formation of others besides the obvious one used in the preceding example. Let us designate (horizontal) alternates by primes. Then selecting any two rows p, s and any two columns q, r we obtain from the definition of a fundamental square the relations:

$$(p, q) + (p, q)' = (p, r) + (p, r)'$$
$$(s, r) + (s, r)' = (s, q) + (s, q)'$$
$$(p, q) + 2N - (p, q)' = (s, q) + 2N - (s, q)'$$
$$(s, r) + 2N - (s, r)' = (p, r) + 2N - (p, r)'$$

Add and obtain:

$$(p, q) - (p, r) = (s, q) - (s, r)$$

so that the intervals in the rows will all be the same. Moreover lateral symmetry is required

$$(p, q) - (p, r) = (p, r)' - (p, q)'$$

and similar relations hold for the vertical differences. Various fundamental squares will be shown in connection with the 64-cell square.

1	2	3	4	5	*6*
7	*8*	9	10	*11*	12
13	14	*15*	*16*	17	18
19	20	*21*	*22*	23	24
25	*26*	27	28	*29*	30
31	32	33	34	35	*36*

R Numbers

.	*2*	.	.	5	.
.	.	*9*	10	.	.
13	.	.	.	.	18
19	.	.	.	.	24
.	.	*27*	28	.	.
.	*32*	.	.	35	.

Partial Alternation.

.	*32*	.	.	5	.
.	.	*27*	10	.	.
19	.	.	.	.	18
13	.	.	.	.	24
.	.	*9*	28	.	.
.	*2*	.	.	35	.

1	.	.	.	.	6
.	8	.	.	11	.
.	.	15	16	.	.
.	.	21	22	.	.
.	26	.	.	29	.
31	.	.	.	.	36

36	.	.	.	.	31
.	29	.	.	26	.
.	.	22	21	.	.
.	.	16	15	.	.
.	11	.	.	8	.
6	.	.	.	.	1

S Numbers

.	.	3	4	.	.
7	.	.	.	.	12
.	14	.	.	17	.
.	*20*	.	.	*23*	.
25	.	.	.	.	*30*
.	.	*33*	*34*	.	.

Partial Alternation.

.	.	3	4	.	.
7	.	.	.	.	12
.	14	.	.	17	.
.	*23*	.	.	*20*	.
30	.	.	.	.	*25*
.	.	*34*	*33*	.	.

36	32	3	4	5	31
7	29	27	10	26	12
19	14	22	21	17	18
13	23	16	15	20	24
30	11	9	28	8	25
6	2	34	33	35	1

Construction of Oddly Even Squares

We shall show how the method of association can be adapted to the formation of oddly even squares. These squares will not be perfect, for we have seen that there are no perfect oddly even squares. People generally ask for the construction of the 100 cell square, so it will be convenient to have a method at hand and not have to explain the distinction between perfect and imperfect squares.

If we attempt to form association patterns for the oddly even squares, we find that we do not have associates available for all the numbers. If we exclude the diagonals, we can associate all the remaining numbers without difficulty. The question arises how to neutralize the diagonals. Consider the four numbers

$$\begin{array}{ll} S, & S' = m - S \\ 2N - S' = 2N + S - m, & 2N - S \end{array}$$

Now let us perform a partial alternation. To do this horizontally, we allow S and S′ to remain as they are, but interchange the other two alternates and so obtain

$$\begin{array}{ll} S & m - S \\ 2N - S & 2N + S - m. \end{array}$$

After performing horizontal alternation in this partial way, the horizontal pairs are still unbalanced by the amount m, but the vertical pairs now sum up to 2N and thus balance themselves. If we perform partial horizontal alternation on the R numbers, partial vertical alternation on the S numbers, and replace each number of the diagonals with its complement, then the R numbers will neutralize the diagonals horizontally and themselves vertically, while the S numbers will neutralize themselves horizontally and the diagonals vertically. In the case of the 100-cell square there may also be a chain of T and U numbers and these can neutralize each other both horizontally and vertically, or they can be self neutralizing in one direction and mutually neutralizing in the other direction.

The construction of the 36-cell square is shown here as an example.

The Lens Formula

T. H. Quigley, S.J.

Since the formula for lenses $1/p + 1/q = 1/f$ is approximate and holds closely only in the case of a *thin* lens of *small aperture*, it is not surprising to find quite a variety of proofs for this formula. In spite of this variety it is agreed that the lens formula is still a bugbear for the average student of College Physics. The present proof, differing somewhat from the proofs generally used, is offered in the hope that it may, without sacrificing rigor, be a little easier for the average college student to grasp.

The first part of the present proof is rigorous and consists in obtaining from Snell's Law a single equation (eq. 1) showing the relation between the index of refraction of the lens and the angles of incidence and refraction at both the first and second faces of the lens.

The remainder of the proof is simply to express this relation *approximately* in terms of the constants of the lens (n, r', r'', f) and the distances of the object and its image from the lens. In this part of the proof the approximations are, as usual, based upon the assumption that the lens is *thin* and that the faces are of *small aperture.*

Finally, if each of the quantities, p, q, r', r'' and f are considered to be positive when they are measured from the lens in the same direction as in the standard case of a double convex lens forming a real image, and if, on the other hand, each of the above quantities are considered negative when measured from the lens in the opposite direction, then the one formula (derived in the case of the double convex lens) will be applicable to all spherical lenses which are thin and of small aperture.

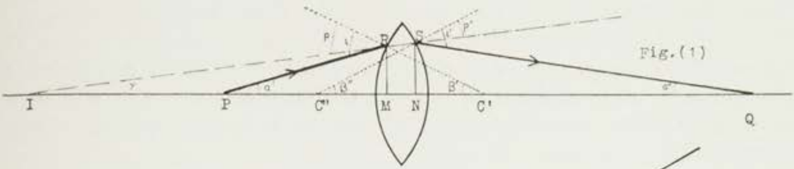


Fig. (1)

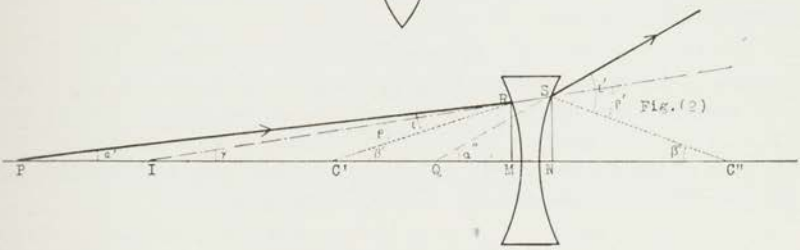


Fig. (2)

Let P be a luminous point, I the virtual image of P formed by the refraction at the first surface of the lens, Q the image formed by the refraction at the second surface, ι and ρ the angles of incidence and refraction respectively at the first surface, ι' and ρ' the corresponding angles at the second surface C' and C'' the centers of curvature of the first and second surfaces respectively, r' and r'' the corresponding radii of curvature, f the focal length of the lens, p and q the distances of the object P and its, image Q respectively from the lens, and n the index of refraction of the lens relative to the medium outside.

From Snell's Law, $\sin\iota = n\sin\rho;\quad \sin\iota' = n\sin\rho'$

Hence $\sin\iota + \sin\iota' = n(\sin\rho + \sin\rho').$ (eq. 1)

From figure (1),
$$\left.\begin{aligned}\iota &= \alpha' + \beta'; & \rho &= \gamma + \beta';\\ \iota' &= \alpha'' + \beta''; & \rho' &= \beta'' - \gamma.\end{aligned}\right\}\qquad \text{(eq. 2)}$$

If the aperture of the lens is small, α', α'', β', β'', and γ will be small angles; and from equations (2) we obtain the following approximate equations:*
$$\left.\begin{aligned}\sin\iota &= \sin\alpha' + \sin\beta'; & \sin\iota' &= \sin\alpha'' + \sin\beta'';\\ \sin\rho &= \sin\gamma + \sin\beta'; & \sin\rho' &= \sin\beta'' - \sin\gamma.\end{aligned}\right\}\qquad \text{(eq. 3)}$$

Substituting these values in equation (1) we have
$$\sin\alpha' + \sin\alpha'' = (n-1)(\sin\beta' + \sin\beta''). \qquad \text{(eq. 4)}$$

Since the lens is thin, RM will be approximately equal to SN. Let d represent this length. Moreover, since the aperture is small, RP and SQ will be approximately equal to p and q respectively.

Then $d/p + d/q = (n-1)(d/r' + d/r'')$

Therefore $1/p + 1/q = (n-1)(1/r' + 1/r'').$ (eq. 5)

* $\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$. If, then, x and y are small angles their cosines are approximately equal to unity and we have the approximate formula: $\sin(x \pm y) = \sin x \pm \sin y$.

To illustrate how this same formula may be derived in the case of any other spherical lens, we may take the double concave lens (figure 2). Here I is the virtual image of P formed by the refraction at the first surface, while Q is the image, (in this case a virtual image), of P formed by the refraction at the surface.

From figure (2),
$$\left.\begin{aligned}\iota &= \beta' - \alpha'; & \rho &= \beta' - \gamma;\\ \iota' &= \alpha'' + \beta''; & \rho' &= \beta'' + \gamma.\end{aligned}\right\}\qquad \text{(eq. 2a)}$$

Reasoning in the same manner as was done in the case of the double convex lens, we obtain $1/q - 1/p = (n-1)\ (1/r' + 1/r'')$.

(eq. 5a)

The symbols p, q, r′ and r″ in equation (5a) represent arithmetical quantities. We note, however, that q, r′ and r″ in figure (2) are measured from the lens in the direction opposite to that in which these same quantities were measured in the (standard) case of figure (1), while p is measured in the same direction in figure (2) as in figure (1′). Hence, if p, q, r′ and r″ are to be considered as algebraic quantities in accordance with the convention noted above, p will be a positive quantity while q, r′ and r″ will have to be negative. Then equation (5a) becomes

$$-1/q-1/p=(n-1)\,(-1/r'-1/r'') \text{ or } 1/p+1/q=(n-1)\,(1/r'+1/r''.)$$

Since $q = f$ when $p = \infty$, then $1/f = (n - 1)\ (1/r' + 1/r'')$. As n is greater than unity, f will be positive or negative according as $(1/r' + 1/r'')$ is positive or negative. Hence f must be positive in the case of the double convex or any converging lens, and negative in the case of the double concave or any diverging lens.

Substituting $1/f$ for $(n - 1)\,(1/r' + 1/r'')$ in equation (5) we have

$$1/p + 1/q = 1/f. \qquad \text{(eq. 6)}$$

The Straight Line

MR. THOMAS QUIGLEY, S.J.

1) If any two real quantities (positive or negative), are each divided by the square root of the sum of their squares, the quotients are equal respectively to the cosine and sine of some angle.

Proof. The two real quantities (positive or negative), can be represented by the abscissa and ordinate respectively of a point in some one of the four quadrants.

$$\sin\alpha = \frac{\text{ordinate of point on terminal side of } \angle}{\text{radius rector}} = \frac{B}{\sqrt{A^2+B^2}}$$

$$\cos\alpha = \frac{\text{abscissa of point on terminal side of } \angle}{\text{radius rector}} = \frac{A}{\sqrt{A^2+B^2}}$$

q. e. d.

2) The locus of the equation $x \cos\alpha + y \sin\alpha = p$ (where p is positive or zero) is a straight line.

Proof. $p = OR + RS = \text{proj. of } x \text{ on } \overline{ON} + \text{proj. of } y \text{ on } \overline{ON}$

$$= x\cos\alpha + y\cos\left(\frac{\pi}{2} - \alpha\right)$$

$$\therefore p = x\cos\alpha + y\sin\alpha$$

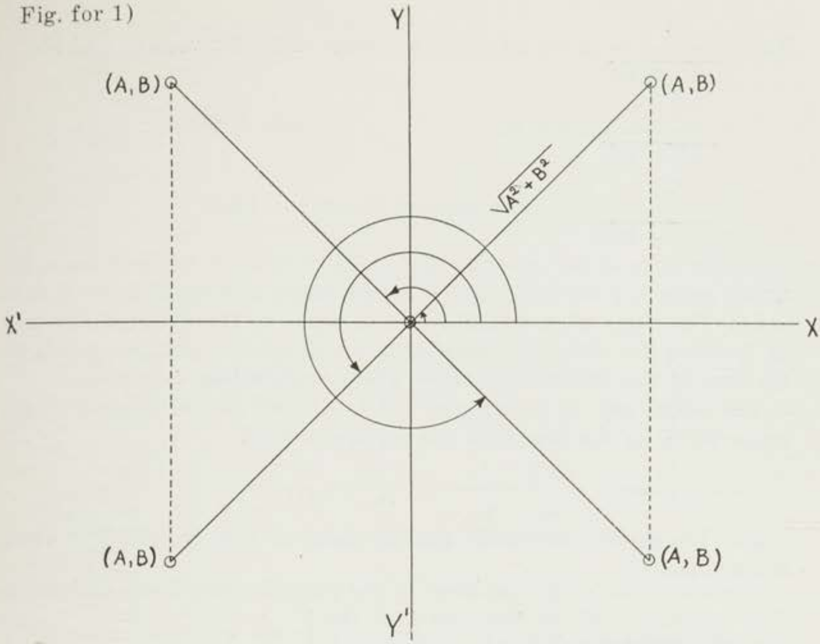
It is clear that, for any positive value of p and for any value of α from 0° to 360°, ($p = x\cos\alpha + y\sin\alpha$) is the locus of a straight line.

The value of p determines the distance of the line from the origin.

The value of α determines the direction of the line (i. e. the angle which the line makes with o x). q. e. d.

3) The locus of the equation $Ax + By = C$ (where C is positive or zero) is a straight line.

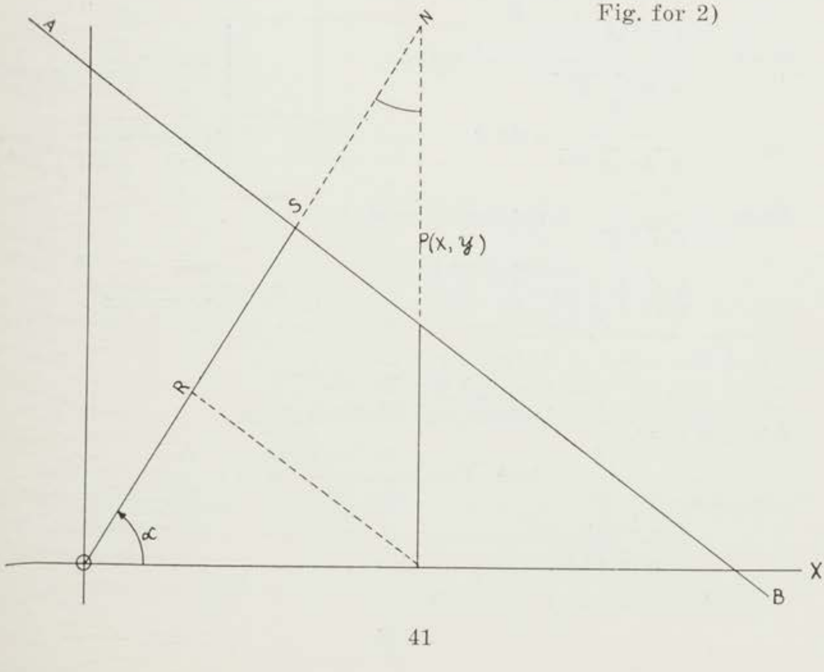
Fig. for 1)



Proof: The locus of the equation $A x + B y = C$ is a straight line if $A x + B y = C$ can be changed to the form $x \cos \alpha + y \sin \alpha = p$
$A x + B y = C$

$$\frac{A}{\sqrt{A^2 + B^2}} x + \frac{B}{\sqrt{A^2 + B^2}} y = \frac{C}{\sqrt{A^2 + B^2}}$$

Fig. for 2)



But $\dfrac{A}{\sqrt{A^2 + B^2}} = \cos \alpha$ (where α is some angle between 0° and 360°)

$\dfrac{B}{\sqrt{A^2 + B^2}} = \sin \alpha$ (cf. 1 above).

$\dfrac{C}{\sqrt{A^2 + B^2}} = p$ (any positive number or zero).

$\therefore$ The locus of the equation $A x + B y = C$ is the locus of the equation $x \cos \alpha + y \sin \alpha = p$ and is consequently a straight line. q. e. d.

4) The slope of a straight line is equal to the quotient obtained by dividing the difference between the ordinates of any two points on the line by the difference between the corresponding abscissae.

The slope (m) of a straight line is defined as the tangent of the angle made by the line with the positive x-axis.

$$\therefore m = \tan \gamma = \frac{y_2 - y_1}{x_2 - x_1}$$ q. e. d.

5) The general constants (parameters) of any straight line. Their determination.

Various forms of the equation of the straight line. Their derivation.

The intercept (a) on the x-axis of the straight line $A x + B y = C$ is value of x when y is 0 $\quad \therefore A x_1 + B(0) = C$

$$\therefore x_1 = \frac{C}{A}$$

$$a = \frac{C}{A}$$

The intercept (b) on the y-axis of the straight line $A x + B y = C$ is the value of y when x is 0

$$\therefore A(0) + B y_1 = C$$

$$\therefore y_1 = \frac{C}{B}$$

$$b = \frac{C}{B}$$

Since $\dfrac{A}{\sqrt{A^2 + B^2}} = \cos \alpha$ and

$\dfrac{B}{\sqrt{A^2 + B^2}} = \sin \alpha$

$$\alpha = \begin{cases} \arccos \dfrac{A}{\sqrt{A^2 + B^2}} \\ \arcsin \dfrac{B}{\sqrt{A^2 + B^2}} \end{cases}$$

Since $\dfrac{C}{\sqrt{A^2 + B^2}}$ is positive or equal to zero then from 2)

$$p = \frac{C}{\sqrt{A^2 + B^2}}$$

$$A x_2 + B y_2 = C$$
$$A x_1 + B y_1 = C$$

$$\therefore A(x_2 - x_1) + B(y_2 - y_1) = 0$$

where (x_1, y_1) and (x_2, y_2) are the coordinates of any two points on the line $A x + B y = C$

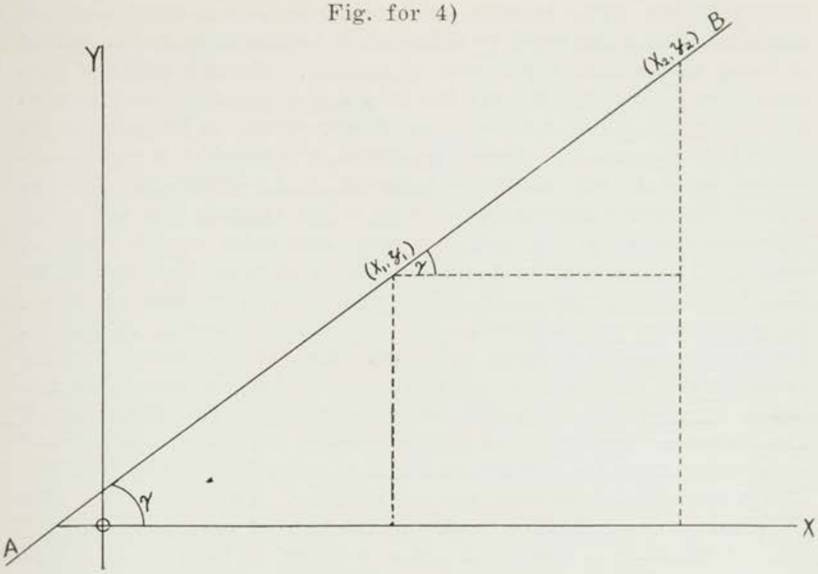
$$\therefore \frac{y_2 - y_1}{x_2 - x_1} = -\frac{A}{B} \therefore \text{from 4)} \; -\frac{A}{B} = m$$

$$m = -\frac{A}{B}$$

$$A x + B y = C$$
$$A x_1 + B y_1 = C$$

$$A(x - x_1) + B(y - y_1) = 0$$

Fig. for 4)



$$\therefore \quad \frac{y - y_1}{x - x_1} = -\frac{A}{B} = m \qquad y - y_1 = m\ (x - x_1) \qquad (1)$$

$$\text{But} \quad m = \frac{y_2 - y_1}{x_2 - x_1} \quad \therefore \quad \frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \qquad \frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \qquad (2)$$

Since $A\,x + B\,y = C$

$$\text{then} \quad y = -\frac{A}{B}x + \frac{C}{B}$$

$$\text{But} \quad -\frac{A}{B} = m \quad \text{and} \quad \frac{C}{B} = b \qquad y = m\,x + b \qquad (3)$$

Since $A\,x + B\,y = C$

$$\text{then} \quad \frac{A}{C}x + \frac{B}{C}y = \frac{C}{C}$$

$$\text{But} \quad \frac{A}{C} = \frac{1}{a} \quad \text{and} \quad \frac{B}{C} = \frac{1}{b} \qquad \frac{x}{a} + \frac{y}{b} = 1 \qquad (4)$$

From 3) $\qquad x \cos \alpha + y \sin \alpha = p \qquad (5)$

"A method of reviewing the discussion of the formulae for the straight line in analytic geometry."

Arithmetical Continuity

FREDERICK W. SOHON, S.J.

In a previous paper I explained the foundations of cardinal number at some length. In the course of the discussion ordinal notions arose, but these were not explained in order that the more interesting parts of the discussion might not be too far separated by so much technical machinery. The question of order is so fundamental that one would be nearer the truth in thinking of mathematics as the science of order rather than the science of number. We talk glibly of constants and variables. We say the height of a parabolic arch is variable. But it really does not change. What changes is the place where I decide to measure it. Hence the change is induced by a purely subjective element, and since mathematics is an eminently objective science, the true mathematical concept of the variable does not really embrace changes at all, but is merely applicable to the study of change. The variable is then something with more than one value. Now we have many examples of concepts with more than one value, human nature, for instance. We call these concepts universals. Hence a variable is a direct universal. But the disjunction between variables and constants is not complete, for the mathematician has had to introduce the term parameter, the so-called variable constant. The essential character of an obscure concept can sometimes be found more easily from the manner in which it is employed than from unhappy attempts at defining the terminology. Parameters are also universals. The difference is that in the case of parameters we are only interested in a particular value, whereas in the case of a true variable we are also thinking of a serial relation between its values. In the universal tendency to generalize, the notion of variable has also been generalized, and so to save the situation let us speak of cardinal and ordinal variables. A variable will then be defined as a direct universal. The field of the variable is the extension of the universal. A cardinal variable will be one in which we are not concerned with any particular order which may or may not exist in the field. An ordinal variable is one whose field is arranged according to some hypothetical order. As the universal exists objectively only in its inferiors there will be no salvation for variables unless their fields are saved.

The scope of this paper must not be exaggerated. It is not possible and would not be wise to develop the field of mathematical analysis in this paper. It is only our purpose to illuminate certain obscure shadows cast by a timid imagination across a laborious path. If then our construction is not detailed enough to be mathematically serviceable, or if for psychological reasons we depart from the beaten path, the reason will be apparent.

The notions of order and relation are correlative. Either may be taken as undefined, and a definition of the other can then be formulated. We might define relation as an ordered couple, *το esse ad*, but as it seems to be the simpler concept of the two, the logical machinery would be less complicated if relations are taken as undefined except in so far as a definition is implied in their postulated properties, and that order should be considered the relation of things according to some relation existing between them. The terms referent, relatum, domain, converse domain and field of a relation have been formerly defined in a previous paper. The relation between Y and X is called the converse of the relation between X and Y. Precedes is the converse of follows. A relation is said to be symmetrical if it implies its converse. A relation is asymmetric if it implies the impossibility of its converse. A relation is said to be transitive if the fact that X has this relation to Y and Y has this relation to Z necessarily implies that X has the same relation to Z. The relations of equality, similarity, etc., are transitive symmetrical relations, while the relations of precedence, inequality and the like are transitive asymmetric relations.

A class is the extension of a concept. Let us call any class that contains at least two members a collection. Our purpose is to shorten diction. To many people the essence of a collection consists in a real physical, actual bond or ligament to give actual physical unity. But here we are satisfied with that conceptual unity that exists between the possible inferiors of a universal concept. A collection is said to be ordered in the technical sense that is here required if a transitive asymmetric relation holds between any couple contained in the collection. In other words an ordered collection is defined to be the field of a transitive asymmetric relation. We might here insert a thesis that ordered collection is not an unsuitable name for the field of a transitive asymmetric relation and show that the vulgar concept of linear order is realized. But this is beside the question, for be the vulgar concept of order what it may, we are interested in the fields of transitive asymmetric relations.

When classes were discussed cardinally nothing was asserted or denied about any order existing among the members. In the discussion of classes from the ordinal standpoint attention is called to one transitive asymmetric relation of hypothesis, but nothing is said about the existence or non-existence of other relations of the same kind having the same field. While it is legitimate to consider now one order and now another, we cannot abstract from the particular kind of order and deduce ordinal properties as such an abstraction was all that was required for cardinal properties. The ordinal properties of a class are the development of the implications of the particular transitive asymmetric relation postulated or discovered in that field.

No claim is advanced for generality in our concept of order. To contradict a universal proposition we only need a particular case. Cyclic order, for example, is excluded, but we are not interested at present in the construction of cyclic fields. We want to show that the assumption that there is no collection longer than an endless collection is unfounded, that the assumption that in an ordered collection every member must have an immediate predecessor or an immediate successor is a bit of imagination, and that the idea that mathematics necessarily obtains its notion of continuity from the observation of extended bodies, is a total misunderstanding of the subject. The tangible proof of our contention is the number system as developed in mathematics. Cardinal numbers are obtained without a discussion of extension. They can be developed from purely spiritual concepts. The finite cardinal numbers form a naturally ordered field, and a generalization of the order found in the scale of finite integers leads to the concept of the arithmetical continuum. Whatever difficulties stand in the way of the mathematician must therefore be purely a question of dialetics, and as there is no doubt about validity of the latter subject, immunity is claimed for mathematics from corrections to meet the results of empirical observation. But to substantiate these claims, it must be explained how this arithmetical continuum is to be constructed logically.

The inadequacy of the scale of finite integers for mathematical purposes is first felt in the fact that subtraction, division, and the extraction of roots is impossible, except in isolated cases. This is remedied by the addition of negative numbers, rational fractions, and finally irrational numbers. Now to augment the extension of a concept means that the concept must be generalized. Some notes must be dropped from the concept of infinite integer. Whatever notes are to be dropped in the course of this generalization, one note must be retained, namely the position or relation of a given finite integer with respect to other finite integers. We have then what is philosophically a new concept which merits a name of its own. Let us therefore define that note or property which an individual has by virtue of which it is said to precede, coincide with or follow some other individual of the same ordered collection to be the scalar value of the individual.

The evolution of the arithmetical continuum proceeds by means of ordered couples. But every relation is objectively an ordered couple, and every ordered couple is implicitly a relation, so that the terms may be interchanged. As relations sound less technical, the definitions are given in terms of relations. The relations between numbers will be formed into an ordered collection by the adoption of a suitable transitive asymmetric relation. The scalar values of these relations will then be our generalized numbers.

Multiplication has always a unique meaning for the finite integers, and the relations of less and greater have also determined meanings. Let us then define a ratio to be a relation between two members of the scale of finite integers a, b which has the property that it is said to precede, coincide with or follow another similar relation between the finite integers a′, b′ accordingly as $a \times b'$ is less than, equal to, or greater than $b \times a'$. A rational number can now be defined as the scalar value of a ratio. With the system of rational numbers division can always be performed, except of course division by zero, but subtraction cannot. Another generalization is therefore in order. Define a rational interval to be a relation between any pair of rational numbers a, b, which has the property that it is said to precede, coincide with, or follow another similar relation between the rational numbers a′, b′ accordingly as $a + b'$ is less than, equal to or greater than $b + a'$. Then define a relative number to be the scalar value of a rational interval. We might have defined first integral intervals, and then interval ratios and obtained the same scalar values as a result. Subtraction is now always possible, but the extraction of roots is not. But we may conveniently stop here to inquire if every member of an ordered collection must have an immediate predecessor or an immediate successor.

If X and Y are individuals of a collection that is ordered so that X precedes Y, and if there is no member of the collection that precedes Y and is preceded by X, then X is said to be the immediate predecessor of Y and Y is the immediate successor of X. The propostion to be refuted asserts that without immediate successors there will be no successors. But the answer is that we have proved that there is no greatest finite integer. From this there follows considering reciprocals that there is no least positive fraction. Now if X and Y are thought to be consecutive, form the difference $X - Y$. Since there is no least fraction a rational number U can always be found such that $0 < U < Y - X$. If we then add X to each member of the inequality we obtain $X < X + U < Y$. In the scale of rational numbers a rational can always be found intermediate between any arbitrarily chosen pair. The proposition stands refuted. There can be successors without immediate successors, but we cannot imagine this because to be imagined a thing must have a certain size and a certain duration.

Since there is no least fraction, there is no greatest rational number less than two. If therefore we take the class consisting of the rationals less than two, we have an endless collection of rationals. If we consider the class of rationals not greater than three, we have a collection with an end, namely three. But the series with a last member is by construction longer than the endless series. We see therefore that the proposition that no collection can be longer than an endless collection really has no foundation, and what was apparently an obstacle to the notion of continuity has proved in fact to be merely another shadow.

An ordered collection in which there is no immedate succession, that is to say, which has a member between any two arbitrarily chosen members, is said to be compact or dense. The scale of rational numbers is a dense set. Now it is quite clear that since there is no least fraction, there can be no extended gaps in the scale of rationals or of relative numbers. There are, however, inextended gaps, corresponding to the scalar values of incommensurable numbers. For instance there is no rational whose square is equal to 2. For if there were we might put $a/b = \sqrt{2}$, where a and b are both integers and prime to each other as it is a simple matter to reduce any fraction to its lowest terms. The supposition that $\sqrt{2}$ is so expressible leads to a dilemma. If a is even let $a = 2c$.

$$\frac{2c}{b} = \sqrt{2}$$

$$c^2 = \frac{1}{2} b^2$$

and therefore b must be even if c is an integer. But for both a and b to be even contradicts the hypothesis that the fraction is reduced to its lowest terms. Therefore a cannot be even. But it cannot be odd because

$$a^2 = 2\,b^2$$

and the square of an odd number is always odd. Since $\sqrt{2}$ cannot be expressed as a fraction, there is no rational whose square is 2. We may now divide the rationals adequately into two mutually exclusive classes, one consisting of the rationals whose squares are less than 2, the other consisting of the rationals whose squares are greater than 2. Now since there is no least fraction there is no greatest rational in the first class, and no least rational in the second class as is easily proved. There is then no rational whose predecessors form one class and whose successors form the other. We have found a division in the scale not marked by a rational number, we have found a scalar value that is not the scalar value of any member of the scale, we have found gaps in the ordering of our collection. It is true the gaps are inextended in themselves, but their importance will be grasped when we say that the cardinal power of the scale of rational numbers is still only $\aleph_0$ (aleph sub-zero), whereas the cardinal power of the class of gaps is the greater transfinite cardinal $2^{\aleph_0}$. (2 to the power aleph sub-zero.) No gap can be discovered that immediately precedes or immediately follows any rational, and while there is a rational between any two gaps and a gap between any two rationals, there is also another gap between any discovered gap and any rational as well as a gap between any two discovered gaps. Here is a situation that the imagination with its experience in handling extended concepts finds itself unable to picture. A higher faculty must be brought into play, but that is another story.

To make a continuous field for our variables, the gaps in the rational scale must be filled. All we require is a universal concept whose inferiors have the scalar values of these gaps. In order to see clearly just how this is done, the necessary logical machinery must first be elaborated. A dichotomy or cut (Schnitt) is a process by virtue of which an ordered collection is divided adequately at least mentally into two mutually exclusive parts one of which wholly precedes the other. The two classes into which the ordered collection is thus divided are called the lower and upper segments of the dichotomy respectively. It is easier to give a technical definition of a lower segment than of a dichotomy. A lower segment may be defined as a class each member of which belongs to a given ordered collection and precedes every member of the ordered collection not also a member of the class itself. This definition makes the lower segment something purely objective. Now the dichotomies will have the same order as their lower segments. There are two accounts of this matter. Dedekind's account uses dichotomies. Russell's account uses lower segments. Let us therefore define a section to be either a lower segment or the dichotomy corresponding to a lower segment. Then we can define a real number as the scalar value of a section of the class of relative numbers. Some of the lower segments will have an upper extremity, that is a member without a successor, while other lower segments will not have an upper extremity. Similar considerations apply to the upper segment. If neither the lower segment has an upper extremity nor the upper segment a lower extremity, then the section will be called irrational, and its scalar value an irrational real number. If either the upper segment has a lower extremity, or the lower segment an upper extremity, the extremity in question will be called a frontier, the section will be called rational, and its scalar value will be called a rational real number.

When we assert that the real numbers form an arithmetical continuum, we assert that the scale of real numbers is compact or dense ordered, and that for every possible dichotomy there will be one and only one real number whose predecessors fall in the lower segment and whose successors fall in the other segment. This means that every dichotomy can be defined by saying that it falls on a real number, or to put it the other way it is impossible to divide the scale except at a real number. It may be objected that the frontier may be assigned to either segment, and that there are two divisions for every number. But this is of no consequence. The numbers are inextended. The interval between 0 and 1/2 if added to the interval between 1/2 and 1 is exactly equal to the interval between 0 and 1. Since the numbers are inextended the transference of the dichotomy from one side to the other of a real number will change the scalar value by the exact scalar extent of a real number, which is exactly zero. As we are interested in the scalar values rather than the logical content, we consider these two sections to be numerically identical. Dense order and a frontier for every section proves to be all but that is required for mathematical purposes, and if one does not like to call that continuity, one can think up another name for it.

It is obvious that the scale of real number is compact, but it is perhaps not so obvious that there will be a frontier for every section. In order to work out a proof of the latter we need the machinery of ordinal similarity, but this is replaced with an imaginary superposition. If such a scheme is objected to, it should not be hard to see how it can be replaced by a net work of relations.

Superpose the scale of real numbers on the scale of rational intervals, keeping each system in its order of magnitude, and do this in such a way that each rational real number will fall on the frontier of that section of the scale of rational intervals to which it corresponds. The irrational real numbers will fall between the rational real numbers and consequently between the members of the rational scale. The first thing to be proved is that each irrational real number falls between the two segments of the rational scale to which it corresponds.

One section of the rational scale precedes another if its upper segment includes the upper segment of the latter. The order of the real numbers is the order of precedence of their corresponding sections of the rational scale. The upper segment of an irrational section includes the upper segments of all those sections of the rational scale that have for frontiers elements that are to be found in the upper segment of the irrational section itself in question Hence the irrational real number must necessarily precede all the numbers of the rational scale found in the upper segment of its corresponding section, for it precedes all the rational real numbers that are superposed on them. In like manner it can be shown that it must necessarily follow all the members of the rational scale found in the lower segment, and from this it follows that the irrational real number must actually fall between the segments of the section of the rational scale of which it is the scalar value.

Between any two real numbers we can find any desired number of members of the rational scale, the proof falling back on the proposition that there is no least fraction.

Now let us divide the scale of real numbers. The same division will automatically divide the scale of rational intervals upon which the real numbers have been superposed. This concomitant section of the scale of rational intervals will either be rational and have a frontier upon which a real number falls, or else it is irrational and its scalar value falls between the segments. In either case there will be no members of the rational scale between the real number and either segment of the scale of rational intervals. But since there is an abundance of rationals between any two real numbers, there will in the latter case be only one real number between the segments of the scale of rational intervals. As the real numbers are all superposed on the scale of rational intervals or fall between them, it follows that the successors of the scalar value of the rational section of the scale of rational intervals all fall in the upper segment and its predecessors in the lower segment of the dichotomy of the scale of real numbers. The existence of a frontier for every section is thereby proved. The gaps in the number system are closed.

The scalar value, it must be confessed, is an invention of my own. There is a system of philosophy in which a direct universal is a disguise for a mere disjunctive predicate consisting of the inferiors of the concept linked together by a chain of *ors*. In this system one should be satisfied in having found the extension of the number concept and it is a rather useless refinement to dig up a comprehension. Community of properties, that is to say obedience to common laws, justifies a common name. Thus finite integers, ratios, rational intervals, sections of the scale of rational intervals are all numbers. But all this is most conveniently summed up in a common attribute which becomes the comprehension of the generalized concept. What is it that all these things have in common? So far as I can see it is only their ordinal disposition, namely that note or property by virtue of which a member precedes, coincides with or follows another member of the same scale. In this the integer 2, the ratio 2, the interval 2 and the section whose frontier is 2 all agree. By defining the number 2 to be the scalar value of any one of these a univocal definition is obtained, otherwise our concept of the number 2 must be equivocal as is indeed claimed.

The number concept still remains in general equivocal. We have cardinal numbers, two kinds of ordinal numbers, and scalar values. Two classes that are cardinally similar have the same cardinal number. To have the same ordinal number in the modern sense, the classes must be cardinally similar and have the same order. The new ordinal number is the predicate of a class. The old ordinal number was the predicate of an individual, namely its relation to an ordered collection, and the fundament of this relation is the scalar value. Transfinite cardinals have been somewhat explained, but the transfinite ordinals we have not touched. Infinity is the attempt to construct a transfinite scalar value, but the attempt is a failure, or at best degenerates into an incomplete symbol like the dot over the letter i, and its meaning must in individual cases be defined from the context. The term infinity is also used in another case as a predicate of a class when we hear a single infinity of points in a line spoken of, and a double infinity of points in a plane. We are not here dealing with cardinal powers, because the points in a finite line segment can be brought into one to one correspondence with all the points in space. As some useful predicate of an ordered class is being signified, I assume that in this sense infinity is used for the transfinite ordinal power.

The field of the variable having been cleared of its shadows, it might be thought logical to continue the process and show that the notion of limit has nothing to do with any methods of approximation, so that if one has such an idea one has missed the whole point of the method. In such a connection we should then show that infinitesimals cannot be small quantities for smallness is relative, and therefore absolute or essential smallness is a contradiction in terms. But these concepts are all explained in the modern standard text-books. It is true that the older books are at fault, but these have been repudiated, and non-contradictory concepts have been in use for a long time. The matter is technical, not philosophical, so may be left to the text-books.

The generalization that we have been describing does not cease at this point. Just as we formed ratios and intervals by means of relations or ordered couples, so ordered couples of real numbers can be formed. Linear order is not then possible so a more general concept of order must be used. In this way the scale spreads out into a field, and a complex number system is produced. Instead of mere couples, triples, quadruples, $n$tuples are used giving vector fields of 3, 4, n dimensions. Choosing ordered couples, triples, etc., in a vector field we obtain dyadics, triadics, polyadics. But these developments have no philosophical significance.

In treating universals, one must necessarily emphasize either the extension or the comprehension of the concept. The emphasis on the comprehension emphasizes the variable, and is a centripetal tendency that finds expression in symbolism, symbolic operations and the like. The emphasis placed on the extension is a centrifugal tendency and finds expression in groups and subgroups and their interrelations. The expression of the tendency to stand things apart that belong together, of mathematical apostasy, if we chose to call it that, is geometry. Thus an $n$tuple may be called a point and the vector co-ordinate of a point. If two $n$tuples have their numbers respectively proportional, they are defined as collinear. Vectors whose differences are collinear are called termino-collinear, and the class termino-collinear vectors is defined to be a line. So the construction proceeds. The expression of mathematical symbolism is of course algebra.

Notes and Corrections

CORRECTION to Fr. Power's Article on Solutions in last issue:

Top of p. 11, 1st equation; $r = \dfrac{amp + bnq}{xr}$

Should be $k = \dfrac{amp + bnq}{xr}$.

Next p. 11 . . . "In this case we have

$a = \dfrac{bn - (q - r)}{r}$ Should be $a = \dfrac{bn\ (q - r)}{r}$.

Just above middle of p. 11, $x = \dfrac{am + b}{a + b}$ Should be $x = \dfrac{am + bn}{a + b}$.

On p. 12 under example 3, $a = \dfrac{bn\ (- r)}{r}$ Should be $a = \dfrac{bn\ (q - r)}{r}$

NOTE to Fr. Roth's article. There is a number of Woodstock Computers and extra celluloid disks of the type invented by the Very Rev. Fr. Provincial of the Md.-N. Y. province, Fr. Edward C. Phillips, in stock, at Woodstock College. Mr. E. J. Nuttall, S.J., will send out any information requested concerning the Computers and take care of any orders for the same. The Computer sells at \$4.50 and the disk at 50¢.

CORRECTIONS to the articles of Fr. Sohon:

No. 2. Page 35. Lines 18, 19 and 20. Z should be z. It is a scalar, not a dyadic.

36. Line 21, Ψ should be lower case ψ.

Line 22, x should be χ.

39. Line 18 from bottom, L should be D.

No. 3. Page 20. Line 6, from bottom. 25-cent should be 25-cell.

24. Last two lines are interchanged.

34. Line 6 2n should be 2^n.

35. Lines 11 and 20, 2n should be 2^n.