A. M. D. G.

BULLETIN

of the

American Association of Jesuit Scientists

(Eastern Section)

For Private Circulation

HOLY CROSS COLLEGE
WORCESTER, MASS.

Vol. VII

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VOLUME VII NO. 3

Father Francis A. Tondorf, S.J., Seismologist

REV. JOSEPH P. MERRICK, S.J.

Friday, November 29, 1929, Fr. Francis A. Tondorf, S.J., of Georgetown University, died of a heart attack. It was an appropriate death for this grand old man of American seismology. During a laborious life he had sketched with cunning and accuracy the heart attacks, the splittings and the rendings of mother earth; in death there was a tearing of the cables and the clay, a rending of the heart and then a silence.

Born in Boston, July 17, 1870, he entered the Society of Jesus from that city. Besides the usual course in the Society he made special studies in physiology at Johns Hopkins University, Baltimore, and in astronomy and physics at Georgetown under the eminent director, Fr. John G. Hagen, S.J., now in charge of the Vatican observatory. His early teaching was chiefly at Loyola College, Baltimore, but in 1903 he went to Georgetown. At his death he might have claimed that he antedated any other member of the faculty in years of continuous service at the University. He might also have boasted, were he given to boasting, that he antedated others in years of toil. For he was accustomed to rise about two or three in the morning, finish his office, say his Mass, then investigate his instruments and get in plenty of study before breakfast. As professor of geology and physiology, he had quite enough to keep him occupied.

Yet it was in the highly specialized field of seismology that he acquired international repute. From 1911, when he first set up his instruments, until his announcement of the Tokyo earthquake in 1923, his accuracy and skill in interpreting records had gradually won recognition of his work among the specialists. But when the telegraphic report confirmed his prediction that the Japan earthquake "was a whopper," he achieved popular and instant applause. What had been the secret of a few became a commonplace among the many, and with his constant reports of tremors to the press, he became the greatest publicity agent seismology has ever had.

He checked up on his instruments many times daily and his monthly reports on earthquakes which he sent to observatories all over the world were held in very high regard. He made science live both to the theorist in earthquakes and to those who are merely potential victims.

It was his lifelong desire to discover a method by which earthquakes could be foretold in time to give warning to the inhabitants of the threatened area. He confessed that such dependable forecasting was many years away, but he held tenaciously to the belief that scientific research would eventually reach that goal.

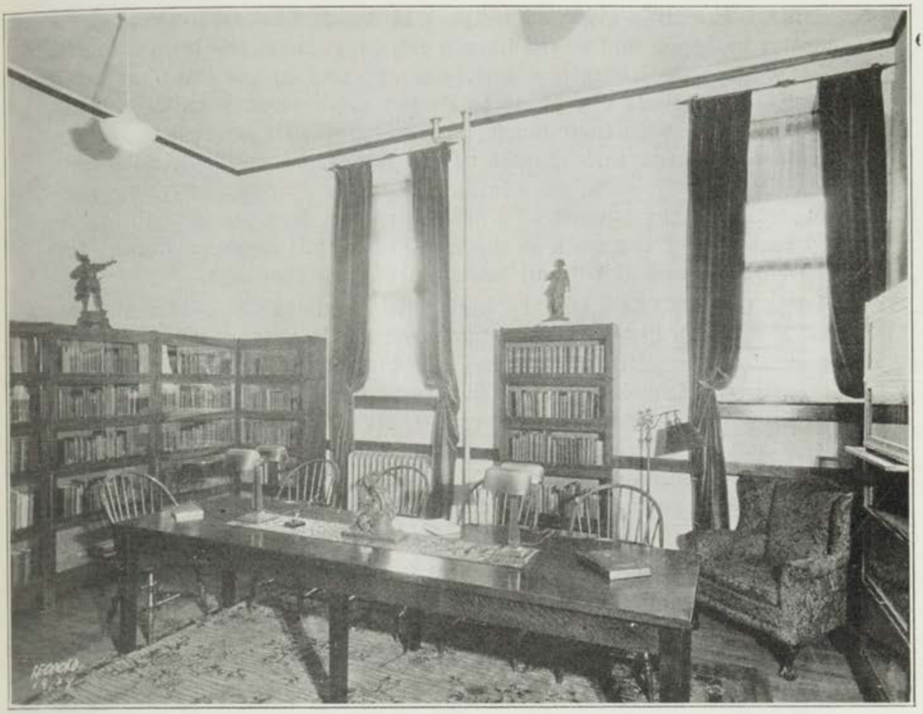
One of the very last scientific accomplishments of Fr. Tondorf, and one for which he received governmental commendation, was the very accurate report he submitted to the United State Coast and Geodetic Survey on the earthquake shocks of November 18. He reported that the center of this disturbance had been in the Atlantic Ocean somewhere southeast of the Newfoundland coast.

Other reports had varied, some estimating the shocks to have centered at the mouth of the St. Lawrence River, and others farther south. But Fr. Tondorf's information was as nearly accurate as is possible to get, government seismologists have announced. The reports of cable ships repairing the broken lines at the scene of the disturbance confirm his computations most exactly.

Fr. Tondorf has been described as the father of seismology in the Jesuit colleges and universities in the United States. There are now Jesuit Seismological Observatories in Washington, New York, New Orleans, St. Louis, Denver, Buffalo, Santa Clara, Spokane, Los Angeles, Chicago, Cleveland, Weston and other cities.

Fr. Tondorf was a member of many scientific societies, among them the American Association for the Advancement of Science, the Geophysical Union, the Washington Philosophical Society, the Seismological Society of America, and the Royal Astronomical Society of Great Britain. His membership in the last organization recalls the fact that he had at one time devoted himself seriously to the study of astronomy, and that during this period he had given to Fr. Hagen, S.J., valuable assistance in his study of the variable stars.

He was buried with a low requiem Mass as is customary for a Jesuit, but the Right Reverend Bishop John McNamara of Baltimore said the Mass and gave the absolution and in the sanctuary was the Apostolic Delegate, Most Reverend Pietro Fumasoni-Biondi, and many illustrious prelates and clergy of high rank, while the little chapel was crowded with students and professional men. The great will miss him but not as much as the little children who used to clean his office and take care of his effects. For he also was a little child, a little child with a heart like Christ's.



THE CHEMISTRY LIBRARY, LOYOLA COLLEGE

The Chemistry Library

REV. R. B. SCHMITT, S.J.

"Measured in human labor a library represents more toil in the gathering of information than the pyramids in the cutting and piling up of stone. The riches in the vaults of the Bank of England are paltry as compared with the treasures stored in a great library. The bank vault is protected with bolts and locks and armed guards lest some one purloin a single gold piece; the library doors open wide and over its portals is inscribed: Whosoever will, let him come and take of the wisdom of life freely." This quotation is from Dr. E. Emmet Reid, of Johns Hopkins University, in "Introduction to Organic Research."

All research laboratories must necessarily have their libraries and they must be readily accessible. Should colleges that are doing undergraduate work only have a chemistry library? We are of the opinion that a departmental library is necessary in order to stimulate the students, to encourage general scientific knowledge, to arouse keener interest in the work of the regular classes and to develop initiative, self-reliance and leadership.

It is one of the essential functions of a scientific course in college to show the student where to find the information he needs and wants. The time given to lectures is not sufficient to cover all the matter assigned, and so the library can supplement the lecture course. The work in the laboratory is often merely mechanical and a matter of routine. Students are prone to do too much work with their hands and too little with their heads; a library of well selected books conveniently located with respect to the laboratory will help to remove this undesirable state of affairs. There is scarcely anything more important in the pursuit of a science than a familiarity with and an appreciation of the work of the masters in that science. Surely, this cannot be acquired without adequate library facilities.

The College Chemistry Library will stimulate the better students to do research in the post-graduate courses in universities or in commercial laboratories. Here they are taught how to consult the vast literature. Knowing how to find facts in a library is as important as knowing the facts themselves. There are at the present time no less than eighty-eight periodicals in chemistry published and recognized by research chemists. This literature gives the information of the immense amount of research that is being done in all parts of the world.

Finally, the successful operation of the seminar or chemists club is assured and made easy with an adequate library.

On November 6th, 1929, at Loyola College, Baltimore, Md., we opened a new chemistry library adjoining the organic laboratory. In the sectional bookcases we have at present about seven hundred volumes and there are twenty-two periodicals of current chemical literature in the magazine rack. The books are catalogued in a suitable Globe-Vernicke steel file; a duplicate copy of the library card will be found in the principal library of the college. The sections of the library include: general literature, theoretical and physical chemistry, laboratory methods, qualitative and quantitative analysis, inorganic chemistry, organic chemistry and crystallography.

Letters of commendation were received from the following professors of Johns Hopkins University: Dr. William M. Thornton, Jr., Associate Professor of Chemistry; Dr. K. F. Herzfeld, Professor of Physics; Dr. F. O. Rice, Associate Professor of Chemistry, and Dr. Neil Gordon, Editor of Chemical Education.

We are also pleased to announce that a Physics-Biology Library has been established here at Loyola College.

A Filing System

REV. JOHN A. FRISCH, S.J.

The filing of pamphlets, such as the Bulletin of the U. S. Dept. of Agriculture, presents several difficulties. They will not stand up by themselves and when stacked together they are hard to get out singly but worst of all the titles are not visible.

The following method was garnered from "Science" years ago. It works best when the pamphlets are all on one subject or specialty, but it will do for a general collection.

Obtain cloth-covered cardboard cases open only at the back and not larger than 12 x 8 x 2½ inches. The pamphlets are filed in these cases not by their titles, but under the author's name. Therefore on the outside of the cases put a label with a large initial letter at the top and below the abbreviations indicating the names of the authors whose papers are filed in that case, e. g.

B
Br.-Bu

If one author's papers are sufficiently numerous to require one or more complete cases, their fronts bear this initial and name and an indication of the years covered by the papers, included, e.g.

B	B
Brown, A.B.	Brown, A.B.
1915-1920	1921-

But to facilitate the location of particular pamphlets a subject index must be maintained. The card will carry only the title and the author's name e.g. Flytraps and Their Operation—F. C. Bishopp. The author's name tells you in what case it is located.

This method makes for ease in filing and ease in locating.

The Determination of Carbon Dioxide in Barium Carbonate

REV. RICHARD B. SCHMITT, S.J.

The following experiment for the class in Quantitative Analysis is useful to teach the students technique in setting up apparatus and careful manipulation of control:

1. Determine the water present by drying a one gram sample of barium carbonate in a porcelain crucible for one hour at 115° C.

2. Apparatus. This consists of a decomposition flask of 250-300 cc capacity, provided with an upright condenser and connected (to the right) with a Drexel bottle; this is connected to a drying tower, containing concentrated sulphuric acid and calcium chloride, respectively. Following this are three U tubes, c and d and e; c is filled with soda-lime; d has soda-lime with the right arm one-third filled with calcium chloride to absorb the water set free by the action of carbon dioxide on the alkaline hydroxides; the e tube is a protection tube whose left arm is filled with calcium chloride and whose right arm contains soda-lime. This last tube is connected to a Mariotte bottle (for regulating the flow), which in turn goes to the suction pump. The reaction flask is provided with a condenser and with a small dropping funnel, containing dilute hydrochloric acid. Finally, connecting the dropping funnel is a purifying tower containing soda-lime. Under the flask is a Bunsen burner.

The calcium chloride is made neutral prior to use by placing the granular salt of commerce in a large tower and connecting it to a supply of carbon dioxide gas. After passing a rapid gas stream for one or two minutes, the exit tube is closed for twelve hours; whereupon the excess carbon dioxide is expelled by drawing through the apparatus purified and dried air for twenty minutes. The soda-lime should be twelve mesh.

3. Procedure. Weigh out 0.5 gram of air dried barium carbonate into the dry flask and wet the powder with 25 cc of freshly boiled water. Connect up the apparatus and draw a slow steady stream of air through the entire train to free it from carbon dioxide. At the end of half an hour, disconnect the U tubes, c and d, and place them in the balance case, then close the outlets which have been opened by the removal of the U tubes. Fill the dropping funnel nearly full with dilute hydrochloric acid (1:3) and insert the stopper immediately. Now weigh the tubes c and d and again join them in place. Start the suction with a small flow and open the stop-cock on the dropping funnel until there is a gentle stream of acid on the carbonate. Regulate the gas current so that three or four bubbles per second of gas pass through. When all the acid has been added, and the water meanwhile turned on through the condenser, heat the flask gently to boiling and maintain the ebullition for five minutes. Remove the heat and allow a current of air to pass through for half an hour (2 or 3 bubbles per second). Then increase the air current to 3 or 4 bubbles per second and continue for another half hour. Detach the U tubes c and d, stopper them and place them in the balance case for twenty minutes to cool. Then weigh the tubes and record the gain in weight as carbon dioxide.

4. References. Treadwell-Hall, Vol. II, p. 333 (1924); Gooch and Whitfield: Geological Survey, Bull. 47, p. 15-17; Hillebrand: Geological Survey, Bull. 700, p. 217-219, and p. 266; Knorr: A.O.A.C., Official and Tentative Methods, p. 277-278; Engelder: Quantitative Analysis, p. 113 (1929).

5. Notes. In case the materials contain sulphides, it is necessary to interpose an absorbent for hydrogen sulphide. For this purpose copper sulphate dehydrated at 150-160° C., may be placed into the drying tower just below the calcium chloride and separated from it by a layer of asbestos.

Every stratum of dry absorbent should be both preceded and followed by asbestos plugs.

See that every connection is a perfect mechanical fit, and do not try to compensate for leaks by coating the joints with paraffin, shellac, collodion or any such material. Use soft elastic rubber tubing and stoppers. On the ground glass connections use a trace of lubriseal.

If the U tube c has a height of about 10 cm and a diameter of 1.2 cm, it will need refilling after every second analysis. The U tube d will last much longer.

The determination of carbon dioxide by gain of weight is almost universally applicable, and when once the process has been mastered, it is capable of yielding accurate results.

The Laboratory in the Service of the Community

REV. FRANCIS W. POWER, S.J.

It may be of interest, especially to those in charge of the material side of administration, to record a few results obtained at Weston and at Boston College.

Four brands of washing powers have been examined in the Weston laboratory, with the following results: Ban (12 cents a pound) and Kalite (7½ cents) are both trisodium phosphate of nearly the same degree of purity—both are in fact nearly chemically pure. This material, in large lots is being quoted on the New York market at 4 cents a pound.

Ban X, although seeming to be a homogeneous substance under the microscope, contains both trisodium phosphate and sodium carbonate, 97.3 per cent of the latter and 2.2 per cent of the former, according to an analysis made in the Weston College laboratory by Mr. Joseph Moynihan, S.J. This analysis is figured to the dry sample; the original substance contains about 25-30 per cent moisture. It sells for 6½ cents a pound in barrel lots but not knowing the moisture content of commercial soda ash I would not care to make an estimate of its cost to the manufacturer. A washing powder, called "Aero Famous Formula," contains about 85 per cent sodium carbonate and about 15 per cent sodium silicate—a little soap powder, however, is present (around one per cent) to make a suds.

A brand of weed killer coming in a gallon can (at about 7 dollars a gallon), is an alkaline solution of sodium arsenite—it has a specific gravity of about 1.28 and the directions call for dilution with 40 gallons of water for use in killing weeds. We have not had a chance to run the arsenic on it quantitatively, but with arsenious oxide selling for 3¾ cents a pound and commercial caustic soda at about 3 cents, both in large quantities, it is clear that the resulting weed killer is not being distributed gratis.

Preliminary tests on a brand of rat and mouse poison, made up in little pellets to simulate grain, indicate that its active ingredient is strychnine.

I am indebted for the following information to Fr. Joseph J. Sullivan, S.J., of Boston College.

A brand of deodorizing crystals used in toilets, etc., which is sold in small quantities for $1.50 a pound turned out to be para-dichlorbenzene which comes on the market in 250 lb. drums at 18 cents a pound —the Dow Chemical Co. manufacture large quantities of it under the name of Paradow—one of its trade names when one pays $1.50 for it is Deodoroma. A little Oil of Cedar leaves or other cheap essential oil is usually mixed with it. A very satisfactory cockroach powder for kitchen and scullery use may be made up by mixing equal parts of sodium fluoride and Dalmatian Powder (Pyrethrum). At Weston we use equal parts of sodium fluoride and ordinary flour, which we found to be the constituents of a very successful and expensive cockroach poison now on the market. As this mixture looks exactly like flour, it must be kept away from the ordinary places of storage as it is also very poisonous to human beings. Finally, two brands of the substance used to clean out sink drains, etc., and which were not supposed to contain any "lye," analyzed 98 per cent sodium hydroxide and 2 per cent sodium carbonate. One had a little aluminum dust in it besides.

At Weston we have worked out a little modification of the usual method for determining the acidity in wine. Most directions prescribe titrating with litmus paper as an outside indicator, but we dilute 5 cc. of the wine with 250 cc. water, add 1 cc. 1 per cent phenolphthalein, and titrate boiling with N/10 alkali, running a blank on the water and indicator used. The end-point is not a true pink, but is easily noted, and the blank takes care of the unusual amount of indicator. The method checked perfectly against a 1 per cent solution of tartaric acid, saturated with CO_2, instead of wine. This of course gives total acidity—the volatile acids may be distilled off if desired and determined in the usual way.

Formulae for Diluting Solutions

REV. FRANCIS W. POWER, S.J.

The question frequently arises in laboratory work of altering the strength of solutions, usually by diluting them with water, to some given concentration, when the strength of the original solution is given. I submit for criticism and suggestion the following dilution formulae. There are a good many different forms given them in the various books, but these are quite simple and direct and lend themselves easily to graphic representation if necessary. The usefulness of some such simple formulae was brought home to me a good many years ago when I was shown up by the family druggist at being unable to tell him offhand how to dilute out an 85 per cent phosphoric acid solution to one of 10 per cent—he knew of course, but I didn't.

First, as regards dilution of solutions:

Nomenclature

	Volume	Specific Gravity	% by weight of active constituent
Weak solution	a	m	p
Strong solution	b	n	q
Desired solution	k	x	r

The fundamental relations are

$$am + bn = kx \text{ and } amp + bnq = kxr$$

whence

$$a = \frac{kxr - bnq}{mp} = \frac{bn\,(q-r)}{m\,(r-p)}$$

$$b = \frac{kxr - amp}{nq} = \frac{am\,(r-p)}{n\,(q-r)}$$

$$r = \frac{amp + bnq}{xr} \quad x = \frac{amp + bnq}{kr}$$

$$r = \frac{amp + bnq}{kx} = \frac{bnq + amp}{am + bn}$$

If the solutions are to be diluted with water, the latter becomes the "weak solution" and $m = 1$, $p = O$. In this case we have

$$a = \frac{bn\text{-}(q\text{-}r)}{r} \qquad k = \frac{bnq}{xr}$$

$$b = \frac{ar}{n\ (q\text{-}r)} \qquad r = \frac{bnq}{kx} = \frac{bnq}{a + bn}$$

All these expressions are exact enough for all practical purposes as they assume no volume change on mixing the solutions, and it is supposed that one has at hand a set of specific gravity tables such as is found in any of the standard chemical handbooks. The following expressions however are inexact except where the volumes of the two solutions differ widely:

$$k = a + b$$

$$x = \frac{am + b}{a + b}$$

Thus, if equal volumes of concentrated sulphuric acid and water are mixed, their volumes are not strictly additive, but come out about 1.87 volumes instead of 2.00, with corresponding changes in the specific gravity and the percent strength.

It should be recalled that the system of percent strength of solutions such as we consider here is figured on a purely weight basis; a 10 per cent solution of silver nitrate means that 90 grams of water (or 90 cc. for all practical purposes), are used to dissolve 10 grams of silver nitrate; in a 50 per cent solution of sodium hydroxide weighing one kilogram there are present 500 grams of water and 500 grams of sodium hydroxide, although the volume of this solution will be, according to the specific gravity tables, 654 cc. Unless otherwise specified, percent strength always means percent by weight—the term per cent by volume is usually restricted to alcohol. In case we are dealing with volumetric solutions which are generally very dilute, the normality may often be substituted for percent strength in the equations and the dilutions figured accordingly.

A few examples will show how the formulae work out.

1) How much concentrated sulphuric acid S.G. 1.84 containing 95.6% H_2SO_4 is needed to make up 10 liters of battery acid S.G. 1.20 containing 27.3% H_2SO_4? Here we have $k = 10$, $x = 1.20$, $r = 0.273$, $q = 0.956$, $n = 1.84$.

$$b = \frac{kxr}{qn} = \frac{10 \times 1.20 \times .273}{.956 \times 1.84} = 1865 \text{ cc.}$$

That is, 1865 cc. conc. H_2SO_4 is added to a sufficiency of water, and the mixture diluted out to 10 liters.

2) What strength alcohol is made by mixing equal volumes of water and commercial ethyl alcohol S.G. 0.804 containing 95.0% C_2H_5OH?

Here we have b = 1, n = 0.804, q = 0.95, a = 1.

$$r = \frac{bnq}{a + bn} = \frac{1 \times .804 \times .95}{1 + (1 \times .804)} = 42.3\%$$

3) How should glacial acetic acid and water be mixed to give an acid containing 28% CH_3COOH? The S. G. of glacial acetic acid is given as 1.055. Here we have b = 1 (let us say), n = 1.055, q = 1.00 (assuming that the acid is 100% acetic), r = 0.28.

$$a = \frac{bn\ (-r)}{r} = \frac{1 \times 1.055 \times .72}{.28} = 2.71 \text{ — that is}$$

to each volume of the strong acetic acid we must add 2.71 volumes of water. Note that here the final volume was not specified, hence it is not necessary to know the specific gravity of the desired acid.

4) A hydrochloric acid is found on standardizing to be 1.150 normal; how much water must be added to each liter of it to bring it to exact normality? The specific gravity of this acid from the tables should be 1.02, so we have b = 1.00, n = 1.02, q = 1.15, r = 1.00

$$a = \frac{bn\,(q\text{-}r)}{r} = \frac{1 \times 1.02 \times 0.15}{1} = 0.153 \text{ cc. water which}$$

may be added from a micro pipette and the acid restandardized.

5) What will be the volume of a mixture of 520 cc. of absolute alcohol and 480 cc. of water?

As the specific gravity of this mixture is not known, it must be looked up in the alcohol tables where it is given as a function of the percent strength of alcohol. This in turn is first calculated as follows: a = 480, b = 520, n = 0.7939, q = 1.00

$$r = \frac{bnq}{a + bn} = \frac{520 \times .7939 \times 1.00}{480 + (520 \times .7939)} = .4625$$

The specific gravity of an alcohol 46.25% by weight is 0.922. Then we have x = .922, r = .4625 and

$$k = \frac{bnq}{xr} = \frac{520 \times .7939 \times 1.00}{.922 \times .4625} = 968 \text{ cc.}$$

Experiment gives 964 cc. This is usually given as the maximum volume contraction for alcohol-water mixtures.

6) How much concentrated hydrochloric acid S. G. 1.18 containing 35.4% HCl must be added to 3 liters of waters to give a normal hydrochloric acid?

In this problem we must express the strength of the desired weak acid in terms of percent HCl by weight: its specific gravity as found in the tables is 1.018; its strength will be

$$\frac{36.46}{1018} = 3.6\% \text{ HCl by weight}$$

Then we have a = 3000, r = .036, n = 1.18, q = .354 and

$$b = \frac{ar}{n(q\text{-}r)} = \frac{3000 \times .036}{1.18 \times (.354 - .036)} = 288 \text{ cc.}$$

In this case the ratio of strong acid to water is appreciable and it is inaccurate to express the quantities q and r as normalities; if this is done the answer comes out too low, viz 243 cc.

Second, as regards making up solutions to a given percentage by weight.

Nomenclature

v = volume of solvent to start with
k = volume of final solution desired
r = percent strength desired in final solution
m = specific gravity of solvent
x = specific gravity of final solution
w = amount of substance in grams to be added to solvent

The three important relations are:

$$w = \frac{rvm}{1 - r} = kxr$$

$$v = \frac{kx(1 - r)}{m}$$

Examples:

1) How much sodium chloride must be added to a liter of water to give a 5% solution of NaCl?
Here v = 1000, m = 1, r = 0.05

$$w = \frac{rvm}{1 - r} = \frac{.05 \times 1000}{.95} = 52.6 \text{ grams}$$

2) How much potassium hydroxide must be added to 500 cc. alcohol S.G. 0.80 to give a 10% alcoholic potash solution? v = 500, r = 0.10, m = 0.8

$$w - \frac{0.1 \times 500 \times 0.8}{0.9} = 44.5 \text{ grams}$$

3) How much sodium hydroxide must be used to make a liter of 50% NaOH solution? k = 1000, x = 1.53 (found in the tables) r = 0.50

$$w = kxr = 1000 \times 1.53 \times 0.5 = 765 \text{ grams.}$$

The 765 grams NaOH are added to 765 cc. water and the volume of the resulting solution will be very nearly one liter.

4) It is desired to make up a liter of 20% KOH solution in alcohol whose S. G. is 0.81. The S. G. of the final solution is 0.985. How much KOH will be required?
k = 1000, x = .985, r = 0.2, w = kxr = 1000 x .985 x .2 = 197 gr.
This should be dissolved in v cc. alcohol:
k = 1000, x = .985, r = 0.2, m = 0.81

$$v = \frac{kx(1 - r)}{m} = \frac{1000 \times .985 \times 0.8}{.81} = 975 \text{ cc.}$$

The figures in this problem are not very exact owing to the uncertainty about the specific gravity of this solution. The value .985 was obtained experimentally using 95% alcohol and "C.P." potassium hydroxide marked to contain 85% KOH, hence 232 grams of the actual substance should be used to take account of this low KOH content.

The Quantum Theory and Energy

JOSEPH T. O'CALLAHAN, S.J.

All are familiar with Millikan's experiments on the Photoelectric effect, and the verification of Einstein's quantum equation. These results may be summed up as follows: The maximum energy of the electron, released by light of frequency (v) is equal to a quantity (hv), whatever the metal used, minus a quantity (P), which with great probability represents the energy used up by the electron in getting free of the metal.

Then, the quantum theory is verified, enthusiasts say! This does not follow. A distinction must be made between the verification of Einstein's equation, $\frac{1}{2}\ mv^2 = h_2 - P$, and the verification of Einstein's Theory—the interpretation of the equation. Such a distinction should be made in all physical experiments, but it is especially necessary when considering the quantum theory. Not only is the theory a novel one, not only does it fail to explain the ordinary phenomena of light, but it seems to be fundamentally untenable.

The hypothesis postulates corpuscular pieces of pure energy, but energy seems to be an ontological accident, and if it is, then bundles of pure energy are simple impossibilities. The reason for thinking that energy is an accident is drawn from the very definition of energy. In all text books it is defined as the capacity for doing work, but "capacity" is an abstract notion and connotes that there is something which has that capacity; just as motion connotes a mover. In scholastic language, the very definition of energy implies that it is an ontological accident, inhering in a substance.

From this it follows that, when scientists speak of any particular kind of energy, electrical energy for example, mere common sense, to say nothing of scientific logic, forces us to define it as electrical capacity for doing work,—and this, of course, connotes that energy is an accident, inhering in a substance. If scientists mean anything else by the term when they speak of electrical energy; if they mean a small particle, or something that has the capacity for doing work, then they are using the term in a double sense,—and such a double use can only lead to greater and greater confusion.

This is not a quibble about the use of words, for we are concerned with the realities behind the words. As far as language is concerned, we could call a small particle that has the ability to do work, by the word "energy," but then by what word would we designate that other reality—the capacity for doing work? And because the word is commonly used to designate one definite thing—capacity for doing work—it should not be used to express an entirely different reality.

Now the question arises, Is the word used to express an entirely different reality? It might appear to some that the definition, when first formed, was based on very crude observations, and that from these, energy did indeed seem to be an accident; but that modern discoveries in physics lead us to conclude that energy is a substance.

Such an assumption seems unfounded, for crude as those observations may have been, they were, nevertheless, true as far as they went; and a fact of observation, once established, remains true, modern discoveries amplify but do not deny previously established facts. From these crude observations, scientists knew that bodies have the capacity of doing work, and they call that capacity "energy." There is, then, a definite reality underlying that word, and modern discoveries do not change it. Hence we are in no way justified in using that term for anything else. Nor does it seem that we can modify or amplify our definition, so that it might express what some modern physicists wish it to express, for the old definition, "capacity for doing work," however modified, can never be twisted into something to which it is entirely opposed. "Accident" can never be modified into meaning "substance."

To sum up then: Energy, according to the definition that scientists themselves give, represents an ontological accident. If, when considering modern physical discoveries, scientists use energy in this ordinary sense, then the postulate of disembodied energy is simply impossible. If scientists wish to designate any other reality, logic compels them to use another word.

Many scientists seem to recognize the problem, but do not face it squarely. Thus Darrow, speaking of Einstein's hypothesis, says: "This idea Einstein offered as an heuristic one, the word, if I grasp its connotation exactly, . . . describes a theory which achieves success, though its author feels at heart that it is really too absurd to be presentable. The implication is that experimenters should proceed to verify the predictions based upon the idea, quite as if it were acceptable, while remembering always that it is absurd. If the successes continue to mount up, the absurdity may be confidently expected to fade gradually out of the public mind." ("Introduction to Contemporary Physics," by K. K. Darrow, Van Nostrand, p. 121.)

The first part of the quotation suggests that Darrow, and Einstein, too, perhaps, saw the fundamental difficulty; the last part seems to be a dangerous attempt to dodge that difficulty. Dr. Arthur Compton seems to fall into the same fault. In his essay, "The Paradox of Light," January, 1929, Scribner's, he says, "We have gradually been accustoming ourselves to the idea of disembodied energy, or rather of energy which is its own body."

This statement may be used in a loose sense, and may mean that we are accustoming ourselves to the novel *fact* that energy is a substance; if that is the meaning, an examination of the facts is necessary, and this shall be made later. If, however, the statement means what it seems to mean, it is very dangerous. We accustom ourselves to *facts*, but we do not accustom ourselves to ideas. We analyze ideas, and if they are fundamentally wrong we reject them. We reject the idea of a square circle, for an analysis shows that it contains a contradiction, and if it were repeated from now till doomsday we would not accustom ourselves to it. When scientists try to "accustom themselves to strange ideas," they are not meeting the problem squarely, and such neglect, it would seem, will only lead to greater and greater confusion in the field of science.

The primary objection to the quantum theory is a philosophical one; it postulates disembodied energy, and the reasons for rejecting this postulate have been given, and yet it is claimed that there is experimental proof for that postulate. Perhaps, Dr. Compton meant to say that we are gradually accustoming ourselves to the fact that energy is localized outside of matter,—as a substance, of course.

The "proof" of the localization of energy outside of matter is drawn from experiments performed before the enunciation of the quantum theory. The argument is given by Rougier, "Philosophy and the New Physics," Chapter III. It is drawn from a consideration of the electric field around a charged conductor, and of a magnetic field produced by a magnet or a current. The argument is identical in both cases; we shall study only the first.

It is known that the electrification of a body determines the behavior of attracting and repelling forces in the surrounding space, under the influence of which, oppositely electrified bodies approach each other, and similarly electrified bodies repel each other, in accordance with Coulomb's law. This region of space in which the phenomenon takes place is called the electric field. It is possible to draw through each point of this electric field a curve whose direction coincides with that of the force capable of thus acting at that point; lines of electric force are thus obtained which have, however, a purely geometric significance. Yet, Faraday made the hypothesis that these lines of force have a real physical significance, they correspond to certain permanent modifications of the ether. This hypothesis does explain the observed facts, and has some confirmation from the theoretical work of Maxwell. From these premises, the proof of the localization of energy outside of matter runs thus: "It is possible to get rid of the consideration of the ether, the existence of which is hypothetical and contradictory, and contemplate nothing but the only positively accessible reality—electrostatic energy localized outside of the conductors in the form of the field. The mechanical work of attraction or repulsion, done by the static forces appearing in an electric field, represents a certain expenditure of energy; that is, the change of a certain amount of potential energy into actual energy of motion. Thus, potential energy localized in empty space around a conductor exists in conformity with Faraday's experiments." ("Philosophy and the New Physics," Rougier, trans. by Masius. Publ. by P. Blakiston's Son & Co., p. 44.)

The "proof" has been quoted verbatim, so that it may be clear from the author's own words that he has presumed precisely what he wished to prove. Passing over this, however, it seems that any attempt of this kind to prove the localization of energy outside of matter must be rejected. And the rejection is based on the nature of physical instruments. The argument given above states that we need "contemplate nothing but the only positively accessible reality." Now to argue from any measurements which our instruments register, that nothing but the measured energy exists, seems illogical. For when we consider only what our instruments measure, our consideration is necessarily inadequate. Physical instruments measure only extension or motion, or say, force and energy; they "prescind," as it were, from any substratum that may or may not be there. To detect that substratum is beyond the scope of the instrument. We say that our senses are affected only by the accidental modifications of the body; substance is not the formal object of any sense. So, too, physical instruments are affected only by accidents, ontological, of course; they give no direct information, and can give no direct information about the underlying reality. If they do not record that underlying reality, and they do not, it is because they "prescind" from it, they are not adapted to recording it; and because they do not record it, one is in no way justified in concluding that there is no underlying reality present.

This discussion has led to the following conclusions: Energy, according to the definition given by scientists, is an ontological accident. The use of the word in any other sense leads to confusion and illogical thinking. The "experimental proofs" that energy is a substance are unsound. Therefore, the postulate of disembodied energy must be rejected.

However, by rejecting the quantum theory, because of this postulate and by insisting that energy, as defined by scientists, is an accident, I do not wish to give the impression that I belittle the work of modern physicists; quite the opposite is the case. Yet physicists themselves are puzzled; they admit that present day physics is in a confused state, fact is far ahead of theory, the experimental foot has taken a great stride into the sub-atomic world, and made discoveries more wonderful than Alice found in Wonderland. It is hard for theory to follow into this new wonderland; and theorists seem to have left the sure road of logic and to have lost themselves in the woods; they seem to be wandering around in circles. Now, no satisfactory advance can be made in this way; all hope of solving the problem of light is lost, if we begin by confusing substance and accident. It appears then that in these conditions the first move is to find our way back to the sure road of logical thinking. We must insist more than ever on clear thought, we must be careful to define our terms, and once defined we must keep to our definitions.

Millikan once said that science advances on two feet, theory and experiment; but neither theory nor experiment, nor both together, will get us anywhere unless we are walking on firm ground; unless we have a sure logical foundation; only in this way can real scientific progress be made.

The Foucault Pendulum Experiment at Weston College

REV. H. M. BROCK, S.J.

On October 30th, 1929, the pendulum experiment devised by that ingenious French Catholic physicist, Léon Foucault, to demonstrate the rotation of the earth on its axis was tried for the first time at Weston College. As is well known, the demonstration depends upon the fact that a pendulum suspended at a point so as to be free to swing in any plane will keep its plane of vibration unchanged provided no other force but gravity acts upon it. This can be shown very simply to a class by attaching a pendulum to a small table capable of rotation. A line is drawn along any diameter and the ball started vibrating along it. The table is then slowly turned. The pendulum continues to vibrate in the same plane and the path of the ball will make an angle with the original line whose magnitude depends upon the rate at which the table is turned. Foucault saw that if the earth were substituted for the table a pendulum could give ocular evidence of its rotation about its axis. Thus if a graduated circle were drawn on the earth with the north pole as the center and a pendulum were suspended over the pole and set in vibration, the earth would turn under it counter-clockwise at the rate of fifteen degrees per hour. To the eye the plane of the pendulum would appear to turn at the same rate in a clockwise direction. At the equator there would be no shift while at any other place whose latitude is L degrees the shift would be $15^{\circ} \sin L$ per hour.

Foucault is said to have tried the experiment first at the National Observatory at Paris. It was so successful that in 1851 he decided to give a public demonstration in the Panthéon. An iron ball weighing over 62 pounds was suspended from the inside of the dome by a wire about 200 feet long. Below it a circular rail was set up on the floor. A ridge of sand was built on this so that a needle attached to the bottom of the ball would pass through it at each vibration. The plane of the pendulum apparently turned through the angle predicted by Foucault, a bit of sand being flicked off at regular intervals. The demonstration aroused great enthusiasm and it was repeated in various parts of the world. Among other places a Foucault pendulum has been set up permanently in the new building of the National Academy of Science at Washington and also in the Museum of Yale University at New Haven, so that one can try the experiment for one's self.

The rotunda at Weston College is admirably adapted for the Foucault experiment, so it was decided to carry it out primarily for the classes of physics and astronomy. There is plenty of room and the inner dome is 71.8 feet above the floor. The three galleries serve as convenient vantage points from which to watch the proceedings. The pendulum used was one recently put on the market by the Eastern Science Supply Company of Brookline, Mass. The bob is a lacquered steel ball six inches in diameter, weighing about 32 pounds. A cylindrical support is provided for holding the ball when not in use and also a handle which can be screwed into it so that it can easily be carried about. The ball is suspended by means of a #22 wire. One end is wound about a small block which fits into a brass plug. This is screwed flush into the ball in the same hole into which the handle fits. The upper end of the wire is attached to a C-shaped bronze yoke which holds a hardened steel pivot. The pivot rests in a conical steel cup held in a second yoke which is screwed on to a steel plate. The whole weight of the pendulum rests on the pivot. It can swing freely in any plane. The fixed yoke carries an inverted cup-like receptacle directly over the pivot pin. This receives an extension of the pivot. While not interfering with the vibrations it prevents the pivot yoke from falling when the pendulum weight is removed. The distance from the pivot to the top of the ball was 67.75 feet and the time of a single vibration was 4.56 seconds.

The steel plate holding the suspension was firmly bolted to a 2 by 2¾ inch wooden beam. The latter was laid across the opening in the dome in place of the metal grating usually in position. A black-topped laboratory table was placed directly under the pendulum on the rotunda floor. Arcs were drawn with chalk about the center of the table and angles laid off at intervals of 10 degrees. A longitudinal crack in the middle of the top served as the zero line. This was placed approximately north and south.

The pendulum was set in variation at 8.00 A. M. and also during the noon and evening recreations. It was started in the usual way by drawing out the ball beyond the top of the table and holding it in position by means of a thread. When it came to rest the string was burned. The table was then adjusted immediately so that the ball moved along the crack. By sighting along the crack it was easy to detect any apparent shift in the plane of vibration. A clockwise deviation could be detected in less than three minutes, showing that the table had skewed around, the south end moving eastward more rapidly than the north end. It was not possible to measure the deviation accurately. It was about 10 degrees per hour. The theoretical value for Weston is 10 degrees 7 minutes. A ridge of sand was also built up through which the needle at the bottom of the ball passed. The pendulum continued to vibrate several hours with diminishing amplitude but the vibrations gradually became decidedly elliptical. In fact, the main difficulty in the experiment is to avoid the formation of an elliptical orbit. Even though the bob is perfectly at rest when started air currents are likely to produce an effect. During the course of the experiment Fr. J. Blatchford took some pictures and also a 16 mm. moving picture film with a Filmo camera.

Note on Relativity

REV. FREDERICK W. SOHON, S.J.

The recent attempt by Einstein to reduce gravitation and the electro-magnetic equations to a single formula does not appear to have been fortunate. It appears that one of the professors at the Massachusetts Institute of Technology after studying the proposed formula arrived at the conclusion that a zero gravitational field resulted. The result was communicated to Einstein and confirmed by him. Einstein then suggested an alteration of the formula, but I understand that the new form gives no better result. Hence no advance has been made on the general theory of relativity.

Meanwhile, the Michelson-Morley experiment has continued to make trouble. I quote from Contribution No. 373 from Mount Wilson.

"Previous to 1925, the Michelson-Morley experiment has always been applied to test a specific hypothesis. . . . Throughout all these observations, extending over a period of years, while the answers to the various questions have been "no," there has persisted a constant and consistent small effect which has not been explained. . . . A complete calculation has now been made, including the observations of both 1925 and 1926, which leads to the following conclusion: The ether-drift experiments at Mount Wilson show, first, that there is a systematic displacement of the interference fringes of the interferometer corresponding to a constant relative motion of the earth and the ether at this observatory of 10 km/sec., with a probable error of 0.5 km/sec.; and second, that the variations in the direction and the magnitude of the indicated motion are just such as would be produced by a constant motion of the solar system in space, with a velocity of 200 km/sec., or more, toward an apex in the constellation Draco, near the pole of the ecliptic, which has a right ascension of 255° (17 hours) and a declination of +68°; and, third, that the axis across which the observed azimuth of drift fluctuates, because of the rotation of the earth on its axis, points in a northwesterly direction, whereas the simple theory indicates that the axis should coincide with the north and south meridian."

The discussion is printed in the Astrophysical Journal, Vol. 68, pp. 341 to 402, Dec. 1928.

The Twenty-Five Cell Squares

Rev. Frederick W. Sohon, S.J.

Proposed Method.

The twenty-five numbers are each decomposed into the sum of two numbers called elements. There are ten elements: 0, 5, 10, 15, 20, and 1, 2, 3, 4, and 5. In order to make each column and each row add up to 65, each column and each row will contain all ten elements. It has not been shown that such a homogeneous distribution of the elements is at all necessary and there may be other perfect 25-cent magic squares that are not made by this method. Other ways of distributing the elements are shown at the conclusion of this paper. The pairs of elements 0 and 20, 5 and 15, 1 and 5, 2 and 4 are styled complementary. They form the basis of complementary numbers. The elements 3 and 10, being self-complementary, are called the middle elements and must be symmetrically disposed. The central cell contains both middle elements $(10 + 3 = 13)$. The arrangement or disposition of any element forms a half-pattern, and half-patterns of complementary elements must be symmetrically disposed in order that numbers diametrically opposite and equally distant from the center may add up to 26. A scheme showing the arrangement of the elements 0, 5, 10, 15, 20 is called a Fundamental Pattern. A scheme showing the arrangement of the elements 1, 2, 3, 4, 5, is called a Supplementary Pattern. The square is finally to be synthesized by superposing a Supplementary Pattern upon a Fundamental Pattern. If the arithmetical progression 1, 2, etc., 24, 25 is produced, the synthesis is successful. Otherwise it is rejected. We should like to know what squares can be constructed by this method.

Nomenclature of Patterns

For the purpose of describing the position of the middle element in any column, the horizontal rows of the square are designated by the five vowels a, e, i, o, u. Positions are only given for the first two columns, the further disposition being determined by the condition of symmetry. Thus the letters *a*e tell us that the middle element occupies the top cell in the first column and the bottom cell in the last column, the next to the top cell in the second column and the next to the bottom cell in the next to the last column. In other words an *a*e pattern has the middle element in a diagonal.

If we omit the middle element there are 24 possible dispositions for the remaining elements consisting of three exterior dispositions containing eight interior dispositions each. In the first exterior disposition the complementary half patterns are mutually exclusive:

1	1	5	5	2	2	4	4	+
5	5	1	1	4	4	2	2	+
2	4	2	4	1	5	1	5	—
4	2	4	2	5	1	5	1	—

This arrangement in a column is designated by either p or b. The interior dispositions do not concern us now.

One pair of complementary half patterns may include the other, thus:

1	1	5	5	2	2	4	4	+
2	4	2	4	1	5	1	5	—
4	2	4	2	5	1	5	1	—
5	5	1	1	4	4	2	2	+

This arrangement in a column is designated by t or d.

The pairs of complementary half patterns may alternate, thus:

1	1	5	5	2	2	4	4	+
2	4	2	4	1	5	1	5	—
5	5	1	1	4	4	2	2	+
4	2	4	2	5	1	5	1	—

Such an arrangement is designated by either k or g.

Only the first two columns are described. In the first column either of the two consonants may be employed to describe the disposition. But if either p, t, or k is used, then the use of one of these three letters to describe the second column will signify that both columns begin with elements from the same complementary pair of half patterns, while the choice of b, d, or g, will signify the contradictory, and correspondingly if b, d, or g is used in the first column. For example, consider the important pattern Tape. The vowels tell us that the middle element occupies a diagonal. T tells us that we have an inclusive arrangement in the first column, while p tells us that in the second column the mutually exclusive arrangement prevails, and that apart from the middle elements, the uppermost element in the second column is drawn from the same pair of complementary half patterns as the uppermost element in the first column. If we represent middle elements by o, one pair of complementary half pattern elements by + and the other by —, then tape will look like this

O	+	—	—	+
+	O	+	—	—
—	+	O	+	—
—	—	+	O	+
+	—	—	+	O

A discussion of transformations materially reduces the number of patterns to be considered.

Transformations.

The 128 simple transformations of the 16-cell Magic Square are also used in the 25-cell square. If H is reflection in a horizontal mirror, A alternation, I inversion, then the affect of applying these is as follows:

O	H	A	I	AI	IA	HA	HI
a	u	a	e	o	e	u	o
e	o	o	a	a	u	e	u
i	i	i	i	i	i	i	i
o	e	e	u	u	a	o	a
u	a	u	o	u	o	a	e

from which some rules may be deduced

$$HA = AH, \ HI = IH, \ HAI = IA$$

but the immediate application is to reduce the fundamental pattern to the *a*e form. We have

$$(*a*e) = (*a*o)A = (*e*a)I = (*e*u)AI = (*o*a)IA = (*o*u)HI$$
$$= (*u*e)HA = (*u*o)H$$

If we decide to adopt *a*e as the standard form for a fundamental pattern we are interested in those transformations that do not affect the diagonals of a square. This leaves us of the superficial transformations only

C = a rotation through 180°

D and S reflection in the diagonals.

Of the linear transformations

double alternation

double inversion.

Concerning their algebra a few rules suffice.

S, D, A, I, and C are all commutative with each other, except

$$AI = CIA$$

The squares are all O, except again

$$(AI)^2 = (IA)^2 = C$$

Finally DS = C, DC = S, CS = D

Our patterns are all symmetrical with respect to the center. Hence for them C is the same as O, D the same as S, AI the same as IA, and the transformations are all commutative. But this is not true in general.

Construction of the Patterns.

We have shown that it is always possible to bring the element 10 into the *a*e position, so that all fundamental patterns are reduced to *a*e patterns. If now the first column be assigned as Ba, Da, or Ga, the i cell of the second column is automatically fixed, because it must be different from the i cell of the first column. The e cell of the second column is also occupied by 10, so that the complement of the element in the i cell of the second column, must be found in either the a, o or u cells. Hence, for each selection of the first column there would appear to be three lines of development open. The pattern Kade has however too many similar elements in the same row, reducing the number of fundamental patterns to eight.

For the supplementary patterns we cannot get the middle element into an arbitrary position without spoiling the fundamental pattern. A few considerations serve to reduce somewhat the number of patterns to be investigated. *a*e, *a*o, *a*u, *o*e, *u*e are excluded because 13 must occur only in the center of the square, and of course *a*a, *e*e, *o*o, *u*u are also excluded so that there remain only *e*a, *e*u, *o*a, *o*u, *u*o. Besides this we add the conditions that the middle row, middle column, and the *a*e diagonal must each contain all five elements. From these considerations 20 supplementary patterns are constructed. The patterns are listed with their transformations:

Original	D or S	Alternation	Inversion
Bage	Bage	Tape	Tape
Bape	Tage	Tage	Bape
Tage	Bape	Bape	Tage
Tape	Tape	Bage	Bage
Kage	Tade	Gabe	Tade
Tade	Kage	Pate	Kage

Gabe	Pate	Kage	Pate
Pate	Gabe	Tade	Gabe
Pubo	Tuko	Tuko	Pubo
Puko	Puko	Tubo	Tubo
Tubo	Tubo	Puko	Puko
Tuko	Pubo	Pubo	Tuko
Teda	Teda	Tobu	Kega
Kega	Kega	Poku	Teda
Keda	Peba	Pobu	Keda
Peba	Keda	Kotu	Peba
Toda	Tebu	Tebu	Keku
Poga	Keku	Keku	Tebu
Poda	Petu	Kebu	Kebu
Koba	Kebu	Petu	Petu
Tebu	Toda	Toda	Poga
Keku	Poga	Poga	Toda
Kebu	Koba	Poda	Poda
Petu	Poda	Koba	Koba
Tobu	Tobu	Teda	Poku
Poku	Poku	Kega	Tobu
Pobu	Kotu	Keda	Pobu
Kotu	Pobu	Peba	Kotu

Synthesis.

By copying the fundamental patterns on tissue paper and superposing each in turn on all the supplementary patterns successively, we find that out of the whole list of 28 patterns, only 4 supplementary and two fundamental patterns can be used. The combinations are

Tubo-tape	Tuko-tage
Petu-tape	Peba-tage

but an insertion of the numbers shows that the squares constructed on tage are all duplicate numbers, and therefore we have only squares constructed on tape. The transformation of these combinations is as follows:

O	D	A	I	AD	AI	ID	AID
Tubo-	Tubo-	Puko-	Puko-	Puko-	Tubo-	Puko-	Tubo-
tape	tape	bage	bage	bage	tape	bage	tape
tape	tape	bage	bage	bage	tape	bage	tape
Petu-	Poda-	Koba-	Koba-	Kebu-	Petu-	Kebu-	Poda-

Resolution into Half Patterns.

We have shown that the squares are to be built up from the fundamental pattern Tape, and from the supplementary patterns Tubo and Petu. The resolution of these patterns into half-patterns is of itself a simple matter, but there are eight ways of doing it, corresponding to eight ways of disposing the elements in a pattern. As we shall have to follow these dispositions in the course of transformation we assign names to them taking care to assign the names in such a way that the algebra of transformations that we are using may still be applied to them. For T patterns the names are assigned as follows:

o	a	i	ai	co	ca	ci	ia
1	1	2	4	5	5	4	2
2	4	1	1	4	2	5	5
4	2	5	5	2	4	1	1
5	5	4	2	1	1	2	4

For P patterns the names are assigned in this way:

1	1	2	4	5	5	4	2
5	5	4	2	1	1	2	4
2	4	1	1	4	2	5	5
4	2	5	5	2	4	1	1

If some transformation of A say AP gives the same square as some transformation of B say BQ then $B = APQ^{-1}$, hence to make sure A and B do not yield duplicates it is only necessary to investigate transformations of A that are likely to reproduce B. We consider only D, S, C, AI, DAI, SAI, CAI since the others change the patterns as we have seen. The following table results

O	o-Tubo	o-Tape	o-Petu
D	co- "	o- "	(-Poda)
S	o- "	co- "	(-Poda)
C	co- "	co- "	co-Petu
AI	ai- "	ai- "	ia-Petu
DAI	ia- "	ai- "	(-Poda)
SAI	ai- "	ia- "	(-Poda)
CAI	ia- "	ia- "	ai-Petu

If the disposition of elements in Tubo is p and the disposition of elements in Tape is q, then we may write the interior disposition of Tubo-tape as (p, q). We have then the following equations:

$$(o, o) \quad D = (co, \ o) \qquad (1)$$
$$(o, o) \quad S = (\ o, co) \qquad (2)$$
$$(o, o) \quad C = (co, co) \qquad (3)$$
$$(o, o) \quad AI = (ai, ai) \qquad (4)$$
$$(o, o) \quad DAI = (ia, ai) \qquad (5)$$
$$(o, o) \quad SAI = (ai, ia) \qquad (6)$$
$$(o, o) \quad CAI = (ia, ia) \qquad (7)$$

From (2) we conclude that the simultaneous introduction of c into the dispositions of both patterns does not give a new square, but the C transformation of the square we already have. Introducing c into both parts of (cp, q) we have

$$(cpc, qc) = (p, qc) = (cp, q)$$

which means that c can be transferred from the fundamental to the supplementary and vice versa without giving a new square. By the same argument using equations (1) and (2) it follows for Tubo-tape that c can be dropped whenever it occurs without giving a new square. The right hand members of (4), (5), (6), and (7), are therefore all the same square. Equation (4) develops into a series of equivalences:

(ai, ai) is equivalent to (o, o)
(i , ai) " (a, o)

so that the dispositions are all paired off. Dropping the c's the dispositions are four for each pattern o, a, i, ai, making 16 for Tubo-tape but these being equivalent in pairs there are eight different interior dispositions in the final count:

(o, o), (o, a), (o, i), (o, ai), (a, o), (a, a), (a, i), (a, ai)

Interior Dispositions of Petu-tape

The equations for Petu-tape are

$$(o, o)C = (co, co) \qquad (3)$$
$$(o, o)AI = (ia, ai) \qquad (8)$$
$$(o, o)CAI = (ai, ia) \qquad (9)$$

We have just discussed the effect of equation (3) and have seen that it enables c to be transferred from one pattern to the other. In this way the right hand member of (8) is the same as (9), so the interior dispositions of Petu-tape drop from 64 to 32 on account of (3), and from 32 to 16 on account of (8) and (9).

Conclusion.

We have 24 squares, 8 of the Tubo-tape patterns, 16 of the Petu-tape pattern, and each of the 24 is subject to 128 transformations. The patterns are given as follows:

o-Tubo

1	2	4	5	3
2	4	5	3	1
4	5	3	1	2
5	3	1	2	4
3	1	2	4	5

o-Tape

10	20	15	5	0
0	10	20	15	5
5	0	10	20	15
15	5	0	10	20
20	15	5	0	10

o-Petu

1	5	4	3	2
3	2	1	5	4
5	4	3	2	1
2	1	5	4	3
4	3	2	1	5

Let it be required to construct (a, i) Tubo-tape.

a-Tubo

1	4	2	5	3
4	2	5	3	1
2	5	3	1	4
5	3	1	4	2
3	1	4	2	5

i-Tape

10	15	20	0	5
5	10	15	20	0
0	5	10	15	20
20	0	5	10	15
15	20	0	5	10

Answer

11	19	22	5	8
9	12	20	23	1
2	10	13	16	24
25	3	6	14	17
18	21	4	7	15

Note 1.

We remarked in the beginning of the study of this square that the theory is not complete because of the nature of the patterns there postulated. I here submit ten other patterns in hopes that some one will find out whether any of them can be used or not.

1bd x 1ff

5	2	2	1	5
4	4	1	3	3
2	1	3	5	4
3	3	5	2	2
1	5	4	4	1

1bd x 4bf

5	5	2	1	2
2	2	3	5	3
1	2	3	4	5
3	1	3	4	4
4	5	4	1	1

1ae x 1bd

5	3	5	1	1
2	2	4	3	4
1	2	3	4	5
2	3	2	4	4
5	5	1	3	1

1ae x 4bf

4	1	1	5	4
3	5	2	1	4
4	3	3	3	2
2	5	4	1	3
2	1	5	5	2

4ac x 4bf

4	2	2	2	5
1	5	3	5	1
4	3	3	3	2
5	1	3	1	5
1	4	4	4	2

4bf x 4bf

1	5	4	4	1
1	4	3	5	2
4	3	3	3	2
4	1	3	2	5
5	2	2	1	5

4ac x 1bd

1	3	1	5	5
5	4	2	2	2
4	3	3	3	2
4	4	4	2	1
1	1	5	3	5

4ac x 1ff

1	5	1	5	3
5	2	2	2	4
4	3	3	3	2
2	4	4	4	1
3	1	5	1	5

4bf x 1ff

3	5	2	1	4
4	2	3	5	1
1	2	3	4	5
5	1	3	4	2
2	5	4	1	3

1ae x 1ff

3	5	1	5	1
2	4	4	3	2
1	2	3	4	5
4	3	2	2	4
5	1	5	1	3

Note 2.

The Tubo-tape pattern is readily extended to all odd squares, and is most easily constructed. In the following patterns, the element is written in italics where the sum of its column and row numbers is odd.

o-Tubo

1	*2*	4	*5*	3
2	4	*5*	3	*1*
4	*5*	3	*1*	2
5	3	*1*	2	*4*
3	*1*	2	*4*	5

o-Tape

10	*20*	15	*5*	0
0	10	*20*	15	*5*
5	*0*	10	*20*	15
15	5	*0*	10	*20*
20	*15*	5	*0*	10

Each element so indicated is translated a distance equal to the length of one edge of the square, and in a direction at right angles to the nearest edge of the square. We then obtain

o-Tubo

				1				
			1		*4*			
		1		4		3		
	1		4		3		*2*	
1		4		3		2		*5*
	4		3		2		*5*	
		3		2		5		
			2		*5*			
				5				

o-Tape

				0				
			15		*0*			
		10		15		0		
	5		10		15		*0*	
20		5		10		15		*0*
	20		5		10		*15*	
		20		5		10		
			20		*5*			
				20				

In others words the pattern Tubo-tape corresponds to a very simple diagonal structure. We may, in fact, start with a fundamental square

				1				
			6		2			
	16	11		7		3		
			12		8		4	
21		17		13		3		5
	22		18		14		10	
		23		19		15		
			24		20			
				25				

whose columns and rows are parallel to the diagonals of the required square. We apply different interior dispositions by merely permuting the columns and rows of the fundamental square. The required magic square is then synthesized by translating the outlying parts of the fundamental square.

The Cardinal Number and Its Generalization

REV. FREDERICK W. SOHON, S.J.

1. Cardinal Similitude

The foundations of mathematics, though satisfactory, are unnecessarily cumbrous for they were built up in spite of a philosophy that refused to consider universals explicitly as such, and a great simplification is introduced if the matter is attacked more directly by conceding the doctrine of universals as taught in scholastic epistemology. On the other hand, let it not be a matter of surprise that concepts become unfamiliar when we enter unfamiliar territory. The nomenclature may be at times unfortunate, but it is only a matter of nomenclature for the monumental work of Whitehead and Russell, Principia Mathematica, is written in symbols rather than words, and words are scarce.

The first notion that we require a name for, is the extension of a concept. The extension of any concept is called the *class* defined by that concept. The concept may be self-contradictory, singular, or universal. If the concept is self-contradictory its class does not contain any member, and does not exist. Such a class is technically called the *null class*. If the class is defined by a singular concept, it contains necessarily but one member, and is called a *unit class*. A class defined by a universal concept contains more than one member, and if translated out of the ideal order would be called a multitude. But to avoid an ignoratio elenchi we shall as pure mathematicians remain wholly in the ideal order. The inferiors of our universal concepts, as far as we are concerned, are contemplated among the possibles with no other bond of union save the common note that is predicable of each one separately.

There are relations of various kinds between individuals and between classes, but these can be described when wanted. In particular if the fact that X and Y have a certain relation R to the same individual necessarily implies that X and Y are identical, and if further any individual having this relation to U and to V necessarily implies that U and V are identical, then the relation R will be called a *one-one* relation. If we care to speak of the extremes of a relation as *referent* and *relatum*, then a one-one relation is a relation of such a nature that the referent corresponding to any given relatum is unique, and the relatum corresponding to any given referent is likewise unique. The referents of a relation as a class are called the *domain* of the relation. The relata as a class are called the *converse domain* of a relation. If the relation is a one-one relation, domain and converse domain are said to be correlated, or to be in *one to one correspondence*. Hence one to one correspondence is a relation that exists between classes as a result of their respective members being referents and relata respectively of a one-one relation.

One to one correspondence is a common enough device and its abstract possibility is all that is being considered. The one-one relation upon which it depends may find its origin in a mere extrinsic denomination. More might be exacted with a loss of generality so that instead of investigating the number concept in general we might confine ourselves to particular systems in the concrete. But to maintain our generality we must not restrict our concepts before the necessity for the restriction has been proved. Classes that can be brought with respect to each other into one to one correspondence, understood in the liberal sense that we have explained, are said to be *cardinally similar*. Cardinal similarity as a relation between classes differs from a state of one to one correspondence in that the latter relation is a denomination of the classes with respect to some particular one-one relation, whereas the former emphasizes the fact that it is quite unimportant how the classes are to be correlated so long as it is possible in the abstract.

It was thought, naturally enough, that the fundament of the relation of cardinal similiarity must be the cardinal number, and so it was called. But the name assigned is unfortunate, because it turns out to be a generalized cardinal number lacking important arithmetical properties. The alternative name commonly used is Cardinal Power (in Latin Index Cardinalis). We may then formulate a definition. The Cardinal Number or Power (Index Cardinalis) of a class is that note or property which a class has by virtue of which it is cardinally similar to another class. It will be observed that we have been able to define a one-one relation without using numbers. If it be objected that the notion of identity implies the number one, we concede this, and we likewise concede that the diversity of different identities implies all the other numbers. It is this implication that makes definition possible. But to show that we have really avoided a vicious circle we can define an individual X to be identical with Y if X is P necessarily implies Y is P and if Y is Q necessarily implies X is Q. Having defined identity without counting, the one-one relation is defined without counting, one to one correspondence is defined without counting, cardinal similitude is defined without counting, and cardinal power is defined without counting. If therefore we succeed in defining ordinary numbers in terms of cardinal powers, we shall have a reduction of our mathematical concepts to purely logical ones. The procedure seems perhaps unnecessarily involved, but it is necessary to separate concepts at great length to show that we have really avoided a vicious circle.

We made the assertion that the cardinal power is a superior genus of cardinal number as commonly known. This is a thesis to be proved, or rather upon clarification it leads to a number of important theses. Supposing then common numbers are known to the reader from some other source, it is fairly obvious that some classes at least that are cardinally similar have the same numerical designation. It is the converse that we are here interested in, namely, every common number is a cardinal power.

The common cardinal number is a note or property of a class, because it is predicable of a class, thus:

Peter and Paul are two.

It exists of course fundamentally in the individuals, each individual by itself being a unit class and having the number one. It adds no new entity to the class, being implied solely by the identity and diversity of the individuals. It does not exist in the class distributively for we cannot say Peter is two. It is a metaphysical accident that exists in a class as a class, and if we destroy the conceptual individuality of a class by taking it distributively we also ipso facto resolve the number into units. The only thing, then, that is not clear is that a class property such as a common number should necessarily entail a relation between individuals such as is required for one to one correspondence. The answer is that it does not pick out a particular one-one relation but makes a one-one relation possible, at least a one-one relation founded on an extrinsic denomination.

Imagine a line to start simultaneously from each member of one class. Let each member from which a line originates be called an enumerator. There will be just as many lines as enumerators. Each member of the other class to be considered will be called an enumerand. Since there are just as many enumerands as enumerators, there are just as many lines as enumerands. Hence it is absolutely possible to imagine the lines simultaneously terminating each in a different enumerand. Since we remain within the order of possibles, we may formulate a definition, and call enumerand and enumerator standing at the extremities of a possible line *proper* to each other. From this definition there results immediately the one-one relation X is the proper enumerand of the enumerator Y. Since one class is domain, and the other converse domain of this relation, the classes are cardinally similar. In those cases then where classes have common cardinal numbers cardinal similarity necessarily exists, so that the note of cardinal similarity must at least be a constituent note of the common cardinal number.

Body of Cardinal Notions.

Perhaps the best way of exhibiting the organic growth of mathematical concepts out of purely logical ones will be to set down the definitions in order as tersely as possible.

Undefined Notions.

Subjects of predication, X, Y, etc.
Predication, X is P, Y is Q, etc.
Relation X has the relation R to Y.

Defined Notions.

Identity.

A subject X is said to be identical with a subject Y if for all predicates of X, X is P always implies Y is P. (The insertion of this is to show that identity can be conceived without an explicit appeal to number.)

Class.

The subjects, if any, having a given predicate are the class defined by that predicate.

The Null Class.

The class L such that the proposition X belongs to the class L is false for all subjects X is called the null class.

A Unit Class.

If a class U to which X belongs has the property that if Y belongs to U, Y is identical with X, then the class U is called a unit class.

Inclusion.

A class M is said to be part of a class N, if the proposition X belongs to M necessarily implies for all subjects X, that X belongs to N.

Exclusion.

The classes M, N are said to be mutually exclusive if for all subjects X, the proposition X belongs to M implies the falsity of the proposition X belongs to N.

Logical Sum.

The logical sum of the classes M, N is the class defined by the predicate which is the disjunction of their respective predicates. In other words it is the class each of whose members either belongs to class M or to class N.

Logical Product, or Common Part.

The logical product or common part of the classes M, N, is the class each of whose members belongs to class M and to class N.

Referent and Relatum.

If X has the relation R to Y, then X is said to be the referent corresponding to Y, and Y is said to be the relatum corresponding to X.

One-one Relation.

If a relation is such that the referents corresponding to a given relatum Y are identical, and the relata corresponding to a referent X are necessarily identical, the relation is called a one-one relation.

Domain, Converse Domain, and Field.

The class of referents is called the domain of a relation, the class of relata the converse domain of the relation and their logical sum the field of the relation.

One to One Correspondence.

The relation between domain and converse domain of a one-one relation, is called one to one correspondence. The logical process of finding or defining a one-one relation whose domain is the class M and whose converse domain is the class N, is called bringing the classes M and N into one to one correspondence.

Cardinal Similarity.

Classes that can be brought into one to one correspondence are said to be cardinally similar.

Cardinal Number or Power.

The predicate of a class implying cardinal similarity, or the possibility of one to one correspondence is called the cardinal number or power of that class.

Zero.

Zero is defined as the cardinal power of the null class.

One.

One is defined as the cardinal power of a unit class.

Addition.

$n + m$ is the cardinal power of the logical sum of the mutually exclusive classes whose respective cardinal powers are n, m.

Finite Integers.

$2 = 1 + 1$, $3 = 2 + 1$, $4 = 3 + 1$, etc. and the class of finite integers is defined as the class of cardinal powers to which zero belongs, and which has the further property that if n is a member of it $n + 1$, provided it has a meaning, must also be a member of it.

Multiplication and involution can be defined, but ordinal notions are needed, so the definitions are not given. Subtraction, division, and evolution, can be defined, will not always be possible, and even then can only be proved unique by mathematical induction.

2. Cardinal Dissimilitude

Two classes have been defined to be cardinally similar if they can be brought into one to one correspondence. Two classes will be said to be cardinally dissimilar if it is absolutely impossible to bring them into one to one correspondence. If a given class M is dissimilar to another class N, and if class M can be brought into one to one correspondence with a part of class N, then the class M is said to be cardinally less and class N is said to be cardinally greater. We are studying the arrangement of classes according to the relations *cardinally* less, *cardinally* similar, *cardinally* greater. A different set of concepts based on the relation of whole and part is sometimes used, and to avoid confusion we shall say that a part of a class not identical with the whole is always *partially* less than the whole. Thus another set of relations, *partially* less, *totally* equal, *partially* greater, might be considered.

In studying a class we may consider various groups of the members belonging to a class. A group will be defined as a class containing no members not belonging to the given class. Thus, if we have a class with three members A, B, C, there will be eight groups: (1) the null group having no members, (2) a unit group with A alone, (3) a unit group with B alone, (4) a unit group with C alone,

(5) a group consisting of A and B, (6) a group consisting of B and C, (7) a group consisting of C and A, and (8) a group consisting of A, B, and C. The addition of a fourth member to the original class would double the number of groups, because the new member could be added or not added to each of the eight groups. In this way it is easily seen that if we have a class with n members there will be $2n$ groups of these n members. We now wish to prove that apart from the null class, every class has more groups than members. Since there are always as many unit groups as members it is evident that the members are *partially* less and the groups *partially* greater. But we want to prove that the members are *cardinally* less and the groups *cardinally* greater. To do this it will be sufficient to prove that it is absolutely impossible to get a one to one correspondence between groups and members, for then the obvious one to one correspondence between members and unit groups will clinch the inequality.

We shall show that the supposition of a one to one correspondence leads to a contradiction. Let each member that corresponds to some group be called the enumerator of that group. Let each group that has a member to correspond to it be called an enumerated group. A group not corresponding to any member is an unenumerated group.

A member may or may not belong to the group to which it is supposed to correspond. If a member happens to belong to the group it enumerates, it is called an included enumerator, and its group is an including group. If the member does not belong to the group it enumerates, it is called an excluded enumerator, and the group it enumerates is called an excluding group. We shall show that the group consisting of the excluded enumerators is always an unenumerated group.

In the first place excluded enumerators must exist. If all the enumerators were included enumerators, each enumerator would have to enumerate the unit group of which it is the only member for otherwise a unit group would have no enumerator. But then there would be no enumerator not employed in enumerating a unit group, and only unit groups would be enumerated. Hence either one to one correspondence is impossible, or else there must be excluded enumerators.

Now consider the group of excluded enumerators. If the group is to have an included enumerator, its own enumerator must be found among its own members. But each member is an excluded enumerator of the group it enumerates. Hence if the group of excluded enumerators has an included enumerator it must be an excluded enumerator, which is a contradiction. Consider the other horn of the dilemma. Suppose the group of excluded enumerators to have an excluded enumerator. The group itself contains all the excluded enumerators and therefore the required excluded enumerator among the others. Hence this excluded enumerator belongs to the group of excluded enumerators which it enumerates and is therefore at the same time an included enumerator. These contradictions show that the group of excluded enumerators cannot have any enumerator either excluded or included. It is therefore an unenumerated group, so that one to one correspondence is shown to be absolutely impossible. Hence apart from the null class, every class has more groups than members.

The consequences of this demonstration are rather far reaching. In the first place consider the class of finite integers. Is there a greatest finite integer? For any given value of n, there are n integers greater than zero, the greatest of which is n. But there are always more than n groups of n things. Hence for every finite integer n there is always another $2n$ greater than n. We have therefore proved that there is no greatest finite integer.

Since there is no greatest finite integer, it is clear that the cardinal power of the class of finite integers cannot itself be a finite integer. Hence either there are cardinal powers that are not finite integers, or we have a class that cannot take the predicate *cardinally similar*. Can cardinal similitude be predicated of a class such as the class of finite integers? It should be pointed out that the existence of the class is not at stake. That has been proved when we showed that $2n$ is always greater than n. The question is whether in drawing up our concepts of cardinal power our definitions were sufficiently general so as not to limit their applicability to those cases only where the class should have a finite integer for a cardinal power. A review of our body of cardinal notions reveals their patent generality. We can go further. Because our classes are the extensions of concepts, and because there are such things as universal concepts, and because we know how to treat universal concepts without having to examine their inferiors one by one, and because a one-one relation is a distributive and not a collective notion, we can apply our terms to classes whose members we cannot count. We shall show that cardinal similitude can be predicated of such a class by working out an example.

Since there is no greatest finite integer, it follows that there is no greatest even integer, and that there is no greatest odd integer. Hence the class of even integers is a class that does not have a finite integer for a cardinal power. We shall show that the class of even integers is cardinally similar to the class of odd integers. Let A be any even integer. Then A-1 is an odd integer. Let A-1 =B. The relation between A and B is a one-one relation, because for each A there is only one B, and for each B there is only one A. Furthermore the class of even integers is the domain of the relation, for there is no even integer for which the relation does not hold. Similarly the class of odd integers is converse domain of the relation. But where one class is domain and the other converse domain of a one-one relation the classes are cardinally similar. Hence the class of even integers is cardinally similar to the class of odd integers. Hence cardinal similarity is really predicable of classes whose members cannot be counted. Hence there exists a predicate of cardinal similarity that is not a finite integer.

A new concept calls for a new name. We therefore define a cardinal power that is not a finite integer to be a transfinite cardinal. The cardinal power of the class of finite integers is defined to be the cardinal power aleph nihil. The groups of finite integers form a class cardinally greater than the class of finite integers, so its cardinal power $C = 2^{\aleph_0}$ will be a transfinite cardinal greater than $\aleph_0$. Our proposition that every class but the null class has more groups than members thus leads to an endless hierarchy of transfinite cardinals, so that there is no absolute maximum cardinal power finite or transfinite. To be philosophically infinite, a thing must be so great that no greater either exists or is conceivable. But there is always a greater than any transfinite cardinal. Hence all the transfinite cardinals are philosophically finite.

The notion of cardinal power might have been suspected as a generalization of cardinal number from the fact that all the properties of common cardinal numbers could not be deduced from it, thus showing that some notes were missing from the comprehension of the concept. But the discovery of transfinite cardinals shows clearly that cardinal power is indeed a generalization, having not only a smaller comprehension, but also a larger extension than the concept finite integer. Cardinal powers represent finite integers to some extent, and a futile sort of cardinal arithmetic can be constructed, but one must not be mislead into stressing the resemblance between the concepts. When the full significance of the cardinal power dawns upon us, we shall probably prefer to think of it as a quality than as a quantity.

To bring out this last point more vividly, consider the one-one relation $A = 2B$. The domain is the class of finite integers, but the converse domain is the class of even integers. Hence the class of even integers has the same cardinal power as the class of finite integers. In other words a part can be cardinally similar to the whole just as a part can be qualitatively similar to the whole. This fact is usually made the definition of transfinite cardinal, a somewhat arbitrary procedure, it must be admitted.

Finite integers and transfinite cardinals, it will be argued, form a dichotomy of the class of cardinal powers. Does there exist a frontier between the segments of the dichotomy? The answer depends on the way in which the class of cardinal powers is put in order. If it is put in order by means of the relations *partially* less than, *totally* equal, *partially* greater, then there is no frontier. If numbered groups of members are progressively removed from transfinite classes the resulting classes fall between the original classes and the segment of finite integers, but there is no limit to this process, and the lower segment of the dichotomy is never reached. If the order is established by means of the relations *cardinally* less, *cardinally* similar, *cardinally* greater we have a frontier. All classes cardinally similar to the same class then fall in the same place and $\aleph_0$, the least of the transfinites, is the frontier. Increasing or decreasing a transfinite class by mathematically finite groups does increase or decrease the *entity* of the transfinite class. But the transfinite cardinal does not attempt to *measure* this entity. It cannot be measured. The transfinite cardinal does tell us something very definite about this entity, and its relation to the entity of other classes. It should be pointed out that the impossibility of passing from finite integer to transfinite integer is entirely a question of method. As the proposed method is nothing but the definition of the concept finite integer, it cannot possibly reach the transfinite cardinal.

One must not be led into thinking that the transfinite cardinals will permit a revamping of the old idea of infinity. The difference between any transfinite and any finite variable is and always remains greater than any finite integer however great. Hence the definition of a limit cannot be applied. And even if it could, which transfinite would be selected? And should the transfinites be arranged according to cardinal relations, or according to the relations based on the notion of whole and part? It seems therefore that the concept of limit not only does not apply, but the concept cannot be generalized so as to obtain any useful notion that might apply. To close the discussion we assert that variables are ordinal notions, and when you begin to talk about the values of a variable you cease to talk about cardinal powers as such. Cardinal powers are still a long way from the number system of mathematics.

Many ingenious difficulties can be proposed in this connection, only one of which is serious. Some, familiar only with the operations of arithmetic and elementary algebra and not having learned that the formal laws of symbolic operations must be modified as one passes from one field of mathematical entities to another, assume that the failure of certain arithmetical laws to hold for transfinites argues a repugnance in the concept. Others, not caring to form clear cut concepts, dismiss the matter with the epithet *indefinite*. Others confuse the transfinite with the philosophical infinite. The proof of the repugnance of the philosophical infinite is the only difficult part of our work. The difficulties urged against the philosophically finite transfinite under the misapprehension that it is philosophically infinite have not even sufficient vigor to help us in proving the repugnance of the philosophical infinite itself. The transfinite is philosophically finite. The cardinal power embracing both finite integers and transfinite cardinals is a concept that is metaphysically univocal, but in its inferiors is physically analogous.

To review our position, we assert that a multitude that is so vast that it can never be completely assembled by the successive addition of single individuals added one by one, cannot be deemed repugnant on the ground of the multiplicity of individuals involved; and if such a multitude cannot actually exist in the physical order it must be either due to lack of material or to lack of an efficient cause capable of producing it. We assert that there is nothing in the notion of such a multitude that would imply that it is a multitude so large that a larger cannot be conceived. The refutation of this notion has illumined a dark shadow cast by the imagination. We assert that the notion of cardinal number is susceptible of an obvious and necessary generalization. This generalization is then capable of being applied to multitudes such as those under discussion and the notion of a transfinite cardinal is something very tangible. We admit that this generalization at first seems violent and seems to destroy the very notion of number itself, but we assert this to be because we have never thought of the cardinal number as the note of cardinal similarity, but always considered it as if its total essence lay in the fact that it obeys the principle of mathematical induction. Transfinite cardinals, not obeying this principle which constitutes the specific difference between themselves and the more common cardinal numbers rightly, impress us as more like descriptive adjectives than like finite integers.

The only real difficulty is connected with the proof that there is no absolute maximum cardinal power. It is admitted that starting with the class of finite integers and taking groups, and then groups of groups, then groups of groups of groups, etc., we get an endless hierarchy of transfinite cardinals, and that there is no greatest in this series. But if we take the class of all classes, then the groups will also be members of the class itself, and have the same cardinal power as the class. If we look for the group of excluded enumerators, it turns out to be the class of all non-self-contained classes. Now it can be shown that the latter class has contradictory predicates. Hence there is no group of excluded enumerators in this case, and the proof falls down. To answer this difficulty we have to put our finger on the precise point where the class of all non-self-contained classes becomes contradictory. We argue that a class is not one entity in the same sense that its members are each one entity. Hence no class can be a member of itself. Therefore all classes are non-self-contained classes, and the class of non-self-contained classes exists, but the word *class* occurs in two different senses in the expression. When the word *all* is inserted, it is implied that the two uses of the word class are in the same sense, because the word *all* is thereupon taken as justification for using the concept as though the word class were univocal. Hence the contradiction arises when the word *all* is inserted to bear this implication. If now we agree that the word *all* is not to have this implication, the contradictions are straightened out, for the class in question is not a self-contained class, there being no self-contained classes, and it is not a non-self-contained class in the same sense that its members are non-self-contained classes, for it is not a class in the sense that its members are classes. If we apply this to the group of excluded enumerators, we find that it does not deny the existence of a group of excluded enumerators, but prevents the group from being itself an excluded enumerator. With this distinction the proof stands, so that there appears to be no absolute maximum cardinal power.

Errata

Owing to a mistake on the part of the editor our own Jesuit magazine, "America," was unintentionally omitted from the list of those magazines carrying special articles on science and scientific research. This the more regrettable as "America" has always been a pathfinder for the advancement of the inductive method and the experimental approach to truth.

Through the fault of the editor, Mr. Quigley's and Mr. Barry's articles were also printed inaccurately.

The following corrections are to be made in Mr. Quigley's article on minimum deviation of prism:

p. 16. line 25,
Read arcsin (n sin r) *instead of*
arcsin (in sin r)
p. 17, line 1,
Read When $r < A/2$ (i.e. when $r < A - r$),
p. 17, line 2,
Read $(n^2 - 1) / \cos^2 (A - r)$ *instead of*
$(n^2 - 1) \cos^2 (A - r)$

A correction to Mr. Barry's article on Calendar Dates.

On page 21, at the end of Rule 2, instead of 16-28-0 read 6-28-0. Apropos of the article entitled "Notes for Authors," Mr. Barry recently received from McGraw-Hill Book Co., 370 Seventh Avenue, N. Y., a booklet entitled "Suggestions to McGraw-Hill Authors." That too may be had free for the asking. It is practically the same as the Wiley publication mentioned in the article.

Acknowledgments

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